ECO2030: Microeconomic Theory II, module 1

Lecture 7

Martin J. Osborne

Department of Economics
University of Toronto

2018.11.20

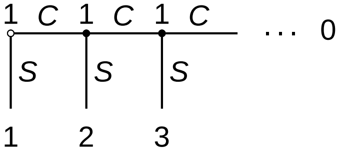
Table of contents

Nash equilibrium and subgame perfect equilibrium

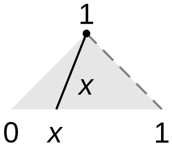
- Every subgame perfect equilibrium is a Nash equilibrium
- A finite game has a subgame perfect equilibrium

$\Rightarrow$ If finite game has unique Nash equilibrium then that equilibrium is subgame perfect

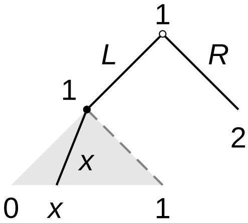
Subgame perfect equilibrium of infinite games



No subgame perfect equilibrium
(and no Nash equilibrium)



Actions: $[0, 1)$
Payoff to action x: x
No subgame perfect equilibrium
(and no Nash equilibrium)



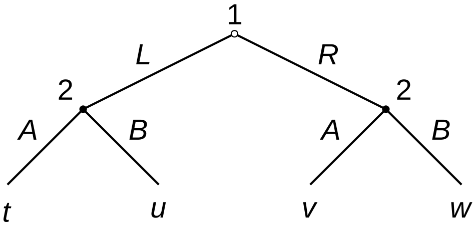
No subgame perfect equilibrium
Nash equilibrium:
(R, x) for any $x \in [0, 1)$

Stackelberg games

Two-player two-stage games in which one player chooses from some set A_1 and then the other chooses from A_2

- $N = \{1, 2\}$ (two players)
- $H = \{\varnothing\} \cup A_1 \cup (A_1 \times A_2)$
- $P(\varnothing) = 1$ and $P(a_1) = 2$ for all $a_1 \in A_1$
- Payoff functions u_i are arbitrary

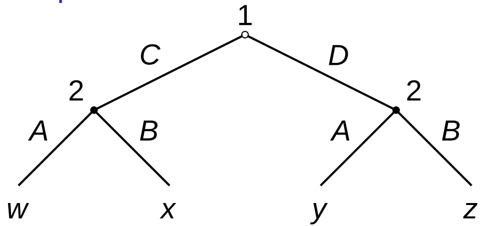
Example



Stackelberg games

Compare Stackelberg game with a related strategic game $\langle N, (A_i)_{i \in N}, (u_i) \rangle$

Example



	A	B
C	w	x
D	y	z

Note: Simultaneous move game is *not* strategic form of extensive game!

Stackelberg games

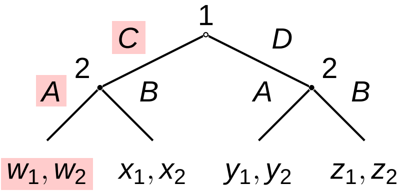
General argument

- Let (a_1^*, a_2^*) be Nash equilibrium of strategic game G
- Consider extensive game Γ in which player 1 moves first
  - Suppose player 1 chooses a_1^*
  - What action does player 2 choose?
  - (a_1^*, a_2^*) is Nash equilibrium of G
    $\Rightarrow a_2^*$ is best response to a_1^*
    $\Rightarrow a_2^*$ is an optimal choice of player 2 following a_1^* in Γ

Example

	A	B
C	w_1, w_2	x_1, x_2
D	y_1, y_2	z_1, z_2

$w_1 \geq y_1$, $w_2 \geq x_2$



Stackelberg games

General argument

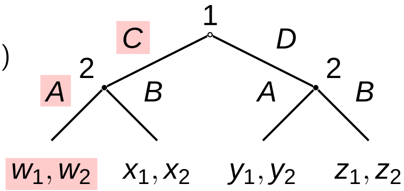
- Let (a_1^*, a_2^*) be Nash equilibrium of strategic game G
- Consider extensive game Γ in which player 1 moves first
    - *If* a_2^* is *only* optimal choice of player 2, player 1's choice of a_1^* *guarantees* her the payoff $u_1(a_1^*, a_2^*)$

