**Economics 2030**

Fall 2018 | Martin J. Osborne

**Problem Set 8**

1. A child's action $a$ (a number) affects both her own private income $c(a)$ and her parent's income $p(a)$; for all values of $a$ we have $c(a) < p(a)$. The child is selfish: she cares only about the amount of money she has. Her loving parent cares both about how much money she has and how much her child has. Specifically, her preferences are represented by a payoff equal to the smaller of the amount of money she has and the amount of money her child has. The parent may transfer money to the child. First the child takes an action, then the parent decides how much money to transfer. Model this situation as an extensive game and show that in a subgame perfect equilibrium the child takes an action that maximizes the sum of her private income and her parent's income. (In particular, the child's action does not maximize her own private income. This result, known as the "rotten kid theorem", is not limited to the specific form of the parent's preferences, but holds for any preferences with the property that a parent who is allocating a fixed amount $x$ of money between herself and her child wishes to give more to the child when $x$ is larger.)

2. Consider the following variant of the bargaining game of alternating offers, when the size of the pie is $100. Neither player discounts future payoffs (i.e. both discount factors are equal to 1), but in any period that an offer is rejected, each player has to pay a penalty of $1 to a third party. (As before, a proposal takes the form $(x, 100 - x)$, where $0 \leq x \leq 100$.) You may assume that a strategy profile is a subgame perfect equilibrium of this game if and only if it satisfies the one deviation property.

   (a) Is the following strategy pair a subgame perfect equilibrium? Player 1 always proposes $(50, 50)$ and accepts an offer $(x, 100 - x)$ if and only if $x \geq 49$, and Player 2 always proposes $(49, 51)$ and accepts an offer $(x, 100 - x)$ if and only if $100 - x \geq 50$.

(b) Is the following strategy pair a subgame perfect equilibrium? Player 1 always proposes $(100, 0)$ and accepts an offer $(x, 100 - x)$ if and only if $x \geq 99$, and Player 2 always proposes $(99, 1)$ and accepts all offers.

(c) Does the game have a subgame perfect equilibrium in which player 2 obtains all the pie in period 1?

3. Consider a variant of the bargaining game of alternating offers in which after an offer in any period $t$ is rejected by player 2, a third player has the option of approaching player 2.

   - If player 3 does not approach player 2 then player 2 makes a counteroffer to player 1's proposal in period $t + 1$, and play continues as in the bargaining game of alternating offers until player 2 rejects an offer, in which case player 3 again has the option of approaching player 2.
   - If player 3 approaches player 2 then player 2 must choose either to stay with player 1 or to switch to player 3.
     - If player 2 stays with player 1, play proceeds as in the bargaining game of alternating offers (with a pie of size 1), without any further possible intervention by player 3 (player 2 makes a counteroffer in period $t + 1$, player 1 either accepts or rejects, ...).
     - If player 2 switches to player 3, players 2 and 3 engage in a bargaining game of alternating offers in which player 3 makes the opening offer (in period $t + 1$), player 1 plays no role, and the size of the pie is $k$, where $k \leq 2$.

   The discount factor of each player is $\delta$, where $0 < \delta < 1$. If players 1 and 2 reach agreement, player 3's payoff is 0; if players 2 and 3 reach agreement, player 1's payoff is 0.

   Find, as a function of $k$, a subgame perfect equilibrium of this game. You need only to describe the equilibrium and sketch the reason it is an equilibrium; you do not need to provide all the details.

4. Consider a variant of the bargaining game of alternating offers in which the size of the pie depends on how much effort player 1 expends. Assume that player 1 first chooses how much effort to expend, and then the two players alternate offers exactly as in the bargaining game of alternating offers. If player 1 expends the effort $e$ then the

size of the pie is $e$. If she expends the effort $e$ and in the agreement ultimately reached she obtains the amount $\alpha$ of the pie in period $t$ then her payoff is $\delta^{t-1}\alpha - e^2$. Player 2's payoff if she obtains the amount of the pie $\beta$ in period $t$ is $\delta^{t-1}\beta$. Find the subgame perfect equilibrium (equilibria?) of this game. Compare the sum of the players' equilibrium payoffs with the maximal possible sum of the players' payoffs.

5. Consider the variant of a bargaining game of alternating offers in which there are *three* players. First player 1 makes a proposal, then players 2 and 3 respond, one after the other in the same period. If they both accept player 1's proposal, the game ends. Otherwise, player 2 make a proposal, to which players 3 and 1 respond one after the other in the same period. If both player 3 and player 1 accept player 2's proposal, the game ends, while if at least one of them rejects the proposal, player 3 makes a proposal, to which players 1 and 2 respond. Upon rejection, player 1 again makes a proposal. The game continues in this way until an offer is accepted by both responders.

   Assume that each player has preferences with discounting, with the same discount factor $\delta$. Find a stationary subgame perfect equilibrium (in which each player always make the same proposal, and always accepts the same set of proposals). (Use the one deviation property, which holds for this game, to show that your strategy profile is a subgame perfect equilibrium.) (The game has, in addition, many *non*stationary subgame perfect equilibria.)