Economics 2030

Fall 2018 Martin J. Osborne

Questions for Tutorial 5

1. Two players use the following variant of the ultimatum game to split \$$c$. Player 1 starts by naming a nonnegative number \$$p$. Then player 2 may either take \$$p$ from player 1 or give \$$p$ to player 1. If player 2 takes \$$p$ from player 1, the two players proceed to divide the \$$c$ by playing an ultimatum game in which player 1 is the proposer. If instead player 2 gives \$$p$ to player 1, the two players proceed to divide the \$$c$ by playing an ultimatum game in which player 2 is the proposer. Both players care about the total amount of money they have at the end of the game. (Suppose, for example, that player 1 names \$$p$, player 2 takes this amount, and player 1 then offers y to player 2. If player 2 accepts y then player 1's payoff is $c - y - p$ and player 2's payoff is $y + p$, and if player 2 rejects y then player 1's payoff is $-p$ and player 2's payoff is p.)

   (a) Model this situation as an extensive game with perfect information. (A diagram is sufficient.)

   (b) Find all subgame perfect equilibria of the game.

2. For a bargaining problem (U, d), define the bargaining solution $S(U, d)$ to be the member u of U with $u_1 - d_1 = u_2 - d_2$ for which u_1 (and u_2) is as large as possible. For each of Nash's axioms (PAR, SYM, INV, and IIA), either show that S satisfies the axiom or show that it does not satisfy the axiom.

3. Consider the infinitely repeated *Prisoner's Dilemma* in which each player's discount factor is δ, with $0 < \delta < 1$, and the stage game payoffs are given in the following figure.

	C	D
C	$2,2$	$0,3$
D	$3,0$	$1,1$

   Consider the following strategies.

(a) (Delayed punishment) Choose C in period 1 and after any history in which the other player chose C in every period except, possibly, the previous period; choose D after any other history. (Punishment is grim, but its initiation is delayed by one period.)

(b) (Relaxed punishment) Choose C in period 1 and after any history in which the other player chose D in at most one period; choose D after any other history. (Punishment is grim, but a single lapse is forgiven.)

(c) (*Pavlov*, or *win–stay, lose–shift*) Choose C in period 1 and after any history in which the outcome in the last period is either (C, C) or (D, D); choose D after any other history. (That is, choose the same action again if the outcome was relatively good for you, and switch actions if it was not.)

For each of these strategies s determine the values of δ, if any, for which the strategy pair (s, s) is a Nash equilibrium of the infinitely repeated game. For each strategy s for which there is no value of δ such that (s, s) is a Nash equilibrium of this game, determine whether there are any other payoffs for the *Prisoner's Dilemma* such that for some δ the strategy pair (s, s) is a Nash equilibrium of the infinitely repeated game with discount factor δ.