      $\Rightarrow$ in any subgame perfect equilibrium, player 1's payoff $\geq u_1(a_1^*, a_2^*)$

Example

	A	B
C	w_1, w_2	x_1, x_2
D	y_1, y_2	z_1, z_2

$w_1 \geq y_1$, $w_2 > x_2$



Stackelberg games

Summary

Let G be two-player strategic game and let Γ be Stackelberg version of G in which player 1 moves first. Then player 1's payoff in every subgame perfect equilibrium of Γ is at least her payoff in any pure strategy Nash equilibrium s^* of G in which s_2^* is the only best response to s_1^*.

Notes

- Matching Pennies?
- Result does not apply to Matching Pennies because that game has no pure strategy equilibrium
- Player 1's payoff in a subgame perfect equilibrium of Γ may be *less* than her payoff in a Nash equilibrium s^* of G in which s_2^* is not the only best response to s_1^*
- Player 1 can be *better off* in every subgame perfect equilibrium of Γ than in the Nash equilibrium of G

Stackelberg games

- Result does not generalize beyond two players
- Consider three-player game:

	L	R
T	$1, 1, 1$	$0, 0, 0$
B	$0, 1, 1$	$0, 3, 0$

L

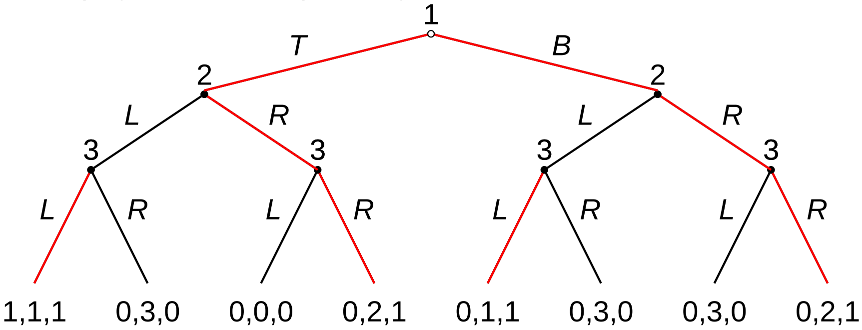
	L	R
T	$0, 3, 0$	$0, 2, 1$
B	$0, 3, 0$	$0, 2, 1$

R

- Game has unique pure strategy equilibrium, (T, L, L), in which player 1's payoff is 1

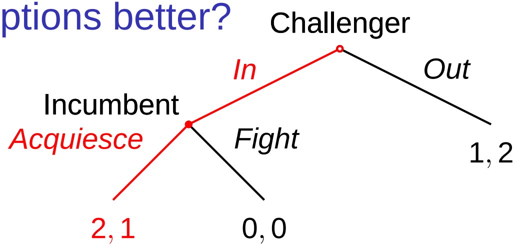
Stackelberg games

- Consider extensive game with perfect information in which players move sequentially



- Game has two SPEs, $(T, RR, LRLR)$ and $(B, RR, LRLR)$, both with payoffs $(0, 2, 1)$
- So player 1 is worse off as first-mover in extensive game than she is in Nash equilibrium of strategic game

Are more options better?



- Unique subgame perfect equilibrium, (*In*, *Acquiesce*)
- If Incumbent's only option is *Fight*, then unique subgame perfect equilibrium (*Out*, *Fight*)
- Incumbent is *better off* in this equilibrium than in the equilibrium of the original game
- So fewer options can be better—commitment has a value
- Challenger is worse off: she prefers Incumbent to have more options
- Sun Tzu's advice in *The Art of Warfare* (written between 500 BC and 300 BC): "in surrounding the enemy, leave him a way out; do not press an enemy that is cornered"

Ultimatum game

- Two players: proposer and responder
- Pie of size c
- Proposer offers an amount of pie (from 0 to c) to responder
- Responder either accepts or rejects offer
  - If responder accepts an offer of x, proposer gets $c - x$ and responder gets x
  - If responder rejects an offer, both proposer and responder get 0

