ECO2030: Microeconomic Theory II, module 1

Lecture 6

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Table of contents

Extensive games with perfect information

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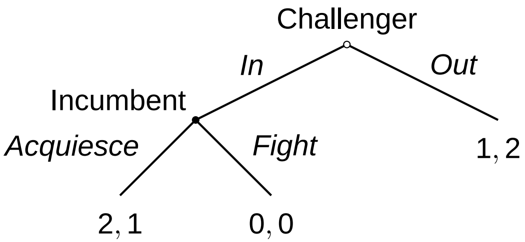
- Actions in strategic game can capture behavior that will unfold over time, but strategic game does not allow reevaluation of choices
- Model that explicitly captures sequential choices: *extensive game*

Example: entry game



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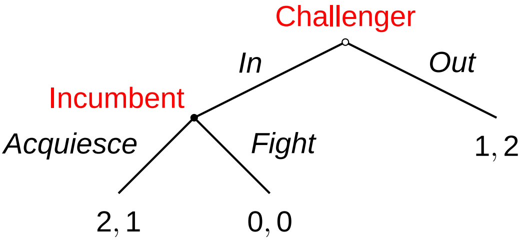

- Two players, Challenger and Incumbent

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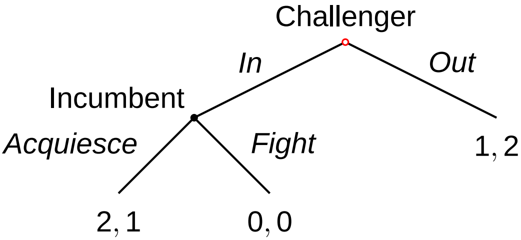

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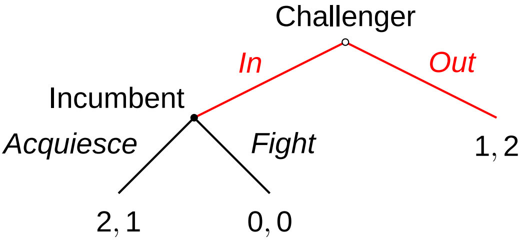

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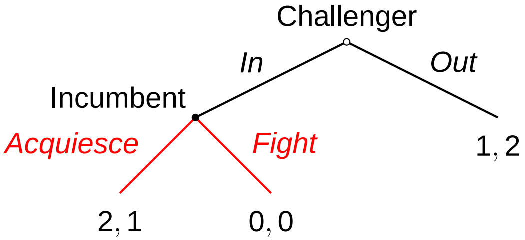

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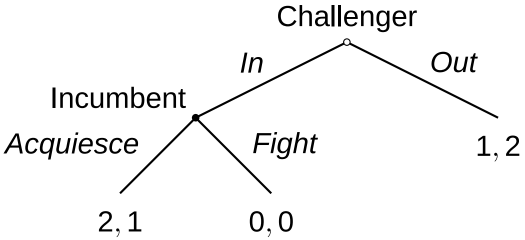

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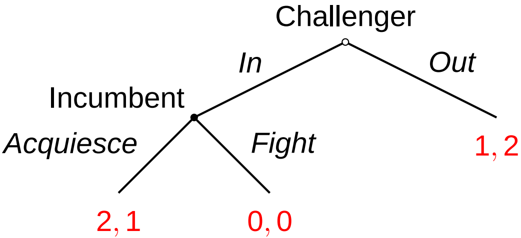

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- First, Challenger chooses *In* or *Out*
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- Payoffs are numbers at bottom (challenger's payoff first)

Extensive games with perfect information

Defined by

- set of players

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- set of possible sequences of actions—*histories*

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- players' preferences over outcomes

Histories

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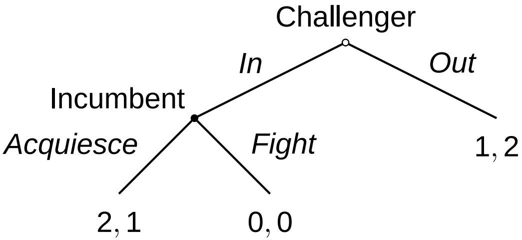

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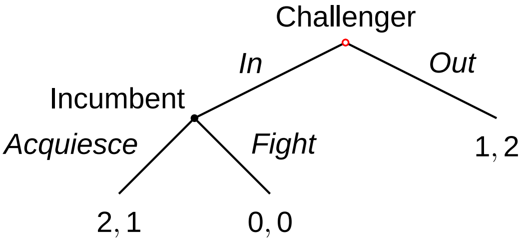

