

Problem Set 10

1. Consider the infinitely repeated game of the strategic game in Figure 1, where the parameters x and y satisfy $1 < x < y$ and each player has the same discount factor δ (with $0 < \delta < 1$). Suppose that each player uses the k -period limited punishment strategy described in class. Find a condition on k for which this strategy pair is a Nash equilibrium of the infinitely repeated game.

	C	D
C	x, x	$0, y$
D	$y, 0$	$1, 1$

Figure 1. The strategic game for Problem 1.

2. Consider an infinitely repeated *Prisoner's Dilemma* in which each player's discount factor is δ , with $0 < \delta < 1$, and the payoffs are given as follows (where $1 < a < 3$).

	C	D
C	a, a	$0, 3$
D	$3, 0$	$1, 1$

Consider the strategy *tit-for-tat*, defined as follows: Play C in the first period; in every subsequent period, choose the action chosen by the other player in the previous period.

Find the range of values of δ , if any, for which the strategy pair in which both players choose this strategy is a Nash equilibrium of the infinitely repeated game.

3. Consider Bertrand's duopoly game in the case that each firm's unit cost is constant, equal to c . Denote the total demand for the good at the price p by $D(p)$, let $\pi(p) = (p - c)D(p)$ for every price p , and assume that D is such that the function π is continuous and has a single maximizer, denoted p^m (the "monopoly price").

- (a) Let s_i be the strategy of firm i in the infinitely repeated game of this strategic game that charges p^m in the first period and subsequently as long as the other firm continues to charge p^m and punishes any deviation from p^m by the other firm by choosing the price c for k periods, then reverting to p^m . Given any value of δ , for what values of k is the strategy pair (s_1, s_2) a Nash equilibrium of the infinitely repeated game?
- (b) Let s_i be the following strategy for firm i in the infinitely repeated game:
- in the first period charge the price p^m
 - in every subsequent period charge the lowest of all the prices charged by the other firm in all previous periods.

(That is, firm i matches the other firm's lowest price.) Is the strategy pair (s_1, s_2) a Nash equilibrium of the infinitely repeated game for any discount factor less than 1?