ECO2030: Microeconomic Theory II, module 1

Lecture 7

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- Every subgame perfect equilibrium is a Nash equilibrium

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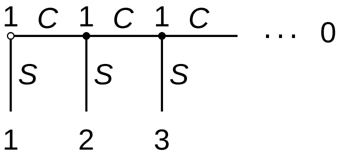
- A finite game has a subgame perfect equilibrium

Nash equilibrium and subgame perfect equilibrium

- Every subgame perfect equilibrium is a Nash equilibrium
- A finite game has a subgame perfect equilibrium

$\Rightarrow$ If finite game has unique Nash equilibrium then that equilibrium is subgame perfect

Subgame perfect equilibrium of infinite games



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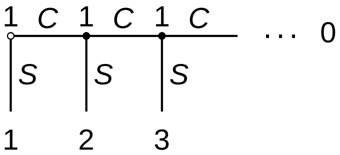
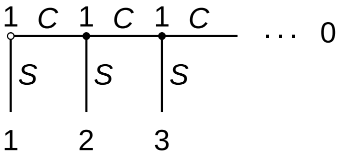


No subgame perfect equilibrium
(and no Nash equilibrium)

Subgame perfect equilibrium of infinite games



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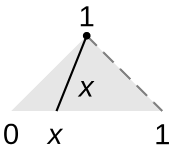
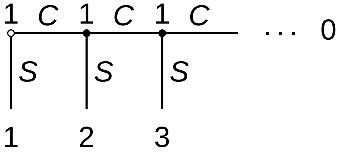


Actions: $[0, 1)$
Payoff to action x: x

Subgame perfect equilibrium of infinite games



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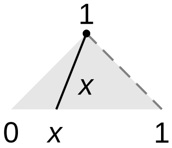

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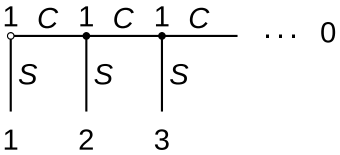

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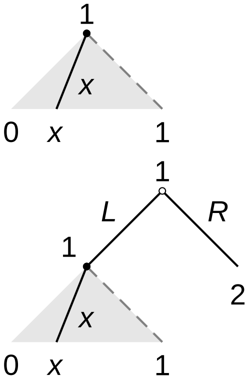

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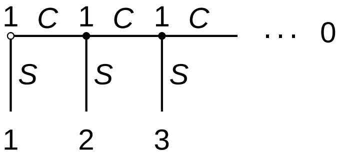

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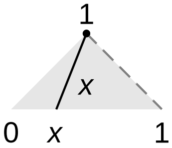

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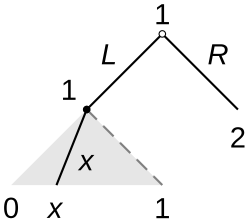
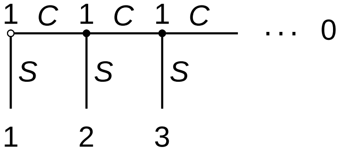


No subgame perfect equilibrium

Subgame perfect equilibrium of infinite games



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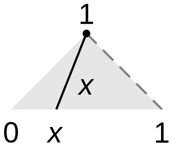

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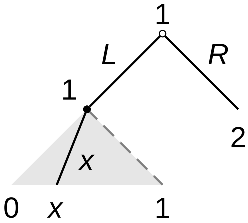
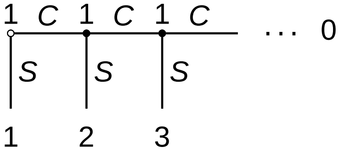


No subgame perfect equilibrium
Nash equilibrium:

Subgame perfect equilibrium of infinite games



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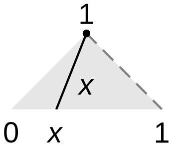

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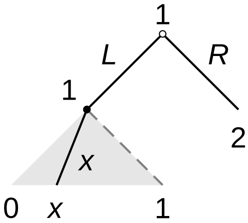

No subgame perfect equilibrium
Nash equilibrium:
(R, x) for any $x \in [0, 1)$

Stackelberg games

Two-player two-stage games in which one player chooses from some set A_1 and then the other chooses from A_2

- $N = \{1, 2\}$ (two players)

Stackelberg games

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Stackelberg games

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- $N = \{1, 2\}$ (two players)
- $H = \{\varnothing\} \cup A_1 \cup (A_1 \times A_2)$
- $P(\varnothing) = 1$ and $P(a_1) = 2$ for all $a_1 \in A_1$

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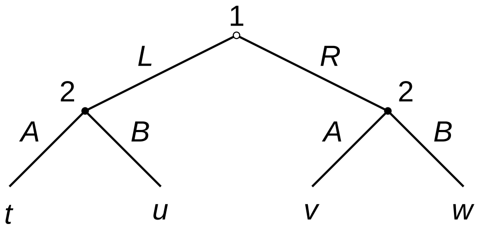
Example



Stackelberg games

Compare Stackelberg game with a related strategic game $\langle N, (A_i)_{i \in N}, (u_i) \rangle$

Stackelberg games

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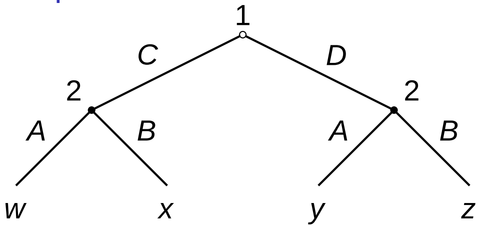
Example

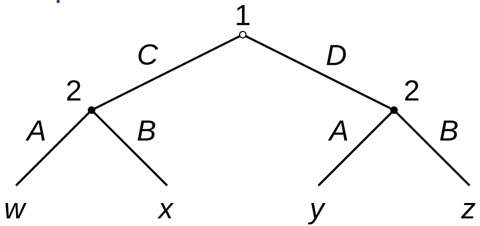


	A	B
C	w	x
D	y	z

Stackelberg games

Compare Stackelberg game with a related strategic game $\langle N, (A_i)_{i \in N}, (u_i) \rangle$

Example



	A	B
C	w	x
D	y	z

Note: Simultaneous move game is *not* strategic form of extensive game!

