ECO2030: Microeconomic Theory II, module 1

Lecture 8

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2018.11.22

Bargaining

Finite horizon

Infinite horizon
- SPE
- Characterization of SPE
- Properties of SPE
- Many players

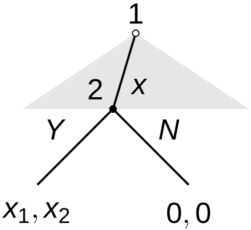
Risk of breakdown

Outside options

Outside options vs. breakdown

Bargaining

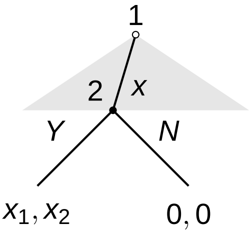
Ultimatum game with pie of size 1



$$x = (x_1, x_2)$$
$$x_1 + x_2 = 1$$

Bargaining

Ultimatum game with pie of size 1



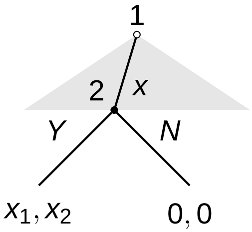
Unique SPE

Offer $(1, 0)$

Accept all offers

Bargaining

Ultimatum game with pie of size 1



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- SPE payoffs: $(1, 0)$

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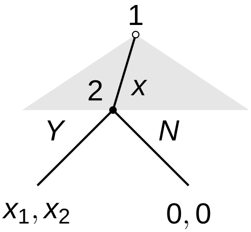

Unique SPE

Offer $(1, 0)$

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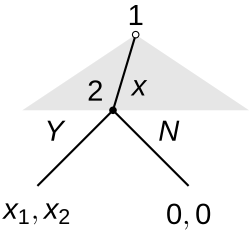
- SPE payoffs: $(1, 0)$
- Why is SPE outcome so one-sided?

Bargaining

Ultimatum game with pie of size 1



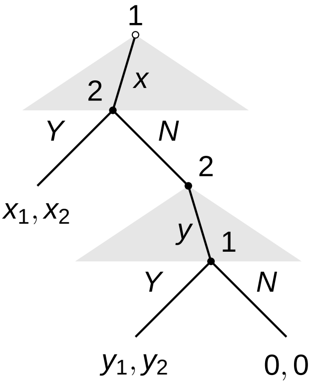
Unique SPE

Offer $(1, 0)$

Accept all offers

- SPE payoffs: $(1, 0)$
- Why is SPE outcome so one-sided?
- Should give player 2 the opportunity to counteroffer?

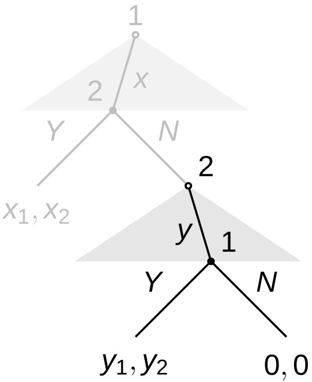
Bargaining: Two-period game



$x = (x_1, x_2)$
$x_1 + x_2 = 1$

$y = (y_1, y_2)$
$y_1 + y_2 = 1$

Bargaining: Two-period game

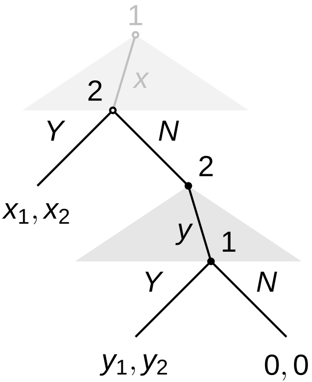


SPE

Offer $(0, 1)$

Accept all offers

Bargaining: Two-period game



SPE

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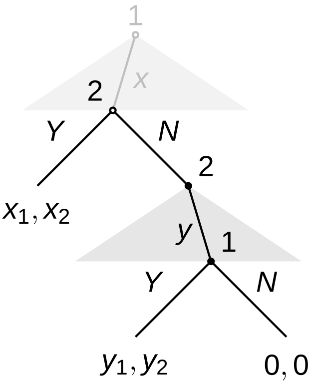
Bargaining: Two-period game



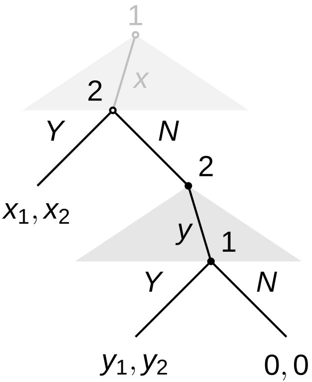
SPE

Reject all offers

Offer $(0, 1)$

Accept all offers

Bargaining: Two-period game



SPE

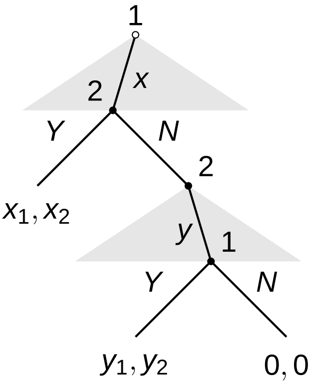
Reject all offers

Reject all offers except $(0, 1)$

Offer $(0, 1)$

Accept all offers

Bargaining: Two-period game



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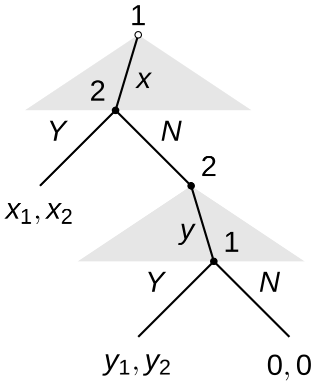


