Economics 2030

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Problem Set 11

1. Consider the infinitely repeated game in which the players' preferences are represented by the discounting criterion, the common discount factor is $\frac{1}{2}$, and the constituent game is the game in Figure 1. Show that $((A, A), (A, A), \ldots)$ is not a subgame perfect equilibrium outcome path.

	A	D
A	$2, 3$	$1, 5$
D	$0, 1$	$0, 1$

Figure 1. The game in Problem 1.

2. Consider the discounted infinitely repeated game of the *Prisoner's Dilemma* in Figure 2.

	C	D
C	x, x	$0, y$
D	$y, 0$	$1, 1$

Figure 2. The general *Prisoner's Dilemma* in Problem 2. The parameters x and y satisfy $1 < x < y$.

   (a) Find the condition on the common discount factor δ under which the strategy pair in which each player uses the grim strategy is a Nash equilibrium of this game.

   (b) Find conditions on the values of x, y, and δ for which the strategy pair in which each player uses the strategy *tit-for-tat* is a subgame perfect equilibrium. (The strategy *tit-for-tat* chooses C in the first period and then, in every subsequent period, the action chosen by the other player in the previous period.)

3. Consider the δ-discounted infinitely repeated game of Bertrand's duopoly game in the case that each firm's unit cost is constant, equal to c. *Assume that prices are discrete*. Specifically, assume that a price must be an integral multiple of the monetary unit k. Denote the total demand for the good at the price p by $D(p)$, let $\pi(p) = (p - c)D(p)$ for every price p, and assume that D is such that the function π has a single maximizer, denoted p^m (the "monopoly price"), $p^m \geq 2k$, and π is increasing up to this maximizer. As in the standard model, in any period if the firms choose different prices, the one choosing the lower price obtains all the demand at that price and if the firms choose the same price then they split the demand equally.

   Let s_i be the following strategy for firm i in the infinitely repeated game:

   - in the first period charge the price p^m
   - in every subsequent period charge the minimum of p^m and the lowest of all the prices charged by either firm in all previous periods.

   Are there any values of $\delta \in [0, 1)$ such that the strategy pair (s_1, s_2) is a subgame perfect equilibrium of the infinitely repeated game? (You can take as given that a strategy pair is a subgame perfect equilibrium if and only if it satisfies the one-deviation property.)

4. Consider a dynamic game with two states, with the strategic game in the two states described by the following matrices.

   State A:

	C	D
C	$3,3$	$0,4$
D	$4,0$	$1,1$

   State B:

	C	D
C	a,a	$0,0$
D	$0,0$	$1,1$

   The initial state is State A. From State A, the state remains State A if the action profile is (C, C), and otherwise transitions to State B. From State B, the state always transitions to State A regardless of the action profile.

   (a) For each discount factor $\delta \in (0, 1)$ and each $a \in \mathbb{R}$, find the set of Markov perfect equilibria of this game.

(b) For $\delta = \frac{3}{4}$ and $a = 1$, prove or disprove whether the set of Markov perfect equilibria is identical to the set of subgame perfect equilibria of this game.