Stackelberg games

General argument

- Let (a_1^*, a_2^*) be Nash equilibrium of strategic game G

Stackelberg games

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Example

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$w_1 \geq y_1$, $w_2 \geq x_2$

Stackelberg games

General argument

- Let (a_1^*, a_2^*) be Nash equilibrium of strategic game G
- Consider extensive game Γ in which player 1 moves first

Example

	A	B
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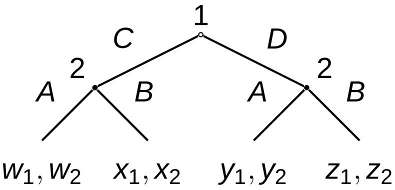
$$w_1 \geq y_1,\ w_2 \geq x_2$$



Stackelberg games

General argument

- Let (a_1^*, a_2^*) be Nash equilibrium of strategic game G
- Consider extensive game Γ in which player 1 moves first
  - Suppose player 1 chooses a_1^*

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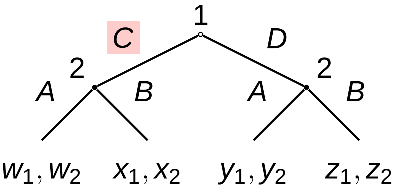

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General argument

- Let (a_1^*, a_2^*) be Nash equilibrium of strategic game G
- Consider extensive game Γ in which player 1 moves first
  - Suppose player 1 chooses a_1^*
  - What action does player 2 choose?

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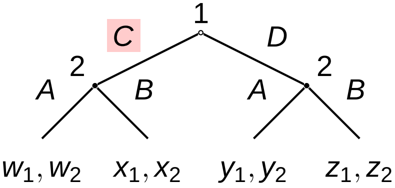

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  - What action does player 2 choose?
  - (a_1^*, a_2^*) is Nash equilibrium of G

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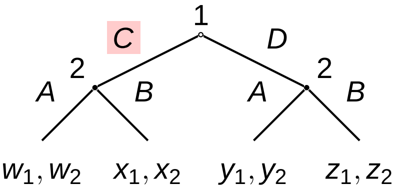

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  - (a_1^*, a_2^*) is Nash equilibrium of G $\Rightarrow a_2^*$ is best response to a_1^*

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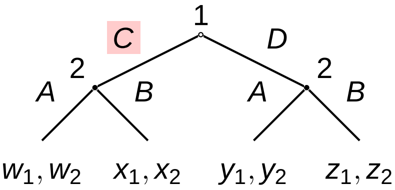

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  - Suppose player 1 chooses a_1^*
  - What action does player 2 choose?
  - (a_1^*, a_2^*) is Nash equilibrium of G
    $\Rightarrow a_2^*$ is best response to a_1^*
    $\Rightarrow a_2^*$ is an optimal choice of player 2 following a_1^* in Γ

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	A	B
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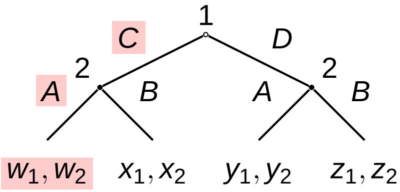
D	y_1, y_2	z_1, z_2

$$w_1 \geq y_1, \; w_2 \geq x_2$$



Stackelberg games

General argument

- Let (a_1^*, a_2^*) be Nash equilibrium of strategic game G
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  - *If* a_2^* is *only* optimal choice of player 2, player 1's choice of a_1^* *guarantees* her the payoff $u_1(a_1^*, a_2^*)$

Example

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C	w_1, w_2	x_1, x_2
D	y_1, y_2	z_1, z_2

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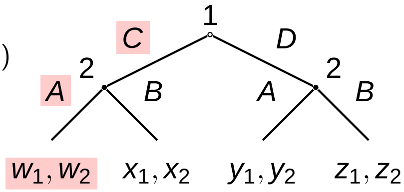

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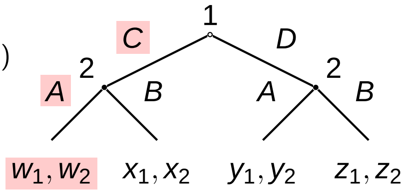
    $\Rightarrow$ in any subgame perfect equilibrium, player 1's payoff $\geq u_1(a_1^*, a_2^*)$

Example

	A	B
C	w_1, w_2	x_1, x_2
D	y_1, y_2	z_1, z_2

$$w_1 \geq y_1,\ w_2 > x_2$$



Stackelberg games

Summary

Let G be two-player strategic game and let Γ be Stackelberg version of G in which player 1 moves first. Then player 1's payoff in every subgame perfect equilibrium of Γ is at least her payoff in any pure strategy Nash equilibrium s^* of G in which s_2^* is the only best response to s_1^*.

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Notes

- Matching Pennies?

Stackelberg games

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- Result does not apply to Matching Pennies because that game has no pure strategy equilibrium

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- Matching Pennies?
- Result does not apply to Matching Pennies because that game has no pure strategy equilibrium
- Player 1's payoff in a subgame perfect equilibrium of Γ may be *less* than her payoff in a Nash equilibrium s^* of G in which s_2^* is not the only best response to s_1^*
- Player 1 can be *better off* in every subgame perfect equilibrium of Γ than in the Nash equilibrium of G

Stackelberg games

- Result does not generalize beyond two players

Stackelberg games

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- Consider three-player game:

	L	R
T	$1, 1, 1$	$0, 0, 0$
B	$0, 1, 1$	$0, 3, 0$

L

	L	R
T	$0, 3, 0$	$0, 2, 1$
B	$0, 3, 0$	$0, 2, 1$

R

Stackelberg games

- Result does not generalize beyond two players
- Consider three-player game:

	L	R
T	$1, 1, 1$	$0, 0, 0$
B	$0, 1, 1$	$0, 3, 0$

L

	L	R
T	$0, 3, 0$	$0, 2, 1$
B	$0, 3, 0$	$0, 2, 1$

R

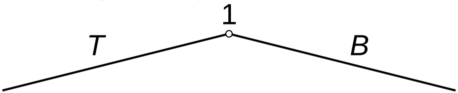
- Game has unique pure strategy equilibrium, (T, L, L), in which player 1's payoff is 1

Stackelberg games

- Consider extensive game with perfect information in which players move sequentially

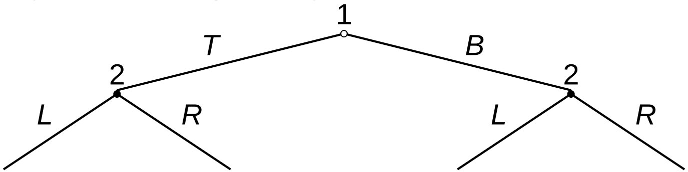
Stackelberg games

- Consider extensive game with perfect information in which players move sequentially