SPE

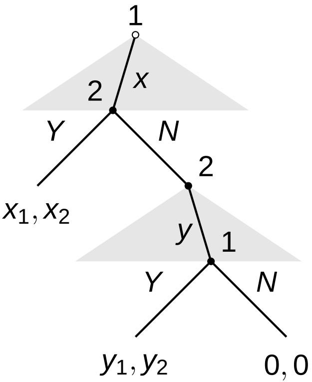
Any offer

Reject all offers | Reject all offers except $(0, 1)$

Offer $(0, 1)$

Accept all offers

Bargaining: Two-period game



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Accept all offers

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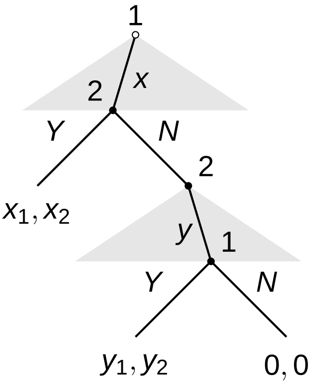

SPE

Any offer	Any offer
Reject all offers	Reject all offers except $(0, 1)$
Offer $(0, 1)$	
Accept all offers	

$\Rightarrow$ in every subgame perfect equilibrium payoffs are $(0, 1)$

Bargaining: Two-period game with cost of delay

- Extreme outcome is result of delay being costless?

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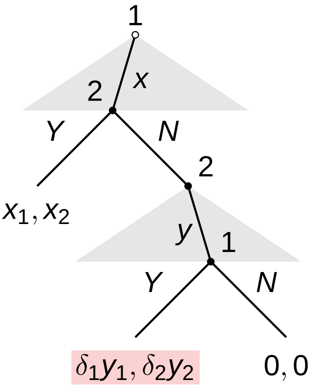

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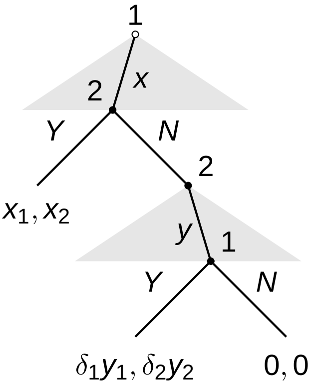

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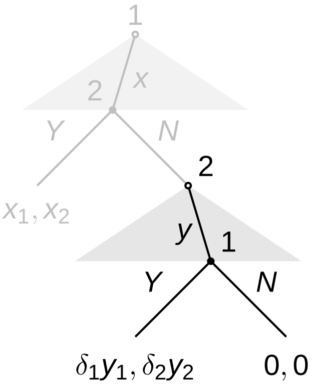

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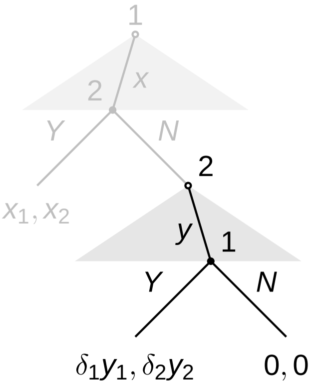

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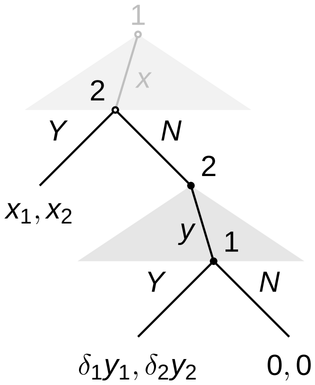

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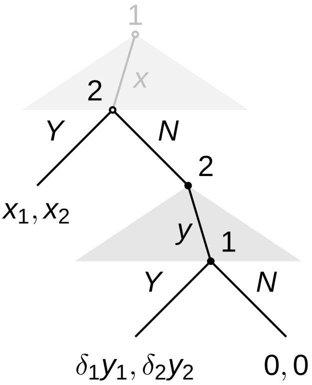


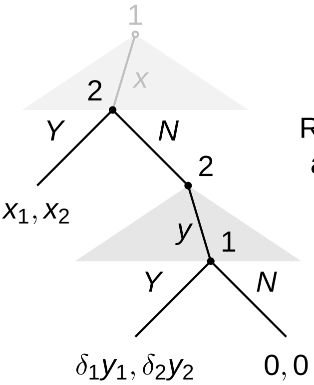
SPE

P2 rejects offer $x \Rightarrow$ gets 1 in period 2 $\Rightarrow$ payoff δ_2

Offer $(0, 1)$

Accept all offers

Bargaining: Two-period game with cost of delay



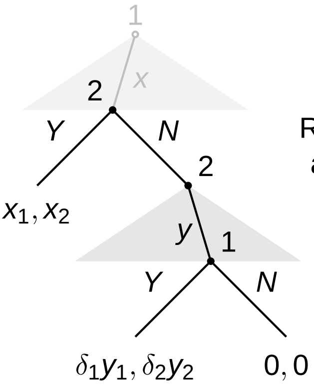
SPE

Reject x if $x_2 < \delta_2$
accept if $x_2 \geq \delta_2$

Offer $(0, 1)$

Accept all offers

Bargaining: Two-period game with cost of delay



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Reject x if $x_2 \leq \delta_2$
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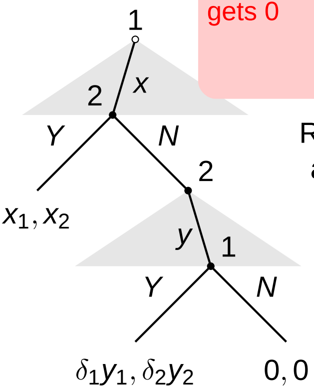
Offer $(0, 1)$