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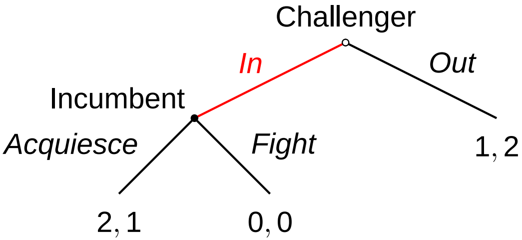

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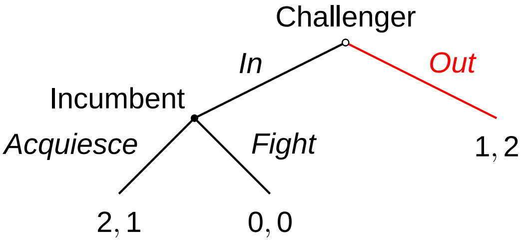

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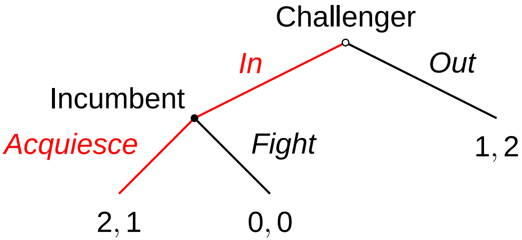

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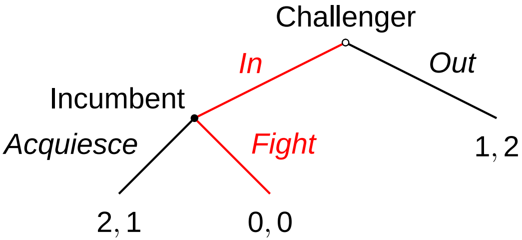

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For set of histories H, denote set of terminal histories $Z(H)$

Extensive games with perfect information

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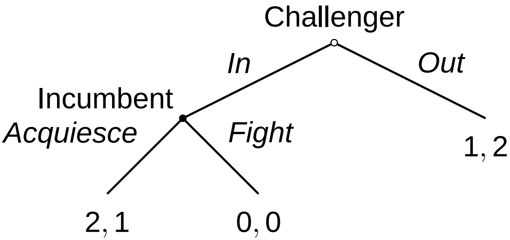
An extensive game with perfect information consists of

- a set N (the set of *players*)
- a set H of histories
- a function $P : H \setminus Z(H) \rightarrow N$ (the *player function*, specifying the player who moves after each nonterminal history)
- for each player $i \in N$, a preference relation over $Z(H)$

Example



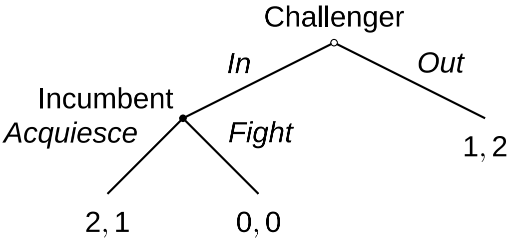
Players $N = \{\text{Challenger}, \text{Incumbent}\}$

Histories

Player function

Preferences

Example



Players $N = \{\text{Challenger}, \text{Incumbent}\}$

Histories $H = \{\varnothing, In, Out, (In, Fight), (In, Acquiesce)\}$

Player function

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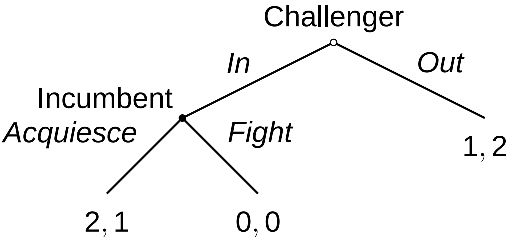

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Histories $H = \{\varnothing, \mathit{In}, \mathit{Out}, (\mathit{In}, \mathit{Fight}), (\mathit{In}, \mathit{Acquiesce})\}$

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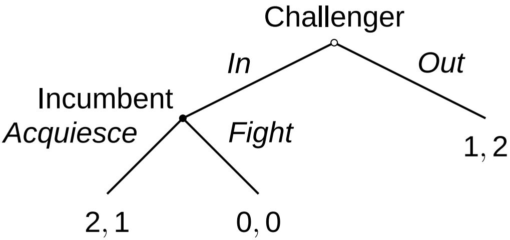
Preferences