Stackelberg games

- Consider extensive game with perfect information in which players move sequentially



Stackelberg games

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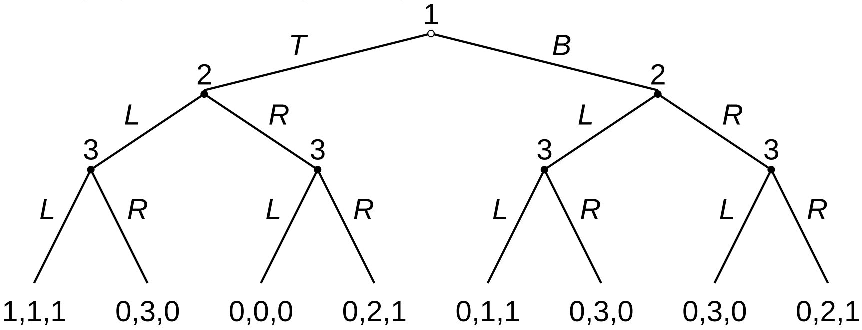

Stackelberg games

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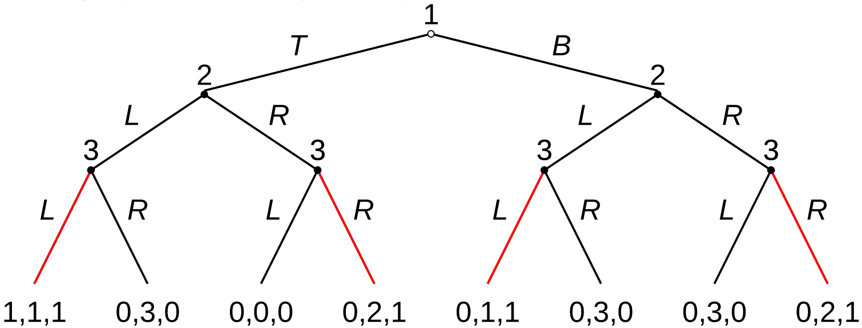

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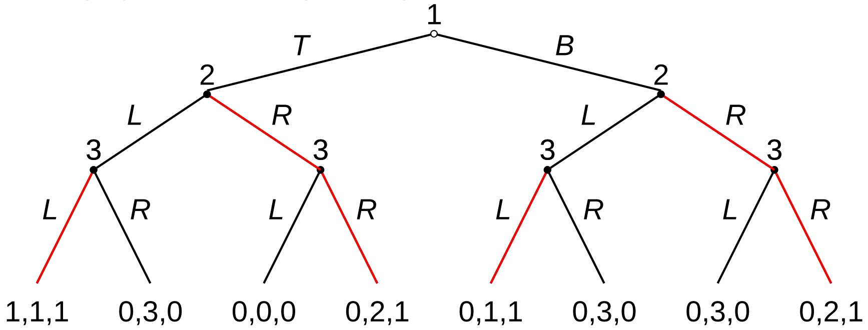

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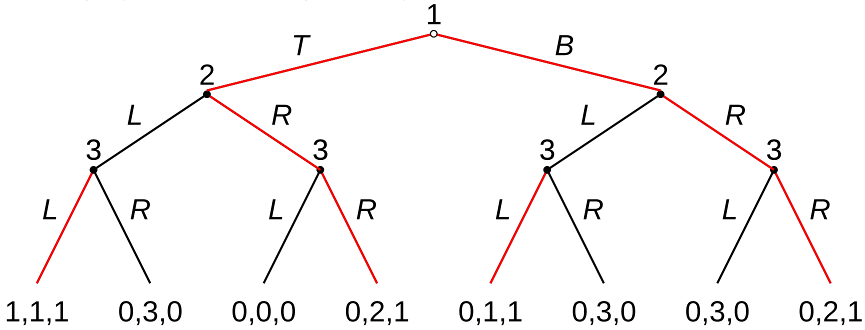

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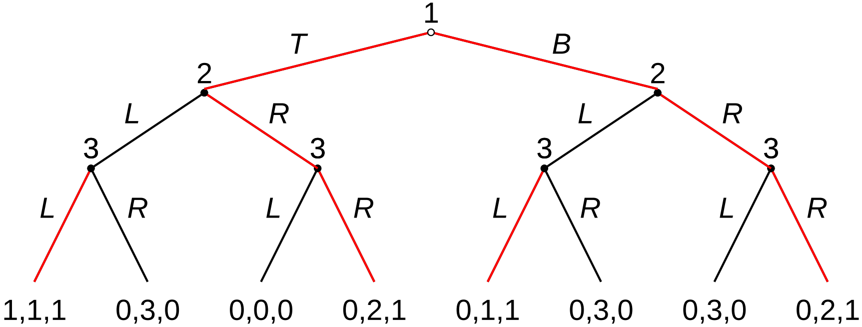
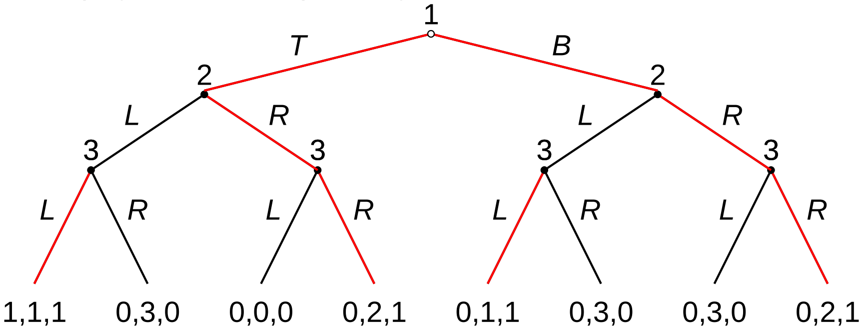


- Game has two SPEs, $(T, RR, LRLR)$ and $(B, RR, LRLR)$, both with payoffs $(0, 2, 1)$

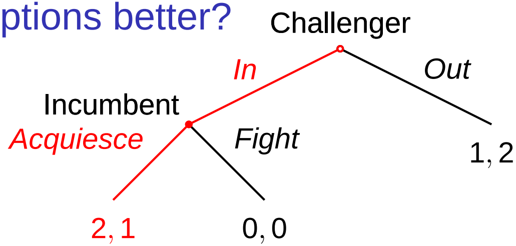
Stackelberg games

- Consider extensive game with perfect information in which players move sequentially



- Game has two SPEs, $(T, RR, LRLR)$ and $(B, RR, LRLR)$, both with payoffs $(0, 2, 1)$
- So player 1 is worse off as first-mover in extensive game than she is in Nash equilibrium of strategic game

Are more options better?



