Economics 2030

Fall 2018 Martin J. Osborne

Questions for Tutorial 4

1. An agent can pursue activity A or activity B. Activity B yields the return 0. Activity A yields the return y but requires both an unrecoverable investment of $c > 0$ and a permit from an official. The permit is free, but the official may demand a bribe of any magnitude; the permit is granted only after the investment has been made and the bribe paid. If the agent pays the bribe b, her payoff is $y - c - b$ and the official's payoff is b.

   Model this situation as an extensive game with perfect information in which the agent first chooses A or B. If she chooses B, the game ends and her payoff and that of the official are both 0. If she chooses A, the official then demands a bribe, which the agent either agrees or refuses to pay. If the agent agrees to pay the bribe, she pursues activity A. If she refuses to pay, she loses her investment of c and pursues activity B (yielding a return of 0).

   (a) Show that this game has a unique subgame perfect equilibrium, in which the agent pursues activity B and the payoffs of both the agent and the official are 0.

   Suppose that activity A takes time and that the official requires a sequence of two permits. One permit is required at the start, after the agent has invested, and another is required after the fraction of time α has passed, during which the activity yields the return αy. In the remainder of the time, the activity yields the return $(1 - \alpha)y$. The official may demand a bribe for each permit. If the agent pays the bribe b_1 demanded initially but does not pay the second bribe, her payoff is $\alpha y - b_1 - c$ and the official's is b_1. If she pays both bribes then her payoff is $y - b_1 - b_2 - c$ and the official's is $b_1 + b_2$, where b_2 is the second bribe.

   (b) Model this situation as the following extensive game with perfect information. The agent first chooses A or B. If she chooses B, the game ends and her payoff and that of the official are both 0. If she chooses A, the official then demands a bribe, which the agent either agrees or refuses to pay. If she refuses to pay, the game ends; she loses her investment of c and pursues activity B (yielding a return of 0). If the agent agrees to pay the bribe, she chooses whether to continue with A or switch to B. If she switches to B, the game ends; she obtains $\alpha y - b_1 - c$ and the official receives b_1. If she continues with A, the official demands a second bribe, which the agent either agrees or refuses to pay. If the agent agrees to pay this second bribe, her payoff is $y - b_1 - b_2 - c$ and the official's payoff is $b_1 + b_2$. If the agent refuses to pay the second bribe, her payoff is $\alpha y - b_1 - c$ and the official's payoff is b_1.

Show that if $\alpha \leq 1 - c/y$ then the game has a subgame perfect equilibrium in which the agent chooses the activity A, pays both bribes, and the official's payoff is $y - c$. Does the game have a subgame perfect equilibrium in which the agent chooses B initially?

2. An incumbent in an industry faces the possibility of entry by a challenger. First the challenger chooses whether or not to enter. If it does not enter, neither firm has any further action; the incumbent's payoff is TM (it obtains the profit M in each of the following $T \geq 1$ periods) and the challenger's payoff is 0. If the challenger enters, it pays the entry cost $f > 0$, and in each of T periods the incumbent first commits to fight or cooperate with the challenger in that period, then the challenger chooses whether to stay in the industry or to exit. (Note that the order of the firms' moves within a period differs from that in the entry game discussed in class.) If, in any period, the challenger stays in, each firm obtains in that period the profit $-F < 0$ if the incumbent fights and $C > \max\{F, f\}$ if it cooperates. If, in any period, the challenger exits, both firms obtain the profit zero in that period (regardless of the incumbent's action); the incumbent obtains the profit $M > 2C$ and the challenger the profit 0 in every subsequent period. Once the challenger exits, it cannot subsequently re-enter. Each firm cares about the sum of its profits.

   (a) Find the subgame perfect equilibria of the extensive game that models this situation.

   (b) Consider a variant of the situation, in which the challenger is constrained by its financial war chest, which allows it to survive at most $T-2$ fights. Specifically, consider the game that differs from the one in part *a* in one respect: the histories in which (*i*) at the start of the game the challenger enters and (*ii*) the incumbent fights in $T-1$ periods, are terminal histories (the challenger has to exit). For the terminal history in which the incumbent fights in the first $T-1$ periods the incumbent's payoff is $M-(T-2)F$ and the challenger's payoff is $-f-(T-2)F$ (in period $T-1$ the incumbent's payoff is 0, and in the last period its payoff is M). For the terminal history in which the incumbent cooperates in one of the first $T-1$ periods and fights in the remainder of these periods and in the last period, the incumbent's payoff is $C-(T-2)F$ and the challenger's payoff is $-f+C-(T-2)F$. Find the subgame perfect equilibria of this game.

3. A group of n players have to divide between themselves a pie of size 1.

   The procedure used to divide the pie potentially has many stages. In each stage, one player proposes a division (an n-vector of nonnegative numbers summing to 1), and then all the players simultaneously vote for or against the division. If a *strict* majority (i.e. *more* than 50%) votes for the division, then that division is the outcome and each player's payoff is the amount of the pie she gets. Otherwise, the proposer is eliminated from future proposals and receives a payoff of zero, and play moves to the next stage. (When one player remains, that player simply takes all of the pie.)

   Player 1 proposes in the first stage, player 2 in stage 2 (i.e. if player 1's proposal is not accepted), player 3 in stage 3, and so on.

   (a) Find the set of all subgame perfect equilibria of the game for $n=2$. (Be careful to specify the equilibrium **strategies**.)

   (b) For $n=2$, does the game have any Nash equilibrium for which the players' payoffs differ from the payoffs in any subgame perfect equilibrium?

   (c) Find the proposal (proposals?) made by player 1 in the subgame perfect equilibria of the game for $n=3$.

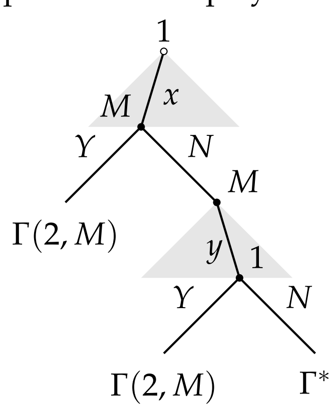
4. Consider the following variant of a bargaining game of alternating offers, denoted Γ^*. There are *three* players, M, 1, and 2, and *two* pies of size 1. Pie 1 is available to be split between players M and 1 and pie 2

is available to be split between players M and 2. Denote by $\Gamma(i, M)$ the standard bargaining game of alternating offers between players i and M in which the first proposal is made by player i.

- Player M starts bargaining with player 1: player 1 proposes a split of pie 1 between her and player M.
- Player M either accepts or rejects 1's proposal.
  - If player M accepts the proposal, players 1 and M get the payoffs player 1 proposed and, starting in period 2, players 2 and M play the game $\Gamma(2, M)$.
  - If player M rejects the proposal, she makes a counterproposal in period 2, which player 1 either accepts or rejects.
    - If player 1 accepts player M's counterproposal, players 1 and M get the payoffs player M proposed and, starting in period 3, players 2 and M play the game $\Gamma(2, M)$.
    - If player 1 rejects player M's counterproposal, the following subgame is exactly Γ^*.

Each player discounts payoffs using the same discount factor δ, with $0 < \delta < 1$. Note that player M's payoff is the sum of her payoffs when bargaining with player 1 and when bargaining with player 2, and that these payoffs are received in different periods, and so will be discounted differently.

The structure of the game is shown in the following diagram, where x and y are divisions of pie 1 between players 1 and M.



Find a subgame perfect equilibrium of Γ^*. (A strategy pair in this game is a subgame perfect equilibrium if and only if it satisfies the one-deviation property.)