UNIVERSITY OF TORONTO
Faculty of Arts and Science

DECEMBER 2018

ECO 2030, unit 1 (Microeconomic Theory II)
Instructor: Martin J. Osborne

Duration: 2 hour 50 minutes

No aids allowed

This examination paper consists of **22** pages and **7** questions. Please bring any discrepancy to the attention of an invigilator. The number in brackets at the start of each question is the number of points the question is worth.

Answer all questions.

TO OBTAIN CREDIT, YOU MUST GIVE ARGUMENTS TO SUPPORT YOUR ANSWERS.

Question	Points
1	15
2	15

Question	Points
3	15
4	15

Question	Points
5	15
6	15
7	10

Total	100

1. [15] For every value of a other than $a = 1$ and $a = 2$, find all the Nash equilibria in pure and mixed strategies of the following strategic game.

	L	C	R
T	$1, 3$	$0, 4$	$2, 2$
M	$0, 2$	$1, 0$	$4, a$
B	$3, 0$	$0, 2$	$1, a$

Space for answer continues on next page

2. A pie of size 1 is available. Simultaneously, each of n players, 1, ..., n, requests an amount of the pie (a number in $[0, 1]$), and the pie is divided as follows. First, the player whose request is smallest is assigned the amount she requested. Then, any remaining pie is assigned to the player with the next smallest request, up to the value of her request. The assignment process continues in the same manner as long as pie remains. Any ties are broken in favor of the player with the smallest index. (For example, if $n = 4$ and the players' requests are $(\frac{1}{4}, \frac{1}{3}, \frac{1}{4}, \frac{2}{3})$, then player 1 is assigned $\frac{1}{4}$, player 3 is assigned $\frac{1}{4}$, player 2 is assigned $\frac{1}{3}$, and player 4 is assigned $\frac{1}{6}$. If the requests are $(1, \frac{2}{3}, \frac{2}{3}, \frac{3}{4})$, then player 2 is assigned $\frac{2}{3}$, player 3 is assigned $\frac{1}{3}$, and players 1 and 4 are assigned 0.)

   (a) [7] Find the set of actions, if any, that are strictly dominated by another action, and the set of actions that are weakly dominated by another action. (If you claim an action is dominated, you need to show why it is dominated. If you claim an action is not dominated, you need to show why it is not dominated.)

Question continues on next page

(b) [8] Find the pure strategy Nash equilibria of the game that models the situation.

3. Two individuals choose how much effort to expend. Effort is a nonnegative number, and player 1's payoff function is $e_1(1 + e_2 - e_1)$, where e_1 is player 1's effort level and e_2 is player 2's effort level. Player 2's cost of effort is either low, in which case her payoff function is $e_2(1 + e_1 - e_2)$, or high, in which case her payoff function is $e_2(1 + e_1 - 2e_2)$. Player 2 knows her cost of effort, but player 1 does not know player 2's cost of effort. Player 1 believes that player 2's cost of effort is low with probability p and high with probability $1 - p$, where $0 < p < 1$.

   (a) [5] Model this situation as a Bayesian game.

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(b) [10] Find the Nash equilibria of the game as a function of p.

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4. Player 1 has one unit of a good. She chooses how much to give player 2. Player 2 transforms any amount x into kx, where $k > 1$, and then chooses how much of this larger amount to give back to player 1. If player 1 initially gives x to player 2 and player 2 returns y to player 1, then the payoff of player 1 is $1 - x + y$ and the payoff of player 2 is $kx - y$.

   (a) [4] Find the subgame perfect equilibria of the extensive game that models this situation. (Be sure to give the equilibrium *strategies*.)

Question continues on next page

(b) [4] Does the game have any Nash equilibrium in which the outcome differs from the outcome of any subgame perfect equilibrium?

Question continues on next page

Now suppose that player 2's payoff is not $kx - y$, but $y(kx - y)$.

(c) [7] The nature of the subgame perfect equilibria under this assumption depend on the value of k. For each possible value of k, give the equilibrium *strategies* as well as the equilibrium outcome.

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5. Consider a variant of the Bargaining Game of Alternating Offers in which each player cares not only about her monetary payoff but also about the monetary payoff of the other player. Specifically, assume that the payoff of each player i for the division of the pie (x_1, x_2) (where $x_1 + x_2 = 1$) received in period t is $\delta^t(x_i + \lambda x_j)$, where j is the other player, $0 < \delta < 1$, and $0 < \lambda < 1$. (The first period is period 0. Both players have the same discount factor, δ.)

   (a) [12] Find a subgame perfect equilibrium of this game. (Be sure to specify the equilibrium **strategies** fully. You may take as given the fact that a strategy pair is a subgame perfect equilibrium if and only if it satisfies the one-deviation property. You need to show that the strategy pair you find is a subgame perfect equilibrium.)

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(b) [3] Does the equilibrium allocation become more or less unequal at λ increases? Why does the model produce this result?

6. (a) [5] Consider the infinitely repeated *Prisoner's Dilemma* in which each player's discount factor is δ, with $0 < \delta < 1$, and the stage game payoffs are given in the following figure.

	C	D
C	$2, 2$	$0, 3$
D	$3, 0$	$1, 1$

Consider the following strategy s. Choose C in period 1 and after any history in which the other player chose C in every period except, possibly, the previous period; choose D after any other history. (The initiation of punishment is delayed by one period.)

Determine the values of δ, if any, for which the strategy pair (s, s) is a Nash equilibrium of the infinitely repeated game.

Question continues on next page

(b) [10] Consider the infinitely repeated game with discounting of the following strategic game.

	A	B	C
A	$4,4$	$3,0$	$1,0$
B	$0,3$	$2,2$	$1,0$
C	$0,1$	$0,1$	$0,0$

When the discount factor is close to 1, does this game have a subgame perfect equilibrium in which the outcome is (A, A) in every odd period and (B, B) in every even period? Either show that it has no such subgame perfect equilibrium or specify a strategy pair that is a subgame perfect equilibrium and find conditions on the discount factor for which the strategy pair is a subgame perfect equilibrium.

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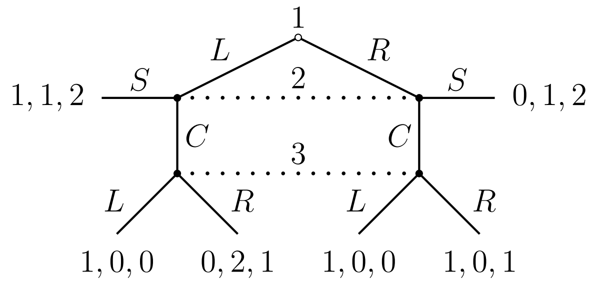
7. Consider the following extensive game.



(a) [5] Find a pure strategy weak sequential equilibrium of this game. (Remember to specify both the strategy profile and the beliefs.)

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(b) [5] Find a pure strategy Nash equilibrium of the game that is not the strategy profile for any weak sequential equilibrium. (Explain why the strategy profile is not part of a weak sequential equilibrium.)

For rough work (will not be graded)

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End of examination
Total pages: 22
Total marks: 100