- Unique subgame perfect equilibrium, (*In*, *Acquiesce*)

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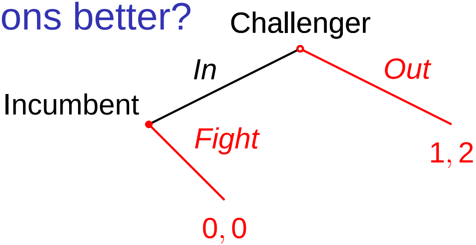

- Unique subgame perfect equilibrium, (*In*, *Acquiesce*)
- If Incumbent's only option is *Fight*, then unique subgame perfect equilibrium (*Out*, *Fight*)

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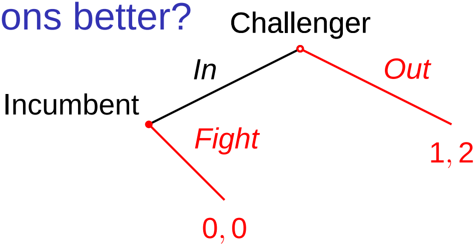
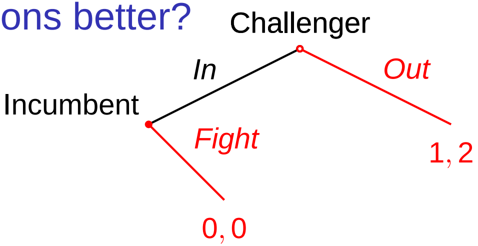


- Unique subgame perfect equilibrium, (*In*, *Acquiesce*)
- If Incumbent's only option is *Fight*, then unique subgame perfect equilibrium (*Out*, *Fight*)
- Incumbent is *better off* in this equilibrium than in the equilibrium of the original game

Are more options better?



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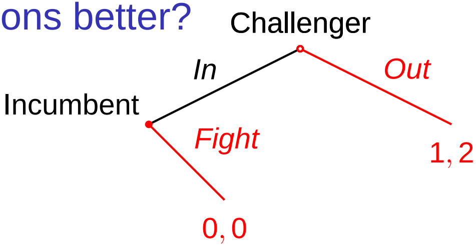

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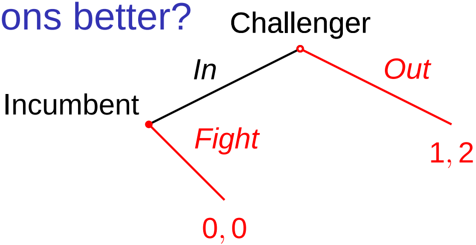

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- Sun Tzu's advice in *The Art of Warfare* (written between 500 BC and 300 BC): "in surrounding the enemy, leave him a way out; do not press an enemy that is cornered"

Ultimatum game

- Two players: proposer and responder

Ultimatum game

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- Pie of size c

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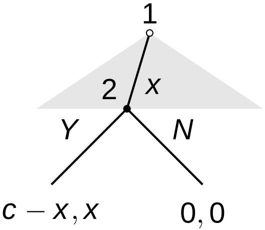
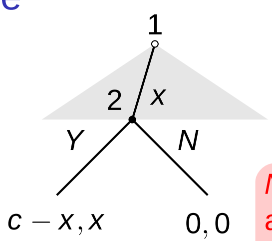
Ultimatum game



$0 \leq x \leq c$
$(c > 0)$

Ultimatum game



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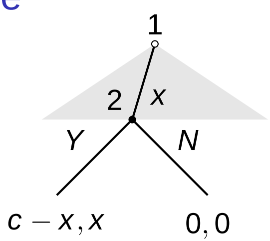
Note: Only *one* representative action of player 1 and *one* subgame of length one are indicated

Ultimatum game



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Extensive game

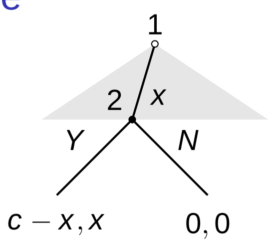
Players $\{1, 2\}$

Terminal histories

Player function

Payoffs

Ultimatum game



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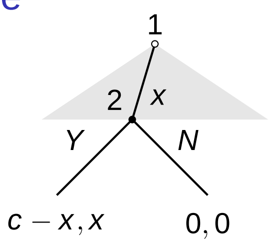

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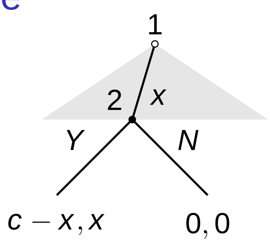

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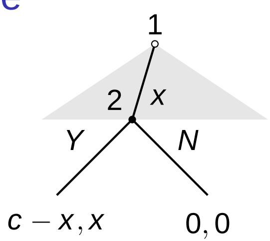
Payoffs $u_1(x, Y) = c - x$, $u_2(x, Y) = x$ for all x, and $u_1(x, N) = u_2(x, N) = 0$

Ultimatum game



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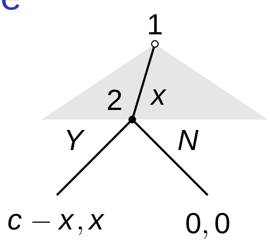
Strategies

Player 1

Player 2

Ultimatum game



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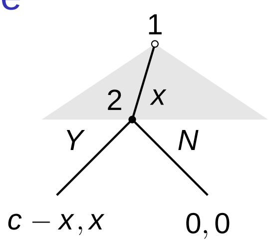

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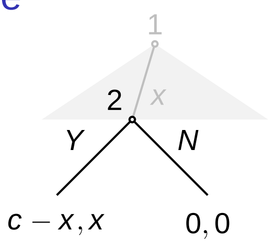
Player 1 $[0, c]$

Player 2 Functions $s_2 : [0, c] \to \{Y, N\}$

Ultimatum game



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Backward induction

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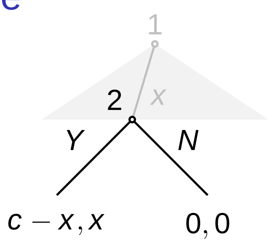
Ultimatum game



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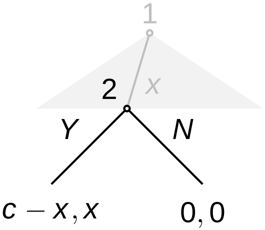
Ultimatum game