Example



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Preferences $(In, Acquiesce) \succ_C Out \succ_C (In, Fight)$,
$Out \succ_I (In, Acquiesce) \succ_I (In, Fight)$

Actions

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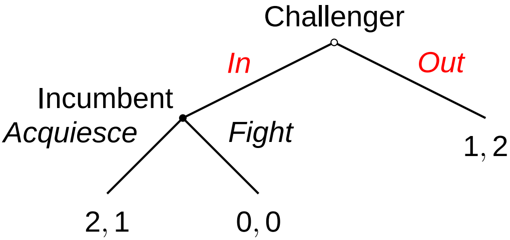
- More precisely, set of actions available to player who moves after history h is

$$A(h) = \{a : (h, a) \in H\}.$$

Example



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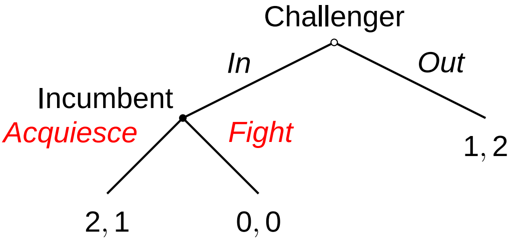

- $A(\varnothing) = \{a : a \in H\} = \{In, Out\}$
- $A(In) = \{a : (In, a) \in H\} = \{Acquiesce, Fight\}$

Finite games

Finite horizon game

Game has finite horizon if every history is finite

Finite game

Game is finite if number of histories is finite

Strategy

Key concept!

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Definition

A strategy of player i in an extensive game with perfect information $\langle N, H, P, (\succsim_i)_{i \in N} \rangle$ is a function that assigns an action in $A(h)$ to every nonterminal history $h \in H \setminus Z(H)$ for which $P(h) = i$

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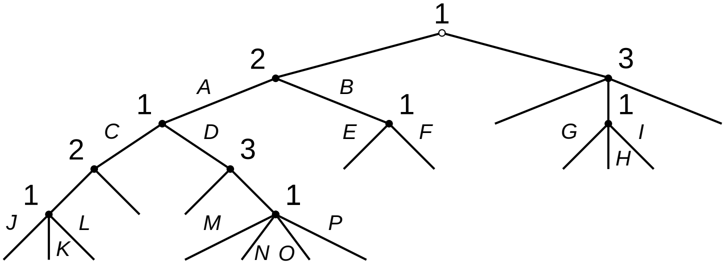
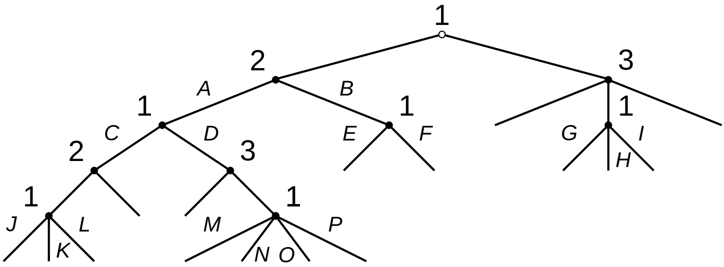
Strategies: example

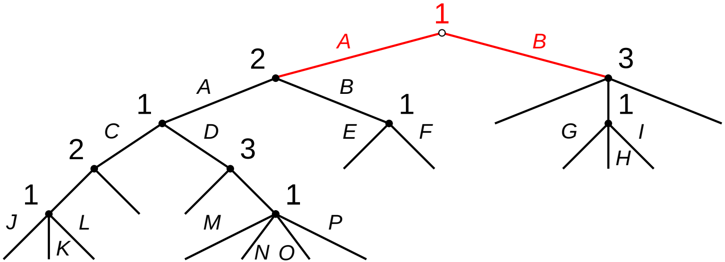


Strategies: example



- Number of strategies of player 1:

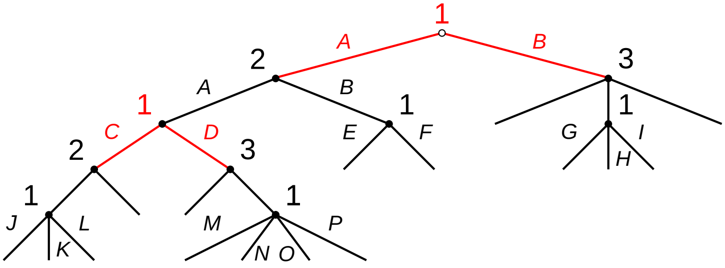
Strategies: example



- Number of strategies of player 1:

$$2 \times$$

Strategies: example



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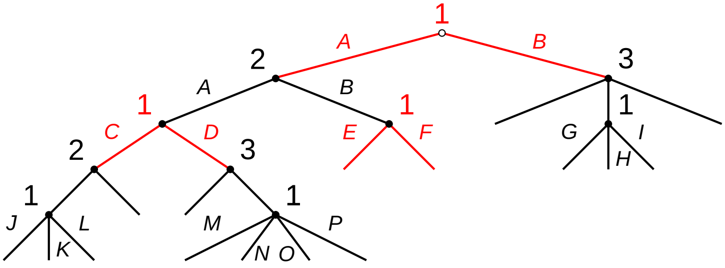

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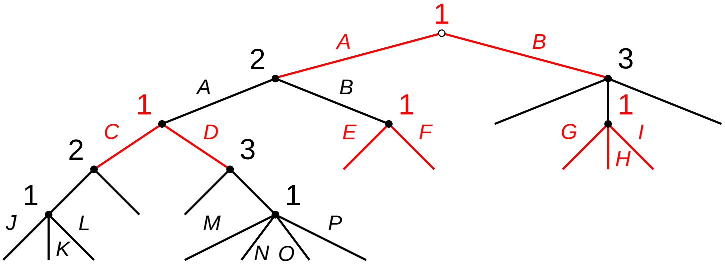

- Number of strategies of player 1:

$$2 \times 2 \times 2 \times 3 \times$$

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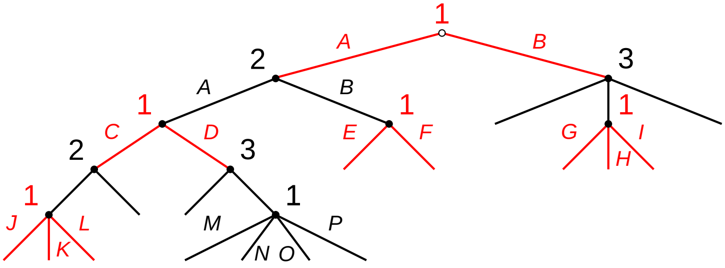

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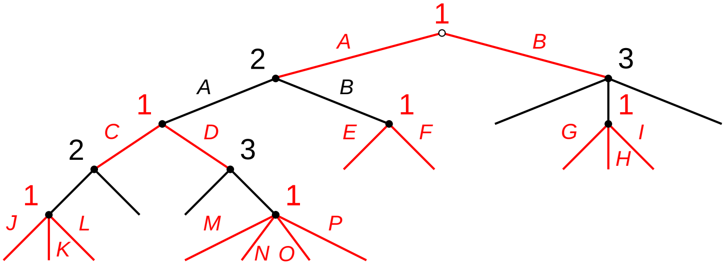
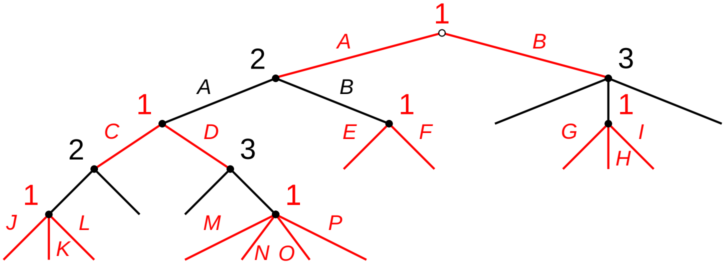


- Number of strategies of player 1:

$$2 \times 2 \times 2 \times 3 \times 3 \times 4$$

Strategies: example



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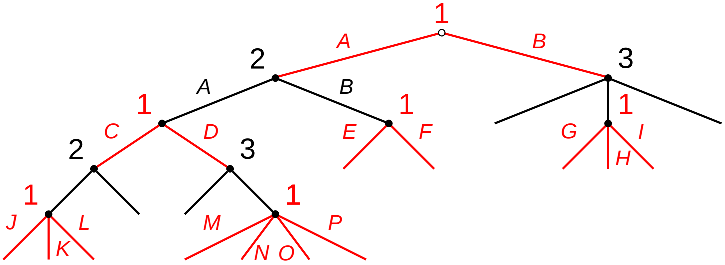

