Economics 2030

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Problem Set 9

1. Show that Nash's bargaining solution satisfies the INV axiom.

2. (a) For any bargaining problem (U, d), let $f(U, d)$ be the point in U on the 45° line through d for which v_1 (and v_2) is as large as possible. Show that the bargaining solution f does not satisfy INV.

   (b) For any bargaining problem (U, d), denote by b^1 the member of U for which $b_2^1 \geq d_2$ and b_1^1 is as large as possible, and by b^2 the member of U for which $b_1^2 \geq d_1$ and b_2^2 is as large as possible. Let $f(U, d)$ be the member of U on the line through d and (b_1^1, b_2^2) for which v_1 (and v_2) is as large as possible. Show that the bargaining solution f does not satisfy IIA. (It satisfies the other three axioms.)

3. Consider the bargaining solution that assigns to the bargaining problem (U, d) the Pareto efficient agreement in U on the line through d and (x_1^*, x_2^*), where x_i^* is the *maximal* payoff of player *i over all agreements in* U.

   For each of Nash's axioms (INV, SYM, PAR, IIA), either show that the solution satisfies the axiom or show that it does not satisfy the axiom.

4. A firm and a union representing L workers negotiate a wage–employment contract. Each worker can obtain the wage w_0 if she does not work for the firm. (Perhaps w_0 is the wage in another firm, or the unemployment benefit.) The firm produces $f(\ell)$ units of output when it employs ℓ workers, where f is an increasing strictly concave function with $f(0) = 0$ and $f(\ell) > \ell w_0$ for some ℓ. The contract (w, ℓ), in which the firm pays the wage w and employs ℓ workers, yields payoffs of $f(\ell) - w\ell$ to the firm and $\ell w + (L - \ell)w_0$ to the union. In the event of disagreement, the firm's payoff is 0 (given $f(0) = 0$) and the union's payoff is Lw_0.

   A pair of payoffs is feasible if it takes the form $(f(\ell) - w\ell, \ell w + (L - \ell)w_0)$ for some (w, ℓ). Notice that the sum of these payoffs is

$f(\ell) + (L - \ell)w_0$, independent of w. Thus, given the Pareto Efficiency of the Nash solution, the value of ℓ in the Nash solution maximizes $f(\ell) + (L - \ell)w_0$. Denote this maximizer ℓ^*. Find the wage rate w in the Nash solution.