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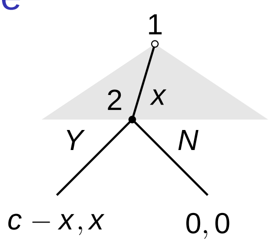

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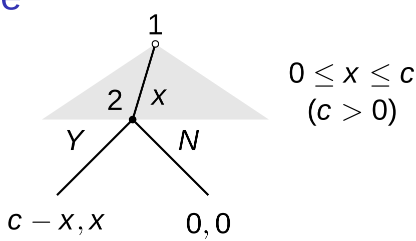

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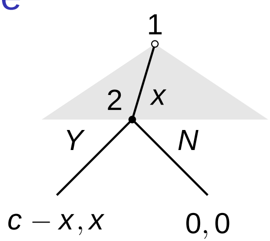

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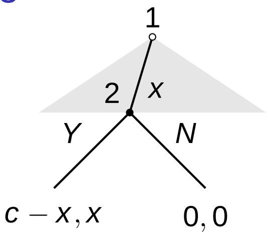

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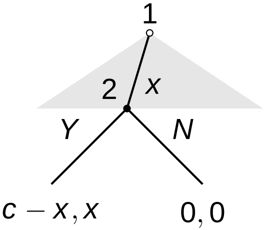

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Subgame perfect equilibria

Hence *unique* subgame perfect equilibrium: $s_1 = 0$ and $s_2(x) = Y$ for all x

Ultimatum game: Experimental evidence

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- Many other experiments yield similar results
- If stakes are high, some evidence that proposers offer lower fraction of pie and fewer offers are rejected

E.g. Andersen et al., *American Economic Review* 101 (2011), 3427–3439

Ultimatum game: Experimental evidence

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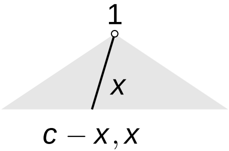

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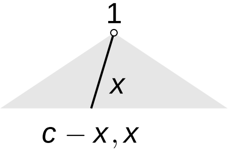

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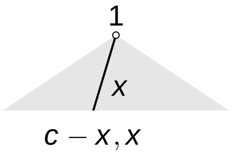

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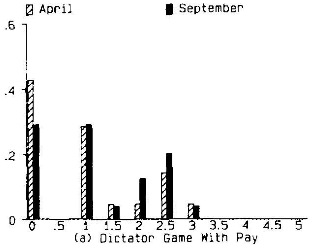
- Unique subgame perfect equilibrium: player 1 offers 0
- If non-zero offers in ultimatum game are result of subjects' concern for fairness, should get similar outcomes in dictator game

Ultimatum game: Experimental evidence

Dictator game

Subjects: students at University of Iowa
Pie size: $5



- Dictators offer less than proposers in ultimatum game, but still offer significant positive amounts

Source: Forsythe et al., *Games and Economic Behavior* 6 (1994), 347–369. See also Bolton et al., *International Journal of Game Theory* 27 (1998), 269–299 and Eckel et al., *Journal of Economic Behavior and Organization* 80 (2011), 603–612.

Ultimatum game: Experimental evidence

Pie size: one or two days' wages

Group	Country	Avg. offer	Rejection rate
Machiguenga	Perú	26%	5%
Torguud	Mongolia	35%	5%
Tsimané	Bolivia	37%	0%
Sangu	Tanzania	41%	10%
Lamalera	Indonesia	58%	0%

Source: Henrich et al., *American Economic Review, Papers and Proceedings* 91 (2001), 73–78

Ultimatum game: Experimental evidence

Another hypothesis

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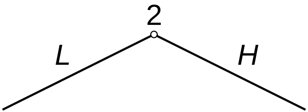
- Why do responders reject offers?
- They may fail to comprehend fully the isolated nature of the interaction, and instead follow their instinct, which is shaped by the long-term relationships to which they are accustomed
- In a long-term relationship, "punishing" a proposer who makes a low offer by rejecting it may have benefit of discouraging low offers in the future

Holdup game

- Before playing ultimatum game, responder decides whether to expend low effort (L) or high effort (H)



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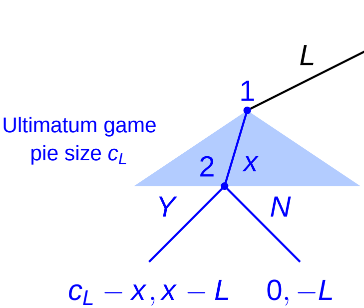

Holdup game

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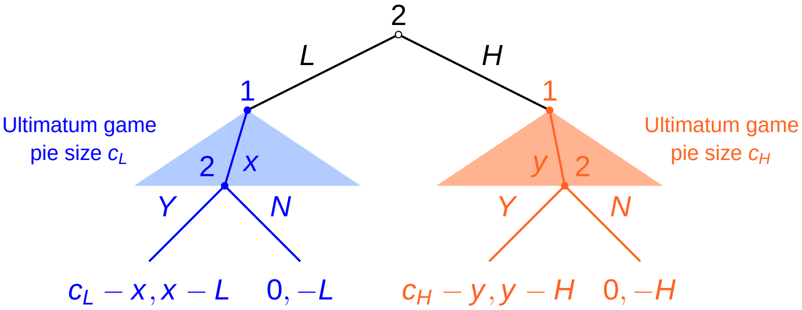
- More effort is more costly, but produces bigger pie: $H > L$ and $c_H > c_L$



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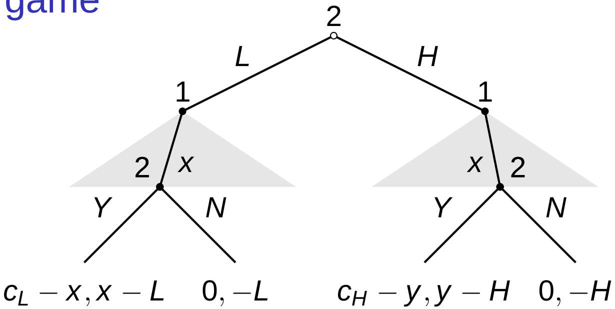
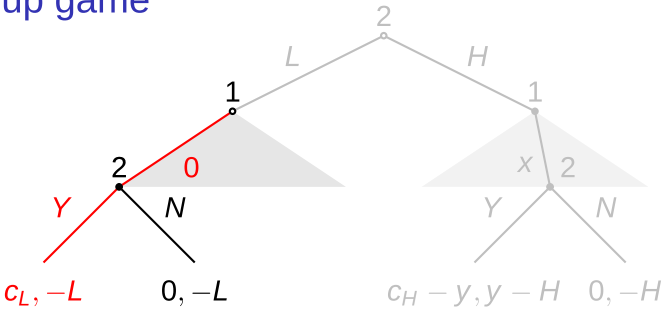