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- *One* strategy: *ACEGJM*

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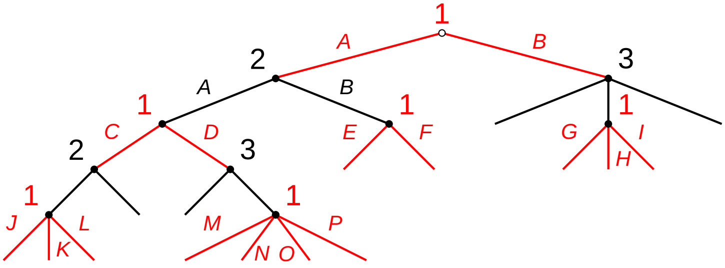

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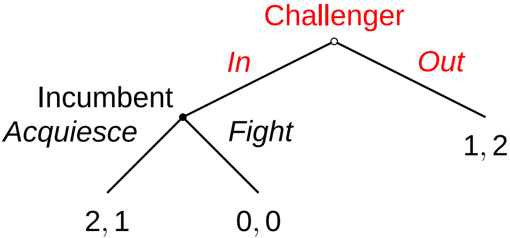
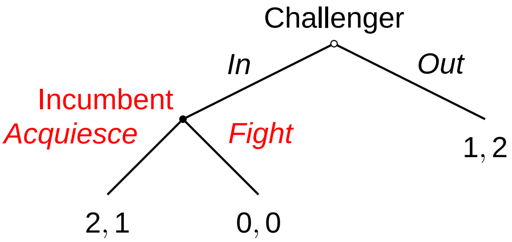
- Let's look at some simpler examples . . .

Strategies: Example



Challenger Moves only after null history. Two actions after this history, so two strategies: *In*, *Out*

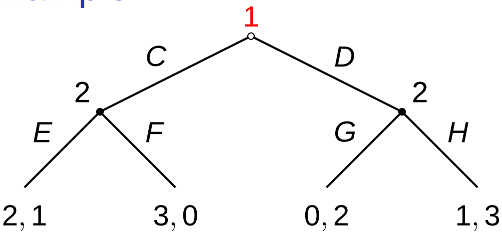
Strategies: Example



Challenger Moves only after null history. Two actions after this history, so two strategies: *In*, *Out*

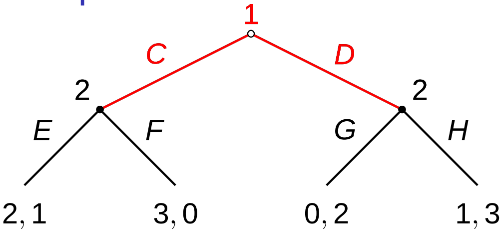
Incumbent Moves only after history *In*. Two actions after this history, so two strategies: *Acquiesce*, *Fight*

Strategies: example



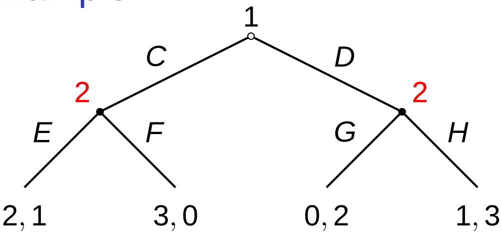
Player 1

Strategies: example



Player 1 Moves only after null history. Two actions after this history, so two strategies: C, D

Strategies: example



Player 2 Moves after two histories:

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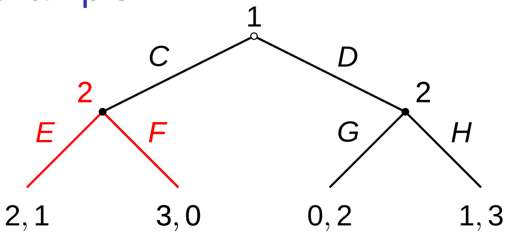

Player 2 Moves after two histories:

C: two actions, *E* and *F*

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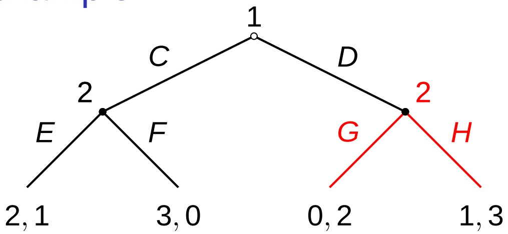
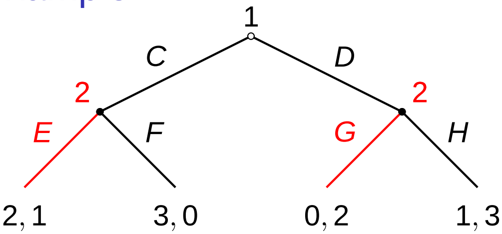


