Economics 2030

Fall 2018 Martin J. Osborne

Solutions for Tutorial 6

1. (a) The outcome path is not a subgame perfect equilibrium. Consider the subgame following the history in which the outcome was (X, X) in periods 1 through $T-1$. If player 1 deviates to Y in period T and subsequently chooses Y regardless of the history, then her payoff exceeds her payoff along the path.

   (b) Consider a history in which the outcome in every previous odd period was (X, X) and the outcome in every previous even period was (Y, Y).

   - After such a history, if the period is even, a player who deviates in that period gets a payoff of 0 rather than 1 in the period and gets 0 subsequently, instead of at least 1 subsequently, and thus is worse off.
   - After such a history, if the period is odd, a player who deviates in that period gets a payoff of 4 rather than 3 in the period and gets 1 in every future period instead of alternating between 1 and 3. Thus the deviation is not profitable if and only if
   $$4 + \frac{\delta}{1-\delta} \leq \frac{3}{1-\delta^2} + \frac{\delta}{1-\delta^2}$$
   which reduces to $\delta \geq \frac{1}{3}\sqrt{3}$.

   After a history in which the outcome in the previous periods did not alternate between (X, X) and (Y, Y), a player who deviates in that period is worse off because she gets 0 rather than 1 in the period and at most 1 subsequently, rather than 1 in every subsequent period.

   We conclude that the strategy pair is a subgame perfect equilibrium if and only if $\delta \geq \frac{1}{3}\sqrt{3}$.

2. This (zerosum) extensive game is shown in Figure 1. The strategic form of this game is given in Figure 2. First note that the strategies

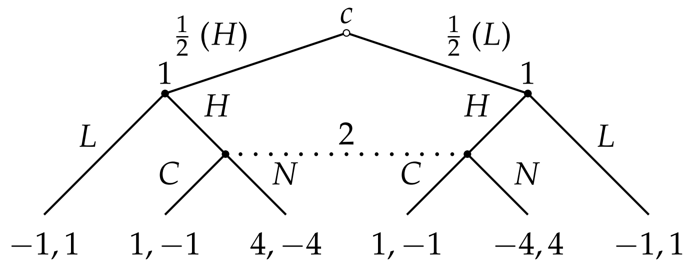

Figure 1. The extensive game for Problem 2.

LH and *LL* are both strictly dominated by *HH*. (I.e. if player 1 gets the high card she is better off not conceding.) Now, there is a unique Nash equilibrium, in which the mixed strategy of player 1 assigns probability $\frac{2}{5}$ to *HL* and probability $\frac{3}{5}$ to *HH* and player 2 concedes with probability $\frac{3}{5}$. (In behavioral strategies this equilibrium is: player 1 chooses *H* when her card is *H* and chooses *H* with probability $\frac{3}{5}$ and *L* with probability $\frac{2}{5}$ when her card is *L*; player 2 concedes with probability $\frac{3}{5}$.)

	C	N
LH	$0, \quad 0$	$-\frac{5}{2}, \quad \frac{5}{2}$
LL	$-1, \quad 1$	$-1, \quad 1$
HL	$0, \quad 0$	$\frac{3}{2}, -\frac{3}{2}$
HH	$1, -1$	$0, \quad 0$

Figure 2. The strategic form of the extensive game in Figure 1.

3. (a) In any equilibrium, player 1 chooses R because it strictly dominates L. Thus player 2's belief assigns probability 1 to the history R.

   Now, if player 3 chooses L, player 2 optimally chooses C, whereas if player 3 chooses R, player 2 optimally chooses S. Denote the probability player 3's belief assigns to (L, C) by p. If $p \geq \frac{1}{2}$, then R is optimal for her. If $p \leq \frac{1}{2}$, then L is optimal.

   Thus the game has two weak sequential equilibria:

   - Player 1 chooses R, player 2 chooses C, $p = 0$, and player 3 chooses L
   - Player 1 chooses R, player 2 chooses S, $\frac{1}{2} \leq p \leq 1$, and player 3 chooses R.

(b) In the first equilibrium, every information set is reached with positive probability, so the equilibrium is a sequential equilibrium.

In the second equilibrium, player 3's information set is not reached. Let player 1's strategy assign ε to L and let player 2's strategy assign probability δ to C. Then as ε and δ approach 1, player 3's belief derived from these strategies assigns probability approaching 1 to the history (R, C), making L the only optimal action of player 3. Thus this equilibrium is not a sequential equilibrium.