Analysis

- SPE:

Holdup game



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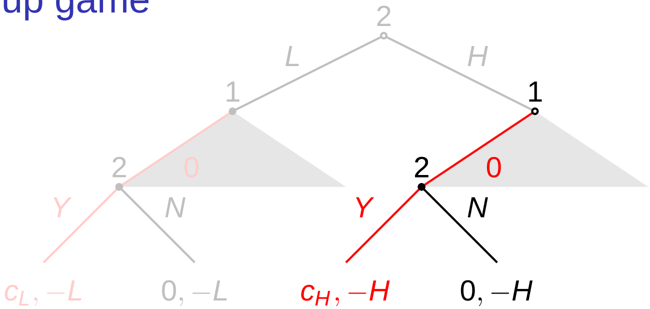

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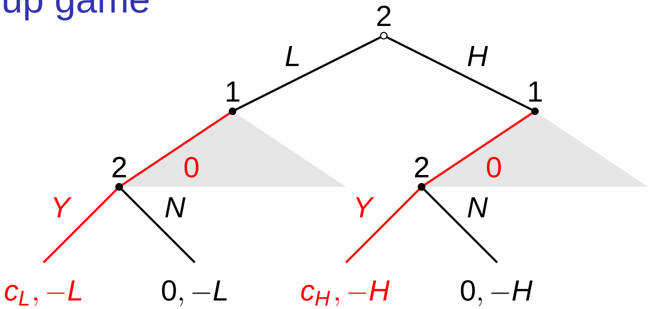

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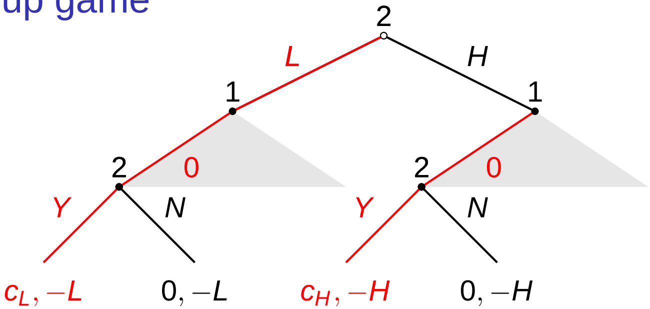
Holdup game



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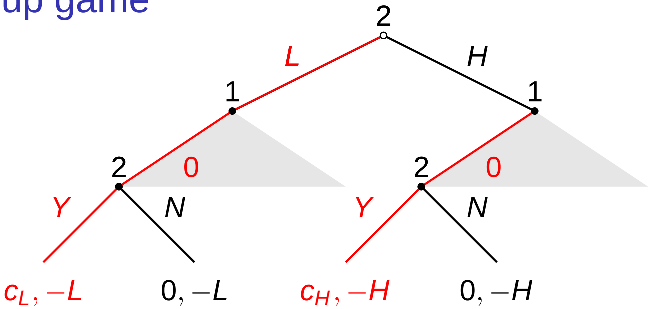

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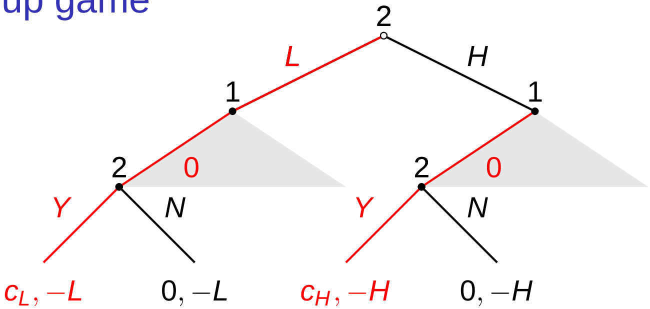

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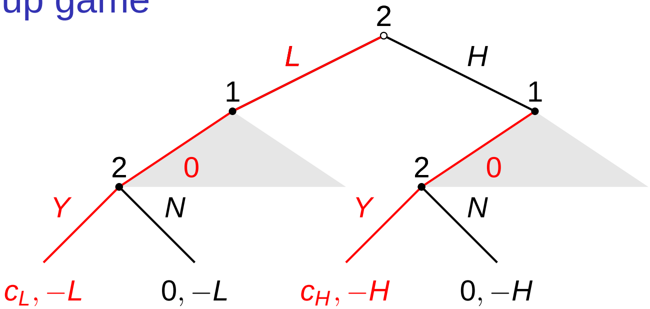

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- Even with less extreme outcome of bargaining, SPE outcome may still be inefficient

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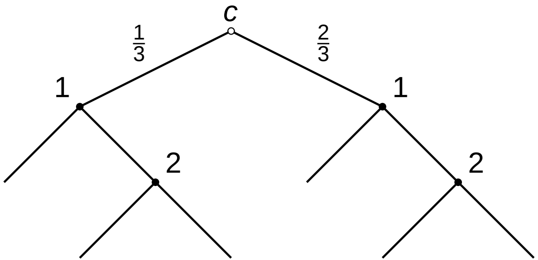

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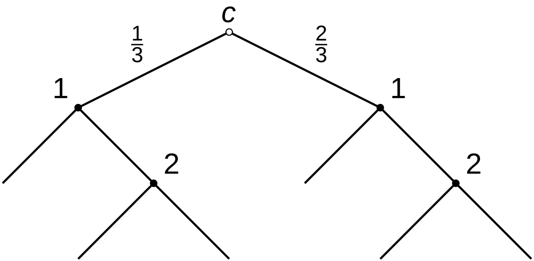

Proposition

Every finite extensive game with perfect information and chance moves has a subgame perfect equilibrium. In a game with a finite horizon the set of strategy profiles satisfying the one-deviation property is the set of subgame perfect equilibria.