Player 2 Moves after two histories:

- C: two actions, E and F
- D: two actions, G and H

Strategies: example



Player 2 Moves after two histories:

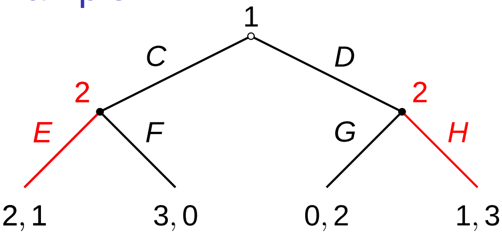
C: two actions, E and F

D: two actions, G and H

Hence four strategies:

- $s_2(C) = E$ and $s_2(D) = G$ (EG for short)

Strategies: example



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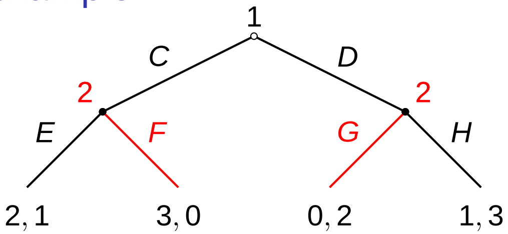

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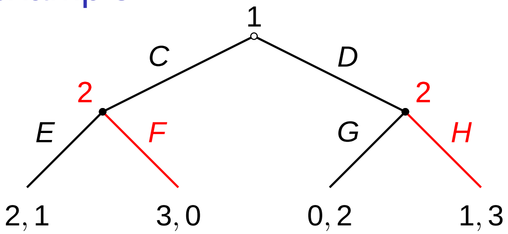
Strategies: example



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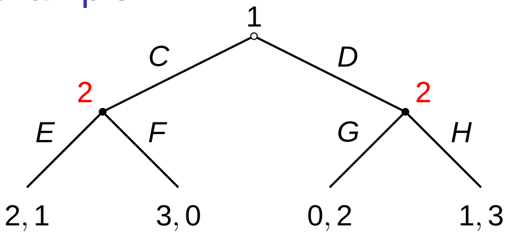
Strategies: example



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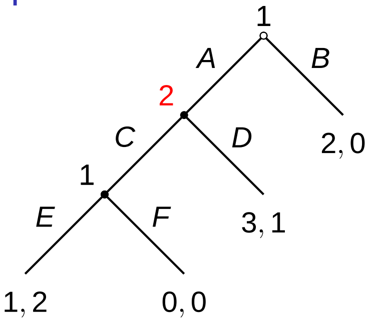
D: two actions, G and H

Hence four strategies:

- $s_2(C) = E$ and $s_2(D) = G$ (EG for short)
- $s_2(C) = E$ and $s_2(D) = H$ (EH for short)
- $s_2(C) = F$ and $s_2(D) = G$ (FG for short)
- $s_2(C) = F$ and $s_2(D) = H$ (FH for short)

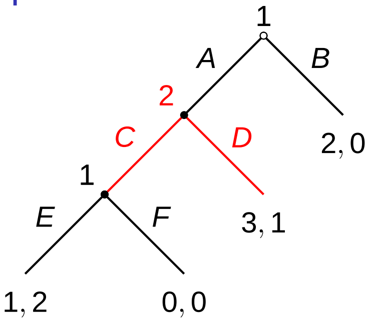
Strategy of player 2 in this game is *plan of action*

Strategies: example



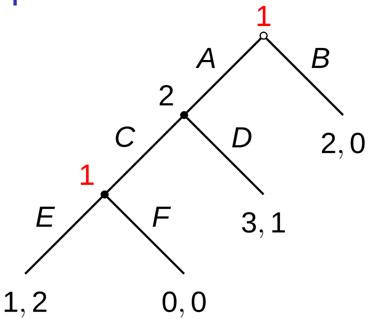
Player 2

Strategies: example



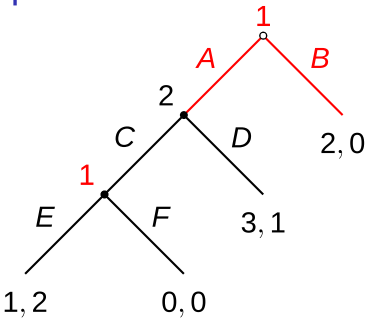
Player 2 Moves after *one* history, A, and has 2 actions, C and D, so 2 strategies: C, D