Accept all offers

Bargaining: Two-p... delay

P1 offers $x_2 < \delta_2 \Rightarrow$ 2 rejects $\Rightarrow$ 1 gets 0



Reject x if $x_2 < \delta_2$ accept if $x_2 \geq \delta_2$

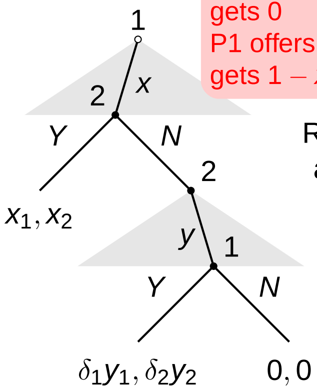
Reject x if $x_2 \leq \delta_2$ accept if $x_2 > \delta_2$

Offer $(0, 1)$

Accept all offers

Bargaining: Two-p … delay

P1 offers $x_2 < \delta_2 \Rightarrow$ 2 rejects $\Rightarrow$ 1 gets 0
P1 offers $x_2 \geq \delta_2 \Rightarrow$ 2 accepts $\Rightarrow$ 1 gets $1 - x_2$



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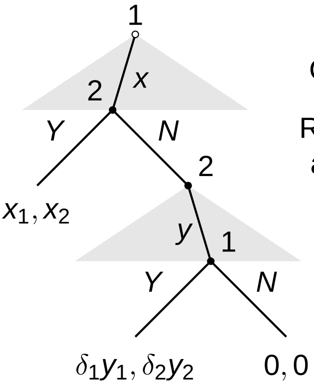
Offer $(0, 1)$

Accept all offers

Bargaining: Two-period game with cost of delay



SPE

Offer $(1 - \delta_2, \delta_2)$

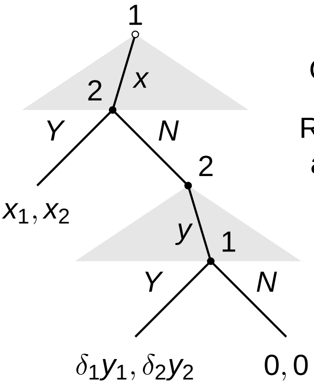
Reject x if $x_2 < \delta_2$
accept if $x_2 \geq \delta_2$

Reject x if $x_2 \leq \delta_2$
accept if $x_2 > \delta_2$

Offer $(0, 1)$

Accept all offers

Bargaining: Two-period game with cost of delay



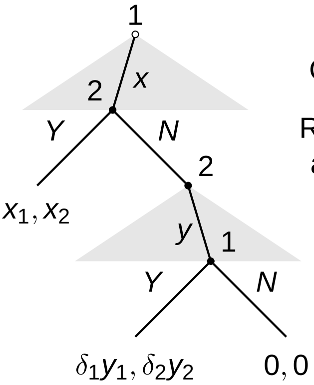
SPE

Offer $(1 - \delta_2, \delta_2)$	No optimal offer
Reject x if $x_2 < \delta_2$ accept if $x_2 \geq \delta_2$	Reject x if $x_2 \leq \delta_2$ accept if $x_2 > \delta_2$

Offer $(0, 1)$

Accept all offers

Bargaining: Two-period game with cost of delay



SPE

Offer $(1-\delta_2, \delta_2)$	No optimal offer
Reject x if $x_2 < \delta_2$ accept if $x_2 \geq \delta_2$	Reject x if $x_2 \leq \delta_2$ accept if $x_2 > \delta_2$

Offer $(0, 1)$

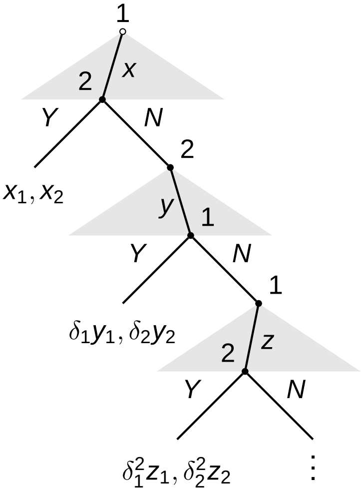
Accept all offers

Unique SPE

Period 1 P1 proposes $(1-\delta_2, \delta_2)$, P2 accepts $(x_1, x_2) \Leftrightarrow x_2 \geq \delta_2$

Period 2 P2 always proposes $(0, 1)$, P1 always accepts all proposals

Bargaining: Infinite horizon



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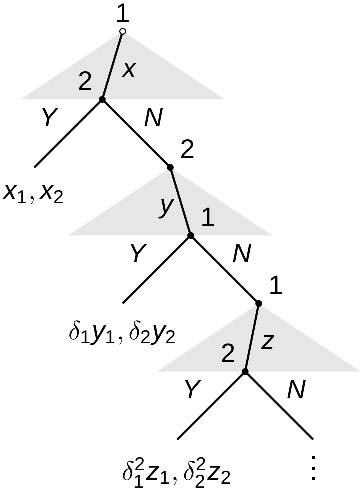
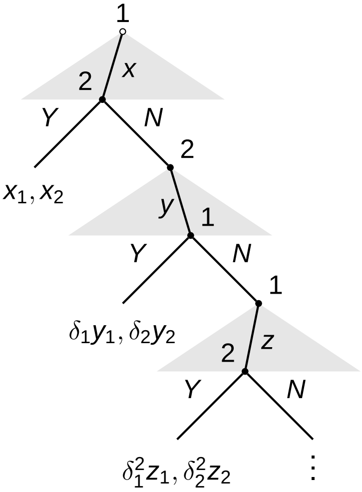