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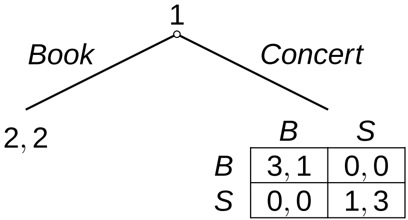
Strategic game Game has single history $\varnothing$, and $P(\varnothing) = N$ (all players move simultaneously at the start of the game)

Simultaneous moves: example



Histories $\{\varnothing, Concert, (Concert, (B, B)),$
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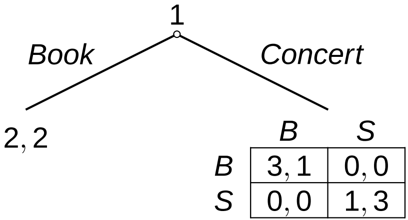
Player function

Simultaneous moves: example



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Player function $P(\varnothing) = 1$, $P(Concert) = \{1, 2\}$

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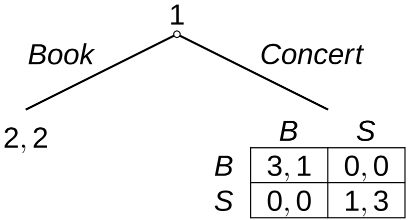

Subgame perfect equilibrium

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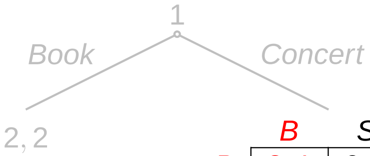

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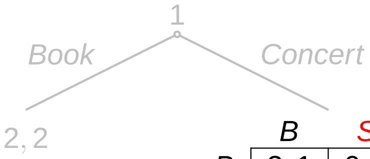

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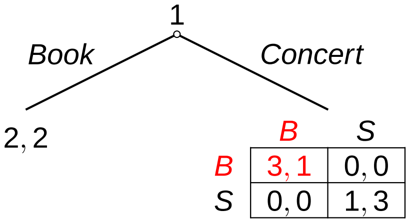

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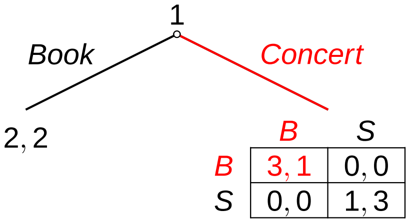

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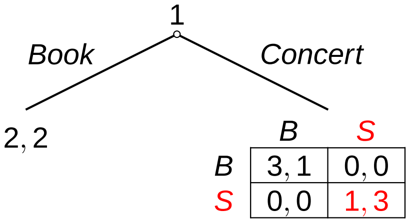

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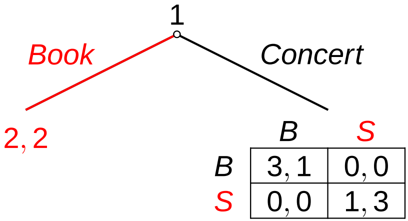

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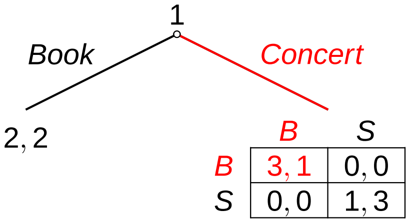

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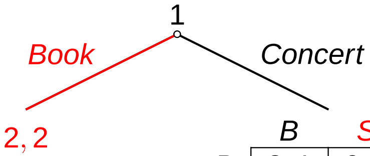

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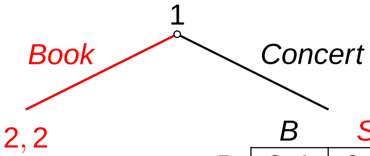

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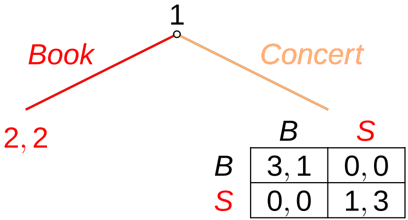

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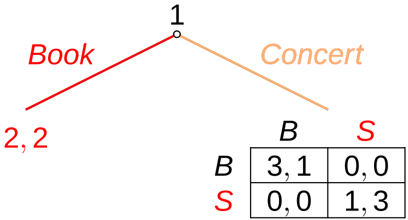

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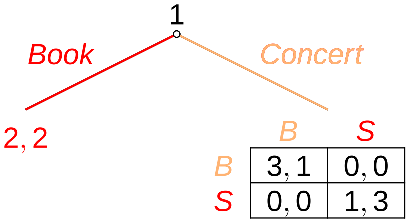

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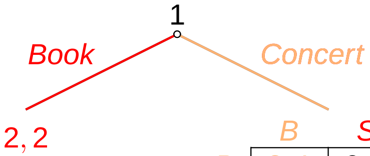

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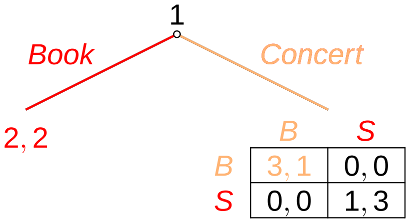

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- P1 can thus reason that deviating from *Book* to *Concert* will increase her payoff

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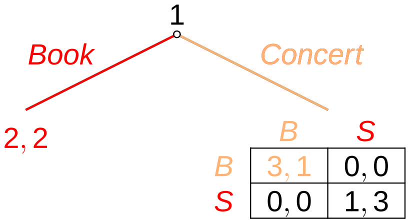

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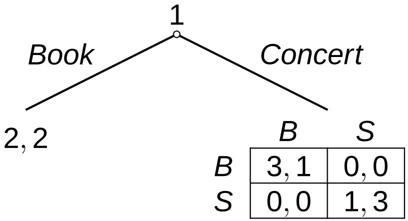

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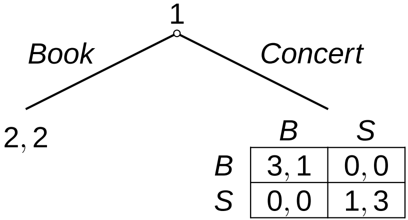

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- Game has two SPEs, $((Concert, B), B)$ and $((Book, S), S)$

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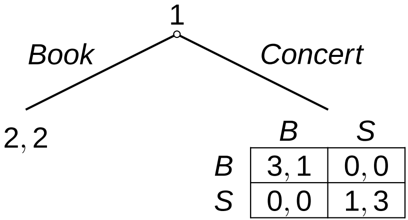

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- SPE $((Book, S), S)$ appears to be not robust to forward induction

Extensive games with perfect information and simultaneous moves

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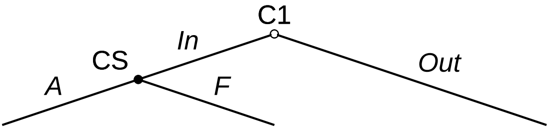