Strategies: example



Player 1

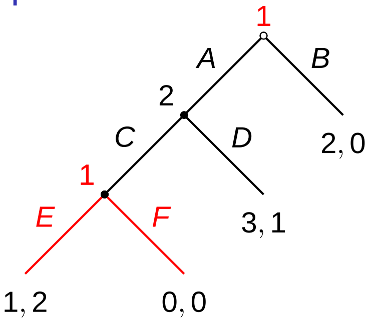
Strategies: example



Player 1 Moves after

- null history: 2 actions, *A* and *B*

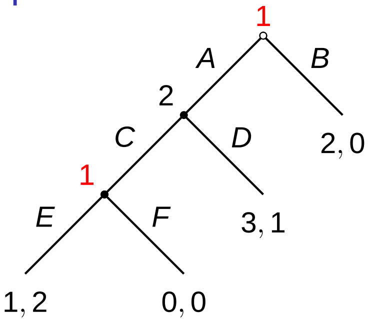
Strategies: example



Player 1 Moves after

- null history: 2 actions, A and B
- history (A, C): 2 actions, E and F

Strategies: example



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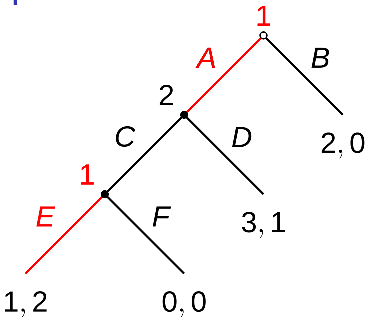
- null history: 2 actions, A and B
- history (A, C): 2 actions, E and F

So 4 strategies:

Strategies: example

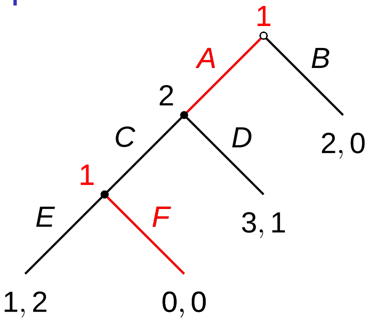


Player 1 Moves after

- null history: 2 actions, A and B
- history (A, C): 2 actions, E and F

So 4 strategies: AE

Strategies: example

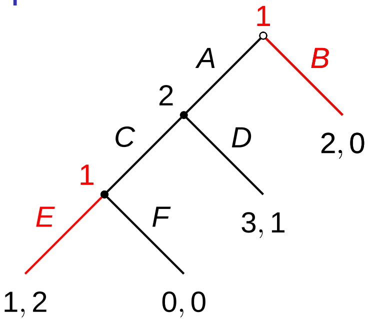


Player 1 Moves after

- null history: 2 actions, A and B
- history (A, C): 2 actions, E and F

So 4 strategies: AE, AF

Strategies: example

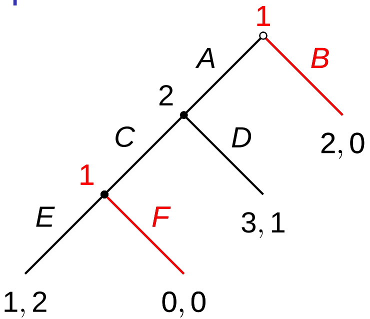


Player 1 Moves after

- null history: 2 actions, A and B
- history (A, C): 2 actions, E and F

So 4 strategies: AE, AF, BE

Strategies: example

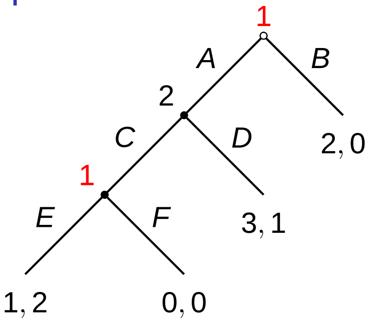


Player 1 Moves after

- null history: 2 actions, A and B
- history (A, C): 2 actions, E and F

So 4 strategies: AE, AF, BE, BF

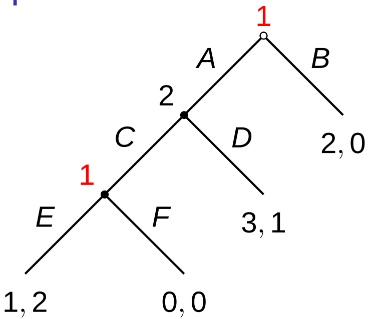
Strategies: example



Note

- Each strategy of player 1 specifies action after history (A, C) *even if it specifies B at beginning of game*!