Notes

- Every subgame starting with proposal by P1 is *identical*

Bargaining: Infinite horizon



Notes

- Every subgame starting with proposal by P1 is identical
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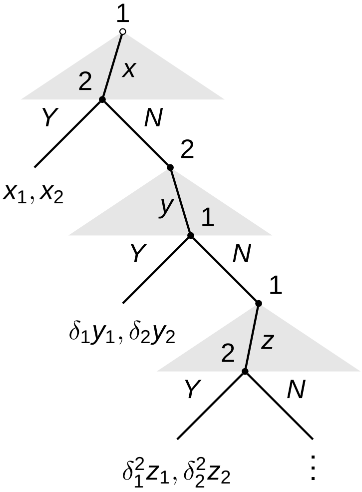
Bargaining: Infinite horizon



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- Every subgame starting with response by P1 to proposal y of P2 is identical

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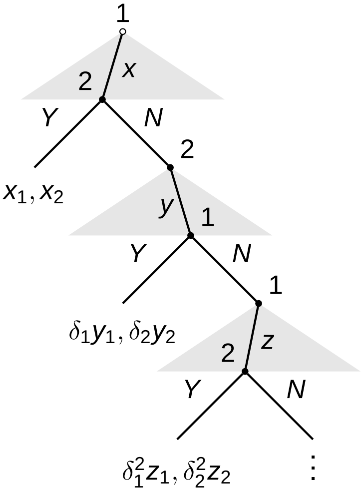

Notes

- Every subgame starting with proposal by P1 is identical
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- Every subgame starting with response by P2 to proposal x of P1 is identical

Bargaining: Infinite horizon

A bargaining game of alternating offers is an extensive game with perfect information with the following components

Players

Histories

Player function

Preferences

Bargaining: Infinite horizon

A bargaining game of alternating offers is an extensive game with perfect information with the following components

Players $N = \{1, 2\}$

Histories

Player function

Preferences

Bargaining: Infinite horizon

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Players $N = \{1, 2\}$

Histories $\varnothing$

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Players $N = \{1, 2\}$

Histories $\varnothing$

every sequence $(x^1, N, x^2, N, \ldots, x^t)$, $t \geq 1$

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every sequence $(x^1, N, x^2, N, \ldots, x^t)$, $t \geq 1$

every sequence $(x^1, N, x^2, N, \ldots, x^t, Y)$, $t \geq 1$

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Terminal every sequence $(x^1, N, x^2, N, \ldots, x^t, Y)$, $t \geq 1$

every sequence $(x^1, N, x^2, N, \ldots, x^t, N)$, $t \geq 1$

every infinite sequence $(x^1, N, x^2, N, \ldots)$

where each x^r is pair of numbers that sum to 1

Player function

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Bargaining: Infinite horizon

A bargaining game of alternating offers is an extensive game with perfect information with the following components

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where each x^r is pair of numbers that sum to 1

Player function $P(\varnothing) = 1$ and for all $(x^1, \ldots, x^t)$

$$P(x^1, N, x^2, N, \ldots, x^t) = P(x^1, N, x^2, N, \ldots, x^t, N) = \begin{cases} 1 & \text{if } t \text{ is even} \\ 2 & \text{if } t \text{ is odd} \end{cases}$$

Preferences

Bargaining: Infinite horizon

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Preferences For $i = 1, 2$, player i's payoff to terminal history $(x^1, N, x^2, N, \ldots, x^t, Y)$ is $\delta_i^{t-1} x_i^t$ $(0 < \delta_i < 1)$,

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Bargaining: Infinite horizon

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Terminal every infinite sequence $(x^1, N, x^2, N, \ldots)$

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Book considers more general preferences

Bargaining game of alternating offers: SPE

Step 1

- Stationary structure of game suggests SPE exists in which
  - P1 always makes same proposal
  - P2 always makes same proposal
  - P1 always responds to a proposal in the same way
  - P2 always responds to a proposal in the same way

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  - P1 always makes same proposal
  - P2 always makes same proposal
  - P1 always responds to a proposal in the same way
  - P2 always responds to a proposal in the same way
- Reasonable guess: game has SPE in which each player's response has cutoff form: accept high offers, reject low ones
- Any strategy pair that satisfies these conditions has form
  - player 1 always proposes x^* and accepts y iff $y_1 \geq y_1^*$
  - player 2 always proposes z^* and accepts w iff $w_2 \geq w_2^*$

  for some proposals w^*, x^*, y^*, and z^*

Bargaining game of alternating offers: SPE

Step 2

- Can we find proposals w^*, x^*, y^*, and z^* such that following strategy pair is SPE?
  - Player 1 always proposes x^* and accepts y iff $y_1 \geq y_1^*$
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- In SPE of two-period game, every equilibrium proposal is accepted ... guess that infinite game has equilibrium with same property