Ultimatum game

Experiment

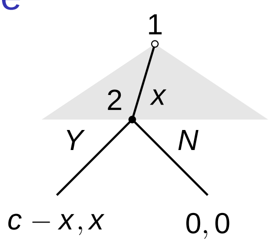
- Pie of size \$20
- Every participant will first act as a proposer
- Every participant will choose an amount from \$0 to \$20 to offer a responder
- After all participants have chosen offers, every participant's offer will be presented to another randomly chosen participant—a responder—who will either accept or reject it
  - If responder accepts an offer of x, proposer will get $\$(20 - x)$ and responder will get $\$x$
  - If responder rejects an offer, both proposer and responder get \$0
  - Your total payoff will be the sum of the payoffs you get as a proposer and as a responder

Ultimatum game

Experiment

- All interaction will be anonymous
- No participant will know identity of participant with whom s/he is matched
- Matching will be random
- If participant A's offer is presented to participant B for a response, then participant B's offer will *not* (except by chance) be presented to participant A for response
- Names of participants with top 3 payoffs will be revealed, but *not* their payoffs

Ultimatum game



$0 \leq x \leq c$
$(c > 0)$

Extensive game

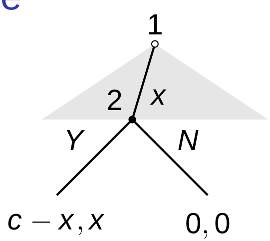
Players $\{1, 2\}$

Terminal histories Set of sequences (x, d) where $0 \leq x \leq c$ and $d \in \{Y, N\}$

Player function $P(\varnothing) = 1$, $P(x) = 2$ for all x

Payoffs $u_1(x, Y) = c - x$, $u_2(x, Y) = x$ for all x, and $u_1(x, N) = u_2(x, N) = 0$

Ultimatum game



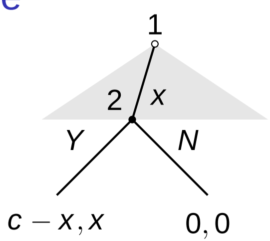
$0 \leq x \leq c$
$(c > 0)$

Strategies

Player 1 $[0, c]$

Player 2 Functions $s_2 : [0, c] \to \{Y, N\}$

Ultimatum game



$0 \leq x \leq c$
$(c > 0)$

Backward induction

In the subgame following x, Y is optimal if $x > 0$, and *both* Y and N are optimal if $x = 0$

So *two* optimal strategies in subgame:

$$s_2^1(x) = Y \text{ for all } x$$

$$s_2^2(x) = \begin{cases} Y & \text{if } x > 0 \\ N & \text{if } x = 0 \end{cases}$$

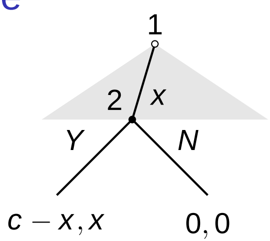
$\Downarrow$

Optimal action of P1 is 0

$\Downarrow$

No optimal action of P1

Ultimatum game



$0 \leq x \leq c$
$(c > 0)$

Subgame perfect equilibria

Hence *unique* subgame perfect equilibrium: $s_1 = 0$ and $s_2(x) = Y$ for all x

Ultimatum game: Experimental evidence

- Experiment at University of Cologne (West Germany) in late 1970s among graduate students of economics (authors say "It is almost sure that none of the students was familiar with game theory"):
  - Size of pie: DM 4–10 (worth $6–14 now)
  - Average offer of player 1 around $0.3c$ to $0.35c$ (versus subgame perfect equilibrium offer of 0)
  - About 20% of offers rejected

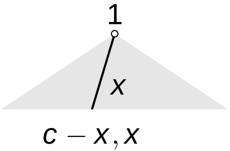
  Source: Güth et al., *Journal of Economic Behavior and Organization* 3 (1982), 367–388

- Many other experiments yield similar results
- If stakes are high, some evidence that proposers offer lower fraction of pie and fewer offers are rejected