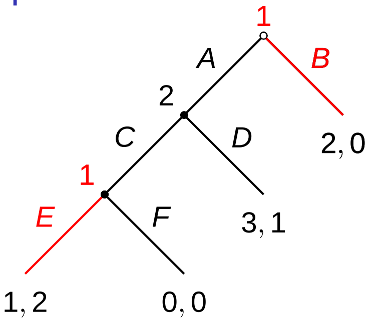
Strategies: example



Note

- Each strategy of player 1 specifies action after history (A, C) *even if it specifies B at beginning of game*!
- In general: definition of strategy requires action to be specified for *every* history after which it is player's turn to move, *even histories not reached if strategy is followed*

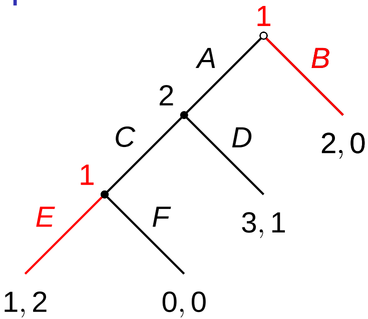
Strategies: example



One interpretation of strategy *BE* of player 1:

1. Action *E* models behavior of player 1 if, by chance, she doesn't choose *B* at start of game (though she intends to)

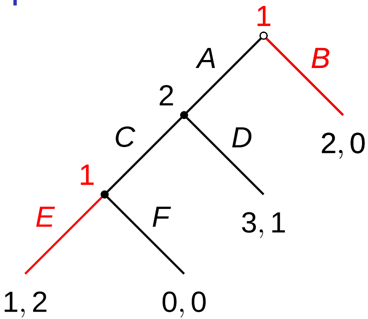
Strategies: example



Another interpretation of strategy *BE* of player 1:

2. When choosing between *A* and *B*,

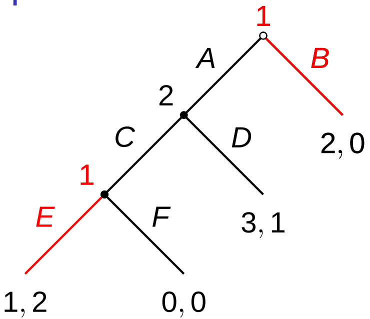
Strategies: example



Another interpretation of strategy *BE* of player 1:

2. When choosing between *A* and *B*,
   - player 1 has to think about action player 2 intends to take

Strategies: example



Another interpretation of strategy BE of player 1:

2. When choosing between A and B,
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   - player 1 knows that player 2's action depends on action player 2 thinks player 1 will take after history (A, C)

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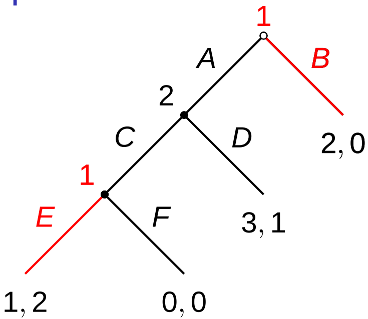

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2. When choosing between A and B,
   - player 1 has to think about action player 2 intends to take
   - player 1 knows that player 2's action depends on action player 2 thinks player 1 will take after history (A, C)

   Component E of player 1's strategy is her belief about player 2's belief about player 1's action after history (A, C)

Strategic form of extensive game

Given any extensive game, can now define strategic game

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- Players: players in extensive game

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Resulting strategic game is strategic form of extensive game

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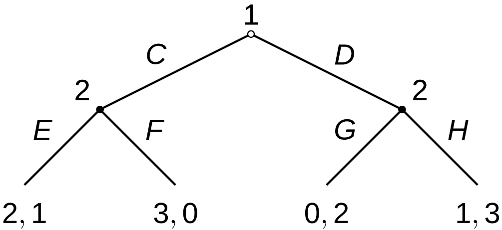
Resulting strategic game is strategic form of extensive game

Extensive game		Strategic form
$\langle N, H, P, (\succsim_i)