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  $\Rightarrow$ In SPE, $x_2^* = w_2^*$
  - Similarly $z_1^* = y_1^*$

Bargaining game of alternating offers: SPE

Step 3

- Can we find proposals x^* and y^* such that following strategy pair is SPE?
  - Player 1 always proposes x^* and accepts y iff $y_1 \geq y_1^*$
  - Player 2 always proposes y^* and accepts x iff $x_2 \geq x_2^*$

Bargaining game of alternating offers: SPE

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  - If P2 accepts x, her payoff is x_2

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  - If P2 accepts x, her payoff is x_2
  - If P2 rejects x, she proposes y^*, which P1 accepts, yielding P2 the payoff y_2^* with one period of delay

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  - So P2 optimally
    - rejects x if $x_2 < \delta_2 y_2^*$

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  - If P2 accepts x, her payoff is x_2
  - If P2 rejects x, she proposes y^*, which P1 accepts, yielding P2 the payoff y_2^* with one period of delay
  - So P2 optimally
    - rejects x if $x_2 < \delta_2 y_2^*$
    - accepts x if $x_2 > \delta_2 y_2^*$
    - is indifferent between accepting and rejecting if $x_2 = \delta_2 y_2^*$
- We seek SPE in which she accepts iff $x_2 \geq x_2^*$, so we need $x_2^* = \delta_2 y_2^*$

Bargaining game of alternating offers: SPE

Step 3 continued

- Similar argument for response by P1 to proposal of P2
  $\Rightarrow y_1^* = \delta_1 x_1^*$

Bargaining game of alternating offers: SPE

Step 3 continued

- Similar argument for response by P1 to proposal of P2 $\Rightarrow y_1^* = \delta_1 x_1^*$
- Thus for strategy pair to be SPE we need

$$x_2^* = \delta_2 y_2^*$$
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Step 3 continued

- Similar argument for response by P1 to proposal of P2 $\Rightarrow y_1^* = \delta_1 x_1^*$
- Thus for strategy pair to be SPE we need

$$x_2^* = \delta_2 y_2^*$$
$$y_1^* = \delta_1 x_1^*$$

- Using $x_2^* = 1 - x_1^*$ and $y_2^* = 1 - y_1^*$, we get

$$x_1^* = \frac{1-\delta_2}{1-\delta_1\delta_2}$$
$$y_1^* = \frac{\delta_1(1-\delta_2)}{1-\delta_1\delta_2}$$

Bargaining game of alternating offers: SPE

Conclusion

Candidate for SPE is strategy pair s^* in which

- player 1 always proposes x^* and accepts y iff $y_1 \geq y_1^*$
- player 2 always proposes y^* and accepts x iff $x_2 \geq x_2^*$

where

$$x_1^* = \frac{1-\delta_2}{1-\delta_1\delta_2} \qquad \text{and} \qquad y_1^* = \frac{\delta_1(1-\delta_2)}{1-\delta_1\delta_2}$$

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Proposition

The strategy pair s^* is the unique subgame perfect equilibrium of the bargaining game of alternating offers

Bargaining game of alternating offers: SPE

Proof that s^* is subgame perfect equilibrium

- In arbitrary infinite horizon game, strategy profiles that satisfy one-deviation property may not be subgame perfect equilibria

Bargaining game of alternating offers: SPE

Proof that s^* is subgame perfect equilibrium

- In arbitrary infinite horizon game, strategy profiles that satisfy one-deviation property may not be subgame perfect equilibria
- But this property *does* hold in bargaining game of alternating offers (in which the single infinite history is the worst terminal history for each player)

Bargaining game of alternating offers: SPE

Proof that s^* is subgame perfect equilibrium

- In arbitrary infinite horizon game, strategy profiles that satisfy one-deviation property may not be subgame perfect equilibria
- But this property *does* hold in bargaining game of alternating offers (in which the single infinite history is the worst terminal history for each player)

Proposition

A strategy profile in the bargaining game of alternating offers is a subgame perfect equilibrium if and only if it satisfies the one-deviation property

Bargaining game of alternating offers: SPE

Proof that s^* is subgame perfect equilibrium

s_1^*: P1 always proposes x^* and accepts y iff $y_1 \geq y_1^* = \delta_1 x_1^*$
s_2^*: P2 always proposes y^* and accepts x iff $x_2 \geq x_2^* = \delta_2 y_2^*$

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- Will show that s^* satisfies one-deviation property

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- Will show that s^* satisfies one-deviation property
- 2 types of subgame: first move offer, first move response

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Subgame in which first move is offer

Bargaining game of alternating offers: SPE

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Subgame in which first move is offer

- Suppose offer is made by P1, and fix P2's strategy at s_2^*

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- Will show that s^* satisfies one-deviation property
- 2 types of subgame: first move offer, first move response

Subgame in which first move is offer

- Suppose offer is made by P1, and fix P2's strategy at s_2^*
- P1 uses $s_1^* \Rightarrow$

Bargaining game of alternating offers: SPE

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- Suppose offer is made by P1, and fix P2's strategy at s_2^*
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- Will show that s^* satisfies one-deviation property
- 2 types of subgame: first move offer, first move response

Subgame in which first move is offer

- Suppose offer is made by P1, and fix P2's strategy at s_2^*
- P1 uses s_1^* $\Rightarrow$ P1 proposes x^*, which P2 accepts $\Rightarrow$ P1's payoff is x_1^*

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- Suppose offer is made by P1, and fix P2's strategy at s_2^*
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- P1 deviates from s_1^* in first period of subgame:

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Proof that s^* is subgame perfect equilibrium

s_1^*: P1 always proposes x^* and accepts y iff $y_1 \geq y_1^* = \delta_1 x_1^*$
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  - P1 offers P2 $> x_2^* \Rightarrow$ P2 accepts $\Rightarrow$ P1's payoff is $< x_1^*$
  - P1 offers P2 $< x_2^* \Rightarrow$ P2 rejects, P2 proposes $y^* \Rightarrow$

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s_1^*: P1 always proposes x^* and accepts y iff $y_1 \geq y_1^* = \delta_1 x_1^*$
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- Suppose offer is made by P1, and fix P2's strategy at s_2^*
- P1 uses s_1^* $\Rightarrow$ P1 proposes x^*, which P2 accepts $\Rightarrow$ P1's payoff is x_1^*
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  - P1 offers P2 $> x_2^*$ $\Rightarrow$ P2 accepts $\Rightarrow$ P1's payoff is $< x_1^*$
  - P1 offers P2 $< x_2^*$ $\Rightarrow$ P2 rejects, P2 proposes y^* $\Rightarrow$

Bargaining game of alternating offers: SPE

Proof that s^* is subgame perfect equilibrium

s_1^*: P1 always proposes x^* and accepts y iff $y_1 \geq y_1^* = \delta_1 x_1^*$
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- Will show that s^* satisfies one-deviation property
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- Symmetric argument if first offer is made by P2

Bargaining game of alternating offers: SPE

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Bargaining game of alternating offers: SPE

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- Symmetric argument applies if responder is P2

Bargaining game of alternating offers: SPE

Conclusion

Strategy pair s^* defined by

- player 1 always proposes x^* and accepts $y \Leftrightarrow y_1 \geq y_1^*$
- player 2 always proposes y^* and accepts $x \Leftrightarrow x_2 \geq x_2^*$,

where

$$x_1^* = \frac{1-\delta_2}{1-\delta_1\delta_2} \quad \text{and} \quad y_1^* = \frac{\delta_1(1-\delta_2)}{1-\delta_1\delta_2}$$

(so that $y_1^* = \delta_1 x_1^*$ and $x_2^* = \delta_2 y_2^*$) satisfies one-deviation property and thus is a subgame perfect equilibrium

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Defining property of SPE: each player indifferent between accepting and rejecting proposal made in equilibrium

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Defining property of SPE: each player indifferent between accepting and rejecting proposal made in equilibrium

Proof that s^* is *unique* subgame perfect equilibrium ($\Rightarrow$ no nonstationary SPE) is a little intricate (see book)

Bargaining game of alternating offers: properties of SPE

Efficiency

- Player 2 accepts player 1's first offer, so agreement is reached immediately; no resources are wasted in delay

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- Intuition relates this feature to perfect information:
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  - given perfect information, players should perceive and pursue this alternative outcome?
- Nevertheless, some variants of model with perfect information have SPEs in which agreement is *not* reached immediately (see, e.g., Exercise 125.2b)

Bargaining game of alternating offers: properties of SPE

$$(x_1^*, x_2^*) = \left(\frac{1-\delta_2}{1-\delta_1\delta_2}, \frac{\delta_2(1-\delta_1)}{1-\delta_1\delta_2} \right)$$

Changes in patience

Bargaining game of alternating offers: properties of SPE

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- Given δ_2, equilibrium payoff x_1^* of P1 increases as $\delta_1 \rightarrow 1$: given patience of P2, P1's share increases as she becomes more patient

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- As P1 becomes extremely patient (δ_1 close to 1), her share approaches 1
- Symmetrically, fixing patience of P1, P2's share increases to 1 as she becomes more patient

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$$(x_1^*, x_2^*) = \left(\frac{1 - \delta_2}{1 - \delta_1 \delta_2}, \frac{\delta_2 (1 - \delta_1)}{1 - \delta_1 \delta_2} \right)$$

First-mover advantage

Bargaining game of alternating offers: properties of SPE

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- $\delta_1 = \delta_2 = \delta \Rightarrow$ only asymmetry in game is that P1 moves first

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$\Rightarrow$ P1's equilibrium payoff is

$$\frac{1-\delta}{1-\delta^2} = \frac{1}{1+\delta} > \tfrac{1}{2}$$

Payoff $\rightarrow \frac{1}{2}$ as $\delta \rightarrow 1$

Bargaining game of alternating offers: properties of SPE

$$(x_1^*, x_2^*) = \left(\frac{1-\delta_2}{1-\delta_1\delta_2}, \frac{\delta_2(1-\delta_1)}{1-\delta_1\delta_2} \right)$$

First-mover advantage

- $\delta_1 = \delta_2 = \delta \Rightarrow$ only asymmetry in game is that P1 moves first

$\Rightarrow$ P1's equilibrium payoff is

$$\frac{1-\delta}{1-\delta^2} = \frac{1}{1+\delta} > \tfrac{1}{2}$$

Payoff $\rightarrow \frac{1}{2}$ as $\delta \rightarrow 1$

- Thus players equally and only slightly impatient $\Rightarrow$ P1's advantage small and outcome almost symmetric

Bargaining game of alternating offers

Many players

- Model can be extended to many players (e.g. all players have to agree to proposal)

Bargaining game of alternating offers

Many players

- Model can be extended to many players (e.g. all players have to agree to proposal)
- With more than two players, game has many SPEs: in fact, for *every* possible agreement there is an SPE in which that agreement is realized immediately