  E.g. Andersen et al., *American Economic Review* 101 (2011), 3427–3439

Ultimatum game: Experimental evidence

- Could a preference for fairness explain the results?
- Consider variant of ultimatum game in which player 2 has no option to reject offer
- Called *dictator game*



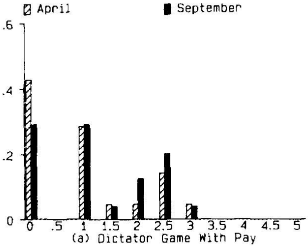
$$0 \le x \le c$$
$$(c > 0)$$

- Unique subgame perfect equilibrium: player 1 offers 0
- If non-zero offers in ultimatum game are result of subjects' concern for fairness, should get similar outcomes in dictator game

Ultimatum game: Experimental evidence

Dictator game

Subjects: students at University of Iowa
Pie size: $5



- Dictators offer less than proposers in ultimatum game, but still offer significant positive amounts

Source: Forsythe et al., *Games and Economic Behavior* 6 (1994), 347–369. See also Bolton et al., *International Journal of Game Theory* 27 (1998), 269–299 and Eckel et al., *Journal of Economic Behavior and Organization* 80 (2011), 603–612.

Ultimatum game: Experimental evidence

Pie size: one or two days' wages

Group	Country	Avg. offer	Rejection rate
Machiguenga	Perú	26%	5%
Torguud	Mongolia	35%	5%
Tsimané	Bolivia	37%	0%
Sangu	Tanzania	41%	10%
Lamalera	Indonesia	58%	0%

Source: Henrich et al., *American Economic Review, Papers and Proceedings* 91 (2001), 73–78

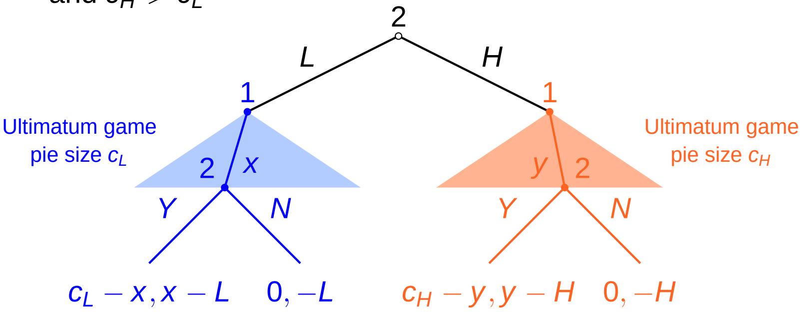
Ultimatum game: Experimental evidence

Another hypothesis

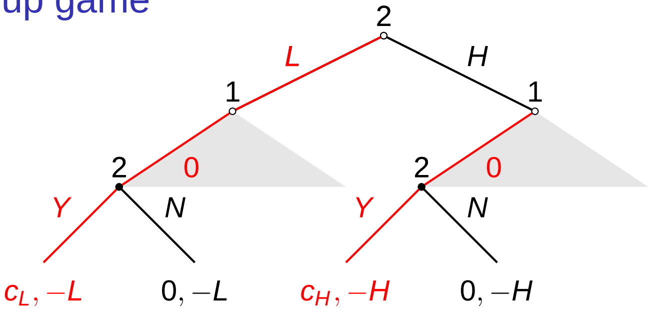
- Significant offers of proposer consistent with proposer's fear that responder will reject offer
- And in fact responders do reject offers
- Why do responders reject offers?
- They may fail to comprehend fully the isolated nature of the interaction, and instead follow their instinct, which is shaped by the long-term relationships to which they are accustomed
- In a long-term relationship, "punishing" a proposer who makes a low offer by rejecting it may have benefit of discouraging low offers in the future

Holdup game

- Before playing ultimatum game, responder decides whether to expend low effort (L) or high effort (H)
- More effort is more costly, but produces bigger pie: $H > L$ and $c_H > c_L$



Holdup game



Analysis

- SPE: in each ultimatum game, P1 offers 0 and P2 accepts all offers
- SPE of whole game: P2 chooses L

