Economics 2030

Fall 2018 Martin J. Osborne

Problem Set 7

1. Two people are initially $n \geq 3$ meters apart, where n is an integer. Each has a gun containing a single bullet. They move alternately, starting with player 1. If, on her turn, the distance between the players exceeds 1 meter, a player can *either* shoot at the other player *or* move a meter closer to the other player. If she shoots, the game ends. If she moves closer, it is the other player's turn to move. When the distance between the players is 1 meter, the player whose turn it is to move has only one option, to shoot. A player who shoots when the distance between the players is k meters hits the other player with probability $p(k)$ and obtains the payoff $p(k)$; the other player's payoff is $1 - p(k)$. Assume that p is a decreasing function (the closer the players, the higher the probability of a shot's hitting its target), with $p(n) < 1 - p(n-1)$ and $p(1) = 1$; assume also that $p(k) \neq 1 - p(k-1)$ for all k. (If you prefer to think about a less violent situation, consider political candidates who alternately have the opportunity to launch attacks against each other. Well, perhaps that's not much less violent.)

   (a) Model this situation as an extensive game. (A diagram suffices.)

   (b) Characterize the subgame perfect equilibria of the game.

   (c) Does the game have a *Nash equilibrium* in which player 1 shoots on her first turn?

2. The voters in Hotelling's model of electoral competition are not players in the game: each citizen is assumed simply to vote for the candidate whose position she most prefers. Consider a model in which the citizens, as well as the candidates, act strategically. Consider specifically the extensive game in which the candidates first simultaneously choose actions, then the citizens simultaneously choose how to vote. Assume that each candidate may either choose a position (as in Hotelling's original model) or choose to stay out of the race, an option

she is assumed to rank between losing and tying for first place with all the other candidates.

Assume that there are $n \geq 3$ candidates and q citizens, where $q \geq 2n$ is odd (so that the median of the voters' favorite positions is well-defined) and divisible by n. Show that the game has a subgame perfect equilibrium in which no citizen's strategy is weakly dominated and every candidate enters the race and chooses the median of the citizens' favorite positions. (You may use the fact that every voting subgame has a (pure) Nash equilibrium in which no citizen's action is weakly dominated.)

3. A seller owns one indivisible unit of a good, which she does not value. Several potential buyers, each of whom attaches the same positive value v to the good, simultaneously offer prices they are willing to pay for the good. After receiving the offers, the seller decides which, if any, to accept. If she does not accept any offer, then no transaction takes place, and all payoffs are 0. Otherwise, the buyer whose offer the seller accepts pays the amount p she offered and receives the good; the payoff of the seller is p, the payoff of the buyer who obtained the good is $v - p$, and the payoff of every other buyer is 0. Model this situation as an extensive game with perfect information and simultaneous moves and find its subgame perfect equilibria. (Use a combination of intuition and trial and error to find a strategy profile that appears to be an equilibrium, then argue directly that it is.)