Bargaining game of alternating offers

Many players

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- Game has only one *stationary* equilibrium (any given player always makes same offer and uses same rule to respond to offers), though it is not clear that this equilibrium is right one to select

Bargaining game of alternating offers with risk of breakdown

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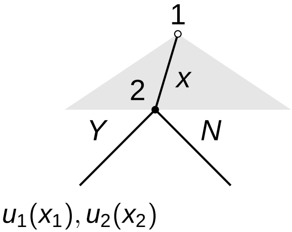
- Will now study variant of model in which motivation is possibility that exogenous event will cause bargaining to break down
- Possibility of breakdown is enough to induce agreement, so assume that discount factors are both 1

Bargaining game of alternating offers with risk of breakdown



Notes: Possibility of breakdown is *exogenous* (move of chance); no discounting; Bernoulli payoff functions u_1, u_2

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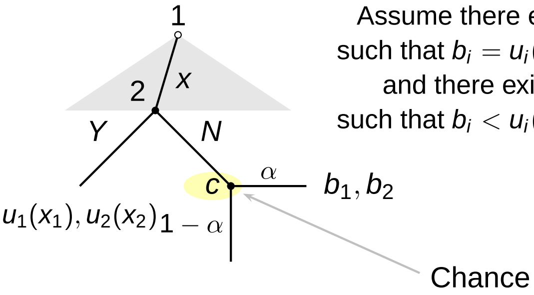

Assume there exists (z_1, z_2)
such that $b_i = u_i(z_i)$ for $i = 1, 2$
and there exists (x_1, x_2)
such that $b_i < u_i(z_i)$ for $i = 1, 2$

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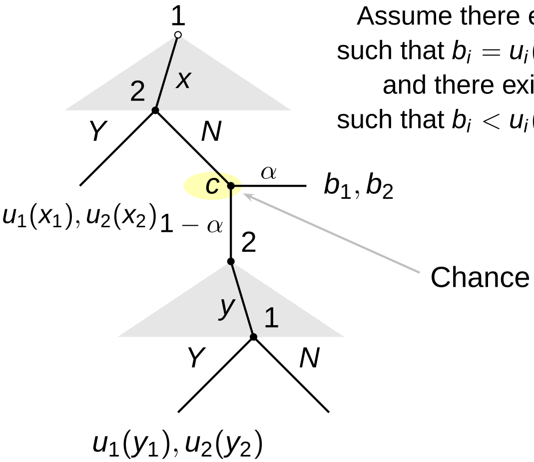

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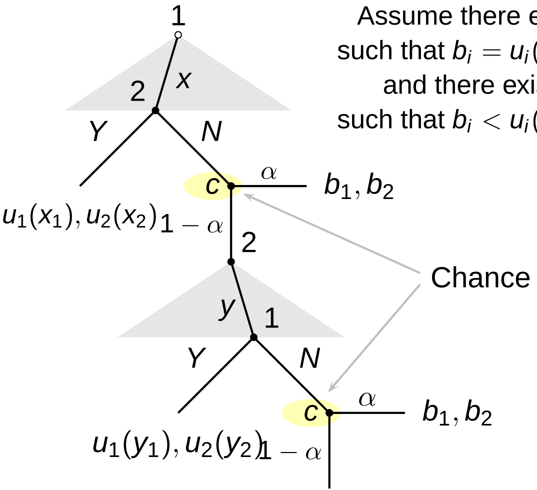

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Bargaining game of alternating offers with risk of breakdown

By logic similar to that for bargaining game of alternating offers, game has unique SPE

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In this equilibrium,

- player 1 always proposes $\hat{x}(\alpha)$ and accepts $y \Leftrightarrow y_1 \geq \hat{y}_1(\alpha)$
- player 2 always proposes $\hat{y}(\alpha)$ and accepts $x \Leftrightarrow x_2 \geq \hat{x}_2(\alpha)$

and each player is indifferent between accepting and rejecting other player's equilibrium proposal, so that

$$u_1(\hat{y}_1(\alpha)) = (1-\alpha)u_1(\hat{x}_1(\alpha)) + \alpha b_1$$
$$u_2(\hat{x}_2(\alpha)) = (1-\alpha)u_2(\hat{y}_2(\alpha)) + \alpha b_2$$

Bargaining game of alternating offers with risk of breakdown: Risk neutral players

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If $u_i(x_i) = x_i$ for each player i then

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so that

$$\hat{x}_1(\alpha) = \frac{1 - b_2 + (1-\alpha)b_1}{2-\alpha}, \quad \hat{y}_1(\alpha) = \frac{(1-\alpha)(1-b_2) + b_1}{2-\alpha}$$

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$$(\hat{x}_1(\alpha), \hat{x}_2(\alpha)) \xrightarrow[\alpha \to 0]{} \left(b_1 + \tfrac{1}{2}(1-(b_1+b_2)), b_2 + \tfrac{1}{2}(1-(b_1+b_2))\right)$$

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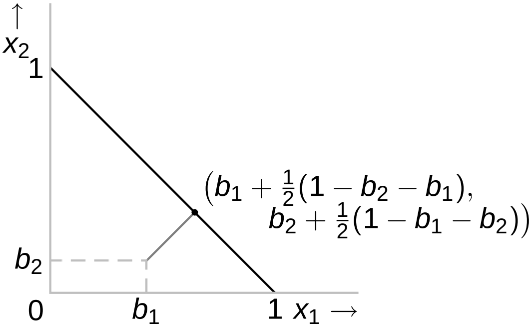
$\Rightarrow$ "split the difference" ($1 - b_1 - b_2$ is surplus over breakdown outcome)