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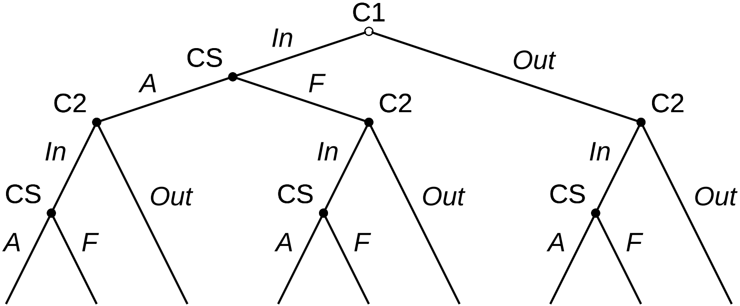

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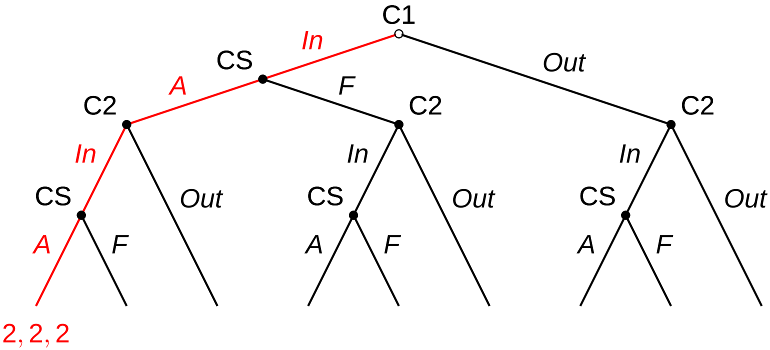

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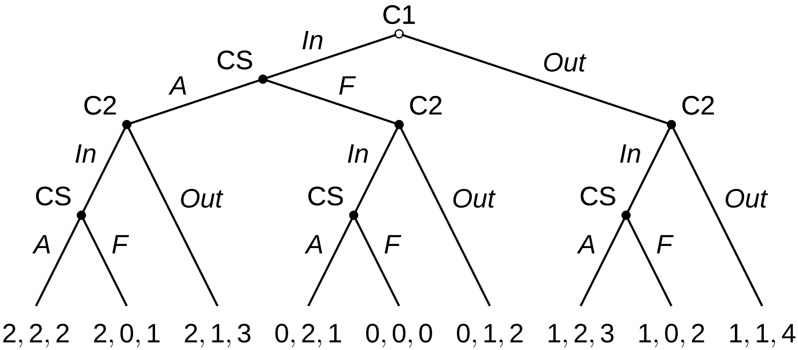

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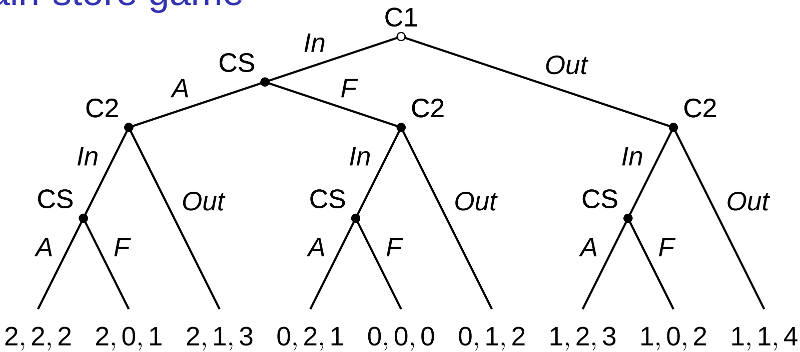

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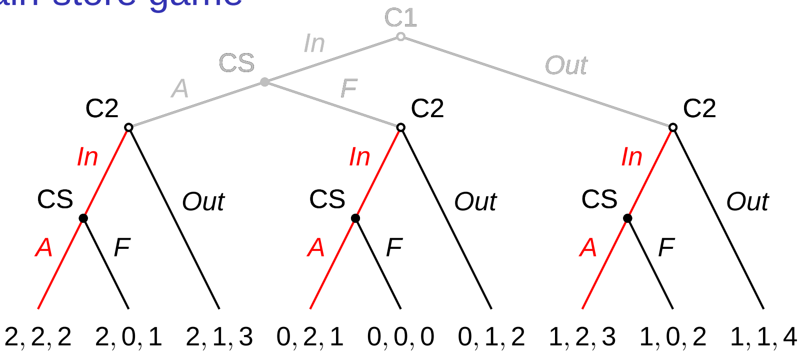

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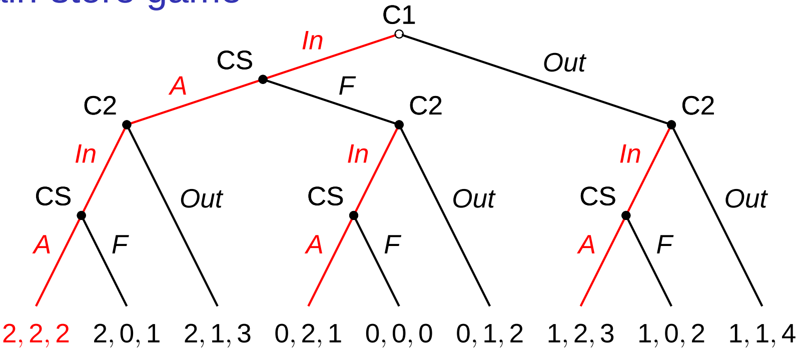

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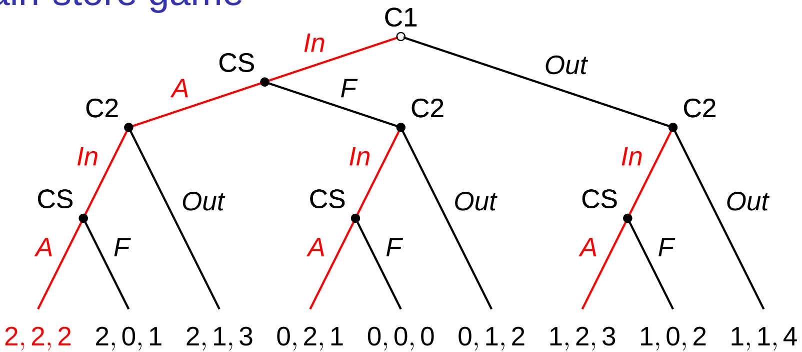

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⇒ unique SPE: All challengers choose *In*, CS always chooses *A*

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- Can chain-store's aggressive behavior in early markets establish for it a reputation for being a fighter?

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