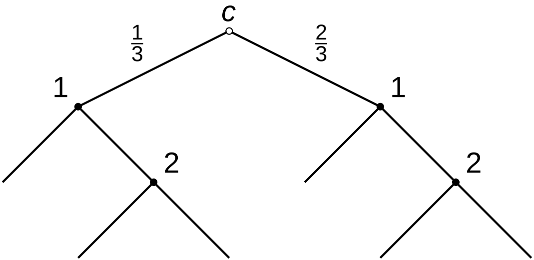
$\Rightarrow$ inefficient outcome if $c_H - H > c_L - L$

- P2 is "held up" for all the surplus her extra effort produces
- Even with less extreme outcome of bargaining, SPE outcome may still be inefficient

Chance moves

- Can allow for existence of random events
- Add “chance” as player whose choices are determined by probability distribution, independently each time it acts

Example



Proposition

Every finite extensive game with perfect information and chance moves has a subgame perfect equilibrium. In a game with a finite horizon the set of strategy profiles satisfying the one-deviation property is the set of subgame perfect equilibria.

Simultaneous moves

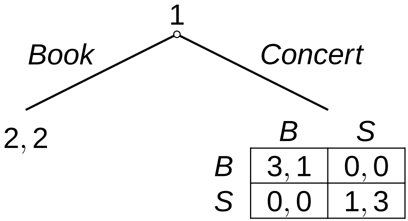
- Can generalize extensive game to allow simultaneous moves
- Main change is that player function assigns a *set* of players to move after each history, instead of a single player

Special cases of extensive game with perfect information and simultaneous moves

Extensive game with perfect information Set of players assigned to each history is a singleton

Strategic game Game has single history $\varnothing$, and $P(\varnothing) = N$ (all players move simultaneously at the start of the game)

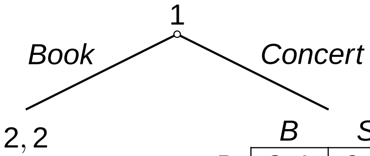
Simultaneous moves: example



Histories $\{\varnothing, Concert, (Concert, (B, B)),$
$(Concert, (B, S)), (Concert, (S, B)),$
$(Concert, (S, S)), Book\}$

Player function $P(\varnothing) = 1$, $P(Concert) = \{1, 2\}$

Simultaneous moves: example



	B	S
B	3, 1	0, 0
S	0, 0	1, 3

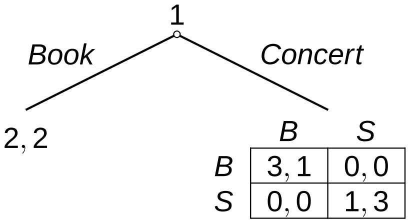
Subgame perfect equilibrium

- Use backward induction
- Subgame following *Concert* has two pure Nash equilibria: (B, B) and (S, S)
- Determine implications of each Nash equilibrium for choice of P1 at start of game:

  (B, B) P1's optimal action is *Concert*

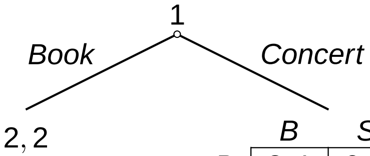
  (S, S) P1's optimal action is *Book*
- So *two* SPEs: $((Concert, B), B)$ and $((Book, S), S)$

Simultaneous moves: example



- Consider the SPE $((Book, S), S)$
- Suppose P1 deviates to *Concert*
- P2 can reason that because P1 has given up a payoff of 2, she intends to choose B (given that P1's payoff to $S \leq 1$)
- If P2 thinks P1 will choose B, she should choose B
- If P2 chooses B, P1 is better off than she would be choosing *Book*
- P1 can thus reason that deviating from *Book* to *Concert* will increase her payoff

Simultaneous moves: example



	B	S
B	3, 1	0, 0
S	0, 0	1, 3

- So the SPE $((Book, S), S)$ seems susceptible to a deviation to *Concert* by P1
- The line of reasoning that leads P1 to conclude a deviation is beneficial is called *forward induction*