Bargaining game of alternating offers

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Bargaining game of alternating offers

Summary

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- Agreement is reached immediately (efficient outcome)
- Player is more patient $\Rightarrow$ higher payoff
- Players equally patient, with δ close to 0 $\Rightarrow$ outcome close to $(\frac{1}{2}, \frac{1}{2})$
- With risk of breakdown (and no discounting), unique SPE with outcome close to "split the difference" solution over breakdown outcome when probability of breakdown close to 0

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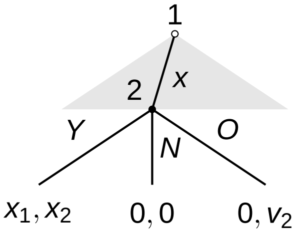

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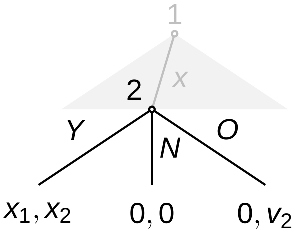

SPE

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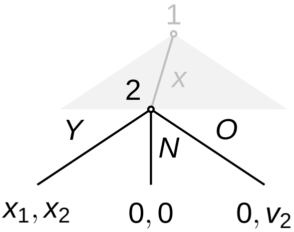

SPE

Y if $x_2 \geq v_2$	Y if $x_2 > v_2$
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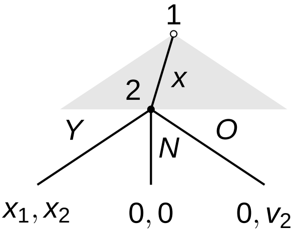

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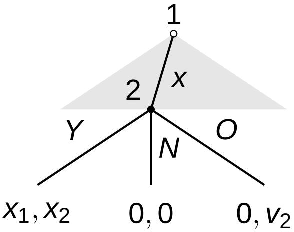

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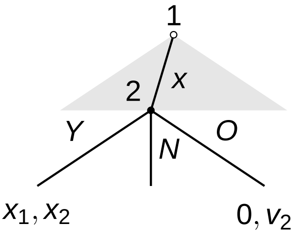
- Without outside option, P2 gets 0 $\Rightarrow$ outside option raises her SPE payoff to v_2

Outside options

Infinite horizon



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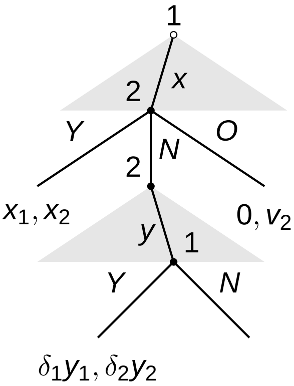

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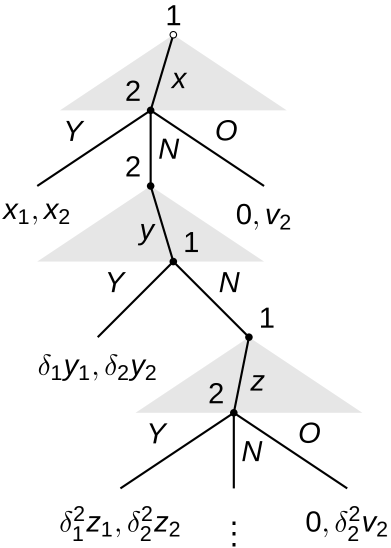

Outside options: Subgame perfect equilibrium

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P2's payoff if she opts out is less than her payoff in SPE of game in which she has no outside option

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If $v_2 > x_2^*$ then the infinite horizon game with an outside option for player 2 has a unique SPE, in which

- player 1 always proposes $(1 - v_2, v_2)$ and accepts $y \Leftrightarrow y_1 \geq \delta_1(1 - v_2)$
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P2's SPE payoff is driven up to her outside option payoff

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- Result says that player's outside option affects SPE only if it is worth more than her equilibrium payoff in its absence
- Uniqueness is sensitive to timing of outside options
  - If player can opt out after opponent rejects offer, get multiple SPEs, with different properties

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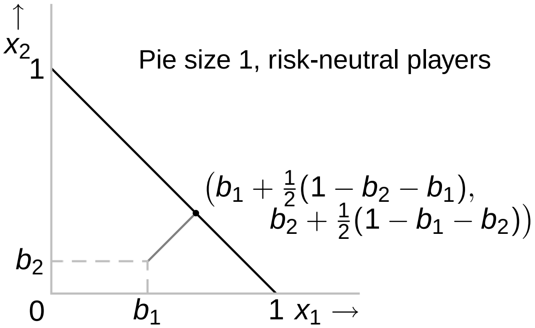

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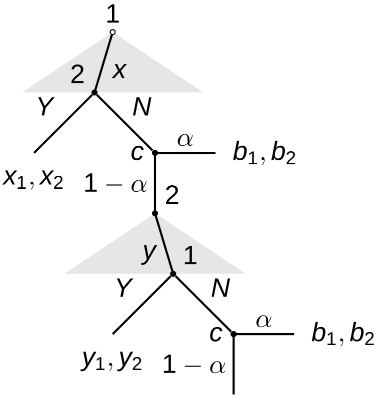

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  - Player rationally takes option only when it benefits her, so option worse than equilibrium payoff in its absence is irrelevant