Economics 2030

Fall 2018 — Martin J. Osborne

Problem Set 6

1. Find the subgame perfect equilibria of the game in Figure 1.

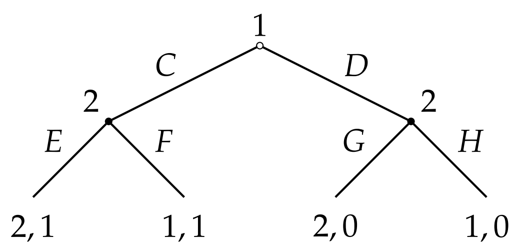

Figure 1. The game in Problem 1.

2. Consider a finite extensive game with perfect information in which no player is indifferent between any pair of terminal histories. Is it possible for such a game to have a Nash equilibrium in which *every* player is better off than she is in the subgame perfect equilibrium? Either provide an example to show that this is possible, or prove that it is not possible.

3. Stackelberg's duopoly game is a sequential variant of Cournot's in which first firm 1 chooses an output, then firm 2 chooses an output. Each unit of output is sold at the price $P(q_1 + q_2)$, where q_1 and q_2 are the outputs chosen by the firms. Consider an example of this game in which the inverse demand function is given by $P(Q) = \alpha - Q$ for all $Q \leq \alpha$ (with $P(Q) = 0$ for $Q > \alpha$), firm 1's cost function is $C_1(q_1) = q_1$, firm 2's cost function is $C_2(q_2) = (q_2)^2$, and $\alpha > \frac{4}{3}$.

   (a) Find the subgame perfect equilibrium (equilibria?) of the game. Specify both the equilibrium **strategies** and the equilibrium outcome.

   (b) Does the game have any Nash equilibrium that yields an outcome different from the subgame perfect equilibrium outcome? If so, specify such an equilibrium. If not, argue that none exists.

4. A firm's output is $L(100 - L)$ when it uses $L \leq 50$ units of labor, and 2500 when it uses $L > 50$ units of labor. The price of output is 1. A union that represents workers presents a wage demand (a nonnegative number w), which the firm either accepts or rejects. If the firm accepts the demand, it chooses the number L of workers to employ (which you should take to be a continuous variable, not an integer); if it rejects the demand, no production takes place ($L = 0$). The firm's preferences are represented by its profit; the union's preferences are represented by the value of wL.

   (a) Formulate this situation as an extensive game with perfect information.

   (b) Find the subgame perfect equilibrium (equilibria?) of the game.

   (c) Is there a feasible outcome of the game that both parties prefer to all subgame perfect equilibrium outcomes?

   (d) Find a Nash equilibrium for which the outcome differs from any subgame perfect equilibrium outcome.

5. An indivisible object is to be sold in an auction. There are two potential buyers, who bid *sequentially* (not simultaneously). Bidder 1 has valuation v_1 and bidder 2 has valuation v_2, where $v_1 > v_2 + 1$ and v_1 and v_2 are nonnegative integers. A bid can be any nonnegative *integer*. First bidder 1 announces a bid. Then bidder 2 either announces a higher bid, or quits; if she announces a higher bid, then bidder 1 either announces a higher bid or quits; and so on until a bidder quits. The bidder who remains (does not quit) obtains the object and pays the price she bid. For every infinite history, both players' payoffs are zero.

   (a) Find a subgame perfect equilibrium of the extensive game with perfect information that models this situation. Specify a complete *strategy* for each player and show that the strategy pair is a subgame perfect equilibrium. To show that the strategy pair is a subgame perfect equilibrium, you may use the fact that a strategy pair in the game is a subgame perfect equilibrium if and only if it satisfies the one deviation property (even though the game does not have a finite horizon).

   (b) Consider the variant of the game in which player 2 (rather than player 1) submits the first bid. Does this extensive game have a *Nash* equilibrium in which player 2 obtains the object? Either specify such an equilibrium or argue that no such equilibrium exists.