Summary

- Game has two SPEs, $((Concert, B), B)$ and $((Book, S), S)$
- SPE $((Book, S), S)$ appears to be not robust to forward induction

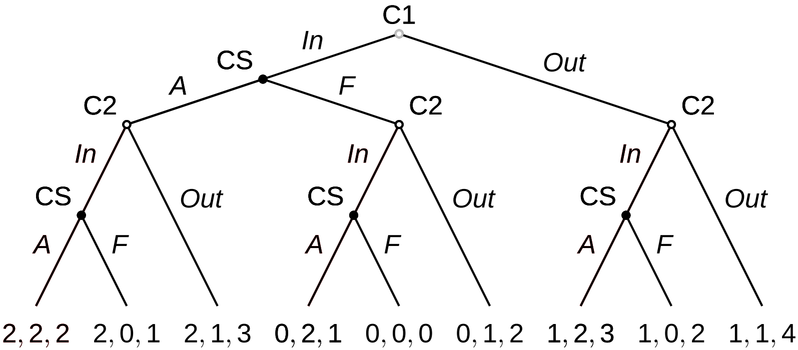
Extensive games with perfect information and simultaneous moves

General results

- A strategy profile is a subgame perfect equilibrium of a finite horizon extensive game with perfect information and simultaneous moves if and only if it satisfies the one-deviation property
- An extensive game with perfect information and simultaneous moves may *not* have a pure strategy equilibrium (even if it is finite)

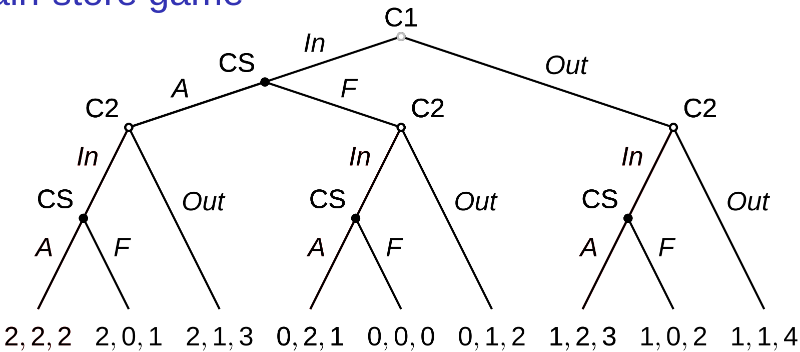
Chain-store game

- Chain-store operates in K markets
- Single challenger in each market, and chain-store and challenger play entry game
- For $K = 2$ (payoff order C1, C2, CS):



Payoffs are sums of payoffs in each period: e.g. (In, A, In, A) yields $(2, 0, 1) + (0, 2, 1) = (2, 2, 2)$

Chain-store game



Subgame perfect equilibria

- Use backward induction
- Last market: unique SPE (In, A), *regardless of history*
- Whole game: outcome in last market is same regardless of actions in first market, so SPE $\Rightarrow$ (In, A) in first market

$\Rightarrow$ unique SPE: All challengers choose *In*, CS always chooses *A*

Chain-store game: Discussion

- Let $K = 100$
- Suppose you are challenger 20
- Suppose every previous challenger has entered and chain-store has fought *every one* (not consistent with SPE, but a possible history)
- According to SPE, chain-store will acquiesce to your entry
- But its actions in previous 19 markets are inconsistent with SPE!
- So is it reasonable for you to expect it to acquiesce to your entry?
- Note that if, by having fought in earlier markets, chain-store persuades you and at least 50 of next 80 challengers to stay out then its profit will exceed its profit in SPE, so that there is some logic to its actions
- Can chain-store's aggressive behavior in early markets establish for it a reputation for being a fighter?

Chain-store game: Discussion

- Given potential advantage to chain-store of persuading later challengers it will fight, fighting at start of game may not be so irrational
- So: not clear that behavior predicted by notion of SPE in this game is reasonable
- Idea that chain-store may be able to earn a “reputation” for fighting is captured in a model in which challengers believe that with small positive probability chain-store *prefers* to fight
- Requires extensive game with *imperfect* information