Economics 2030

Fall 2018 | Martin J. Osborne

Solutions for Tutorial 5

1. (a) See Figure 1.

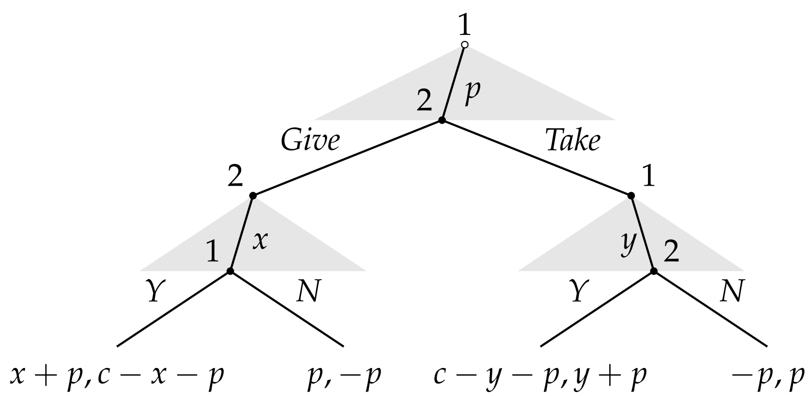

Figure 1. The game in Problem 1.

(b) The subgame following any history in which player 2 chooses *Take* is an ultimatum game in which player 1 is the proposer. Thus in the unique subgame perfect equilibrium of this subgame, player 1 offers the division $(c, 0)$ and player 2 accepts all proposals. Similarly, the subgame following any history in which player 2 chooses *Give* is an ultimatum game in which player 2 is the proposer, so that in the unique subgame perfect equilibrium of this subgame player 2 proposes $(0, c)$ and player 1 accepts all proposals.

Now consider a history after which player 1 names some amount p. The payoff of player 2 is p if she takes p from player 1 and $c - p$ if she gives p to player 1. Thus in any subgame perfect equilibrium player 2 takes p if $p > \frac{1}{2}c$ and gives p to player 1 if $p < \frac{1}{2}c$. If $p = \frac{1}{2}c$, she is indifferent between giving p and taking p.

Finally consider player 1's choice at the start of the game. Her payoff is $c - p$ for any $p > \frac{1}{2}c$, p for any $p < \frac{1}{2}c$, and $\frac{1}{2}c$ for $p = \frac{1}{2}c$. Thus in any subgame perfect equilibrium player 1 names

$\frac{1}{2}c$ at the start of the game, player 2 chooses *Take* if $p > \frac{1}{2}c$ and *Give* if $p < \frac{1}{2}c$, player 1 proposes $(c, 0)$ if she is the proposer and accepts any proposal when player 2 is the proposer, and player 2 proposes $(0, c)$ when she is the proposer and accepts any offer proposed by player 1.

2. SYM is satisfied by definition.

   PAR is not satisfied: Suppose $d = (0, 0)$ and U is the convex hull of $(0, 0)$, $(0, \frac{1}{2})$, $(2, \frac{3}{2})$, and $(2, 0)$. Then $S(U, d) = (1, 1)$, which is not Pareto efficient.

   INV is not satisfied: Suppose that $d = (0, 0)$ and U is the triangle with corners at $(0, 0)$, $(0, 1)$ and $(1, 0)$. The solution assigns to this problem the point $(\frac{1}{2}, \frac{1}{2})$. Now suppose that $d' = (0, 0)$ and U' is the triangle with corners at $(0, 0)$, $(0, 1)$ and $(2, 0)$. The solution assigns to this problem the point $(\frac{2}{3}, \frac{2}{3})$. But $U' = \{(2u_1, u_2) : (u_1, u_2) \in U\}$ and $d' = (2d_1, d_2)$, so INV requires, given the bargaining solution of (U, d), that the bargaining solution of (U', d') be $(1, \frac{1}{2})$.

   IIA is satisfied: If $U' \subseteq U$, $d' = d$, and $S(U, d) \in U'$ then $S'(U', d') = s(U, d)$.

3. (a) A player who adheres to the strategy obtains the discounted average payoff of 2. The best deviation yields the stream of payoffs $(3, 3, 1, 1, \ldots)$, with a discounted average of $3(1 - \delta)(1 + \delta) + \delta^2$. Thus for an equilibrium we require $3(1 - \delta)(1 + \delta) + \delta^2 \leq 2$, or $\delta \geq \frac{1}{2}\sqrt{2}$.

   (b) A player who adheres to the strategy obtains the payoff of 2 in every period. A player who chooses D in the first period and C in every subsequent period obtains the stream of payoffs $(3, 2, 2, \ldots)$. Thus for any value of δ a player can increase her payoff by deviating, so that the strategy pair is not a Nash equilibrium. Further, whatever the one-shot payoffs, a player can increase her payoff by deviating to D in a single period, so that for no payoffs is there any δ such that the strategy pair is a Nash equilibrium of the infinitely repeated game with discount factor δ.

   (c) A player who adheres to the strategy obtains the discounted average payoff of 2 (the outcome is (C, C) in every period). If player 2 deviates to D in every period then she induces the outcome to alternate between (C, D) and (D, D), yielding her a discounted average payoff of $(1 - \delta) \cdot (3 + 3\delta^2 + 3\delta^4 + \ldots) + (1 - \delta)(\delta + \delta^3 +$

$\delta^5 + \ldots) = (1-\delta)[3/(1-\delta^2) + \delta/(1-\delta^2)] = (3+\delta)/(1+\delta)$. For all $\delta < 1$ this payoff exceeds 2, so that the strategy pair is not a Nash equilibrium of the infinitely repeated game.

However, for different payoffs for the one-shot *Prisoner's Dilemma*, the strategy pair *is* a Nash equilibrium of the infinitely repeated game. The point is that the best deviation for player 2 leads to the sequence of outcomes that alternates between (C, D) and (D, D). If the average payoff of player 2 in these two outcomes is less than her payoff to the outcome (C, C) then the strategy pair is a Nash equilibrium for some values of δ. (For the payoffs in the question, the average payoff of the two outcomes (C, D) and (D, D) is exactly equal to the payoff to (C, C).) Consider the general payoffs in Figure 2. The discounted average payoff of the sequence of outcomes that alternates between (C, D) and (D, D) is $(y+\delta)/(1+\delta)$, while the discounted average of the constant sequence containing only (C, C) is x. Thus for the strategy pair to be a Nash equilibrium we need

	C	D
C	x, x	$0, y$
D	$y, 0$	$1, 1$

Figure 2. A *Prisoner's Dilemma*.

$$\frac{y+\delta}{1+\delta} \leq x,$$

or

$$\delta \geq \frac{y-x}{x-1},$$

an inequality that is compatible with $\delta < 1$ if $x > \frac{1}{2}(y+1)$—that is, if x exceeds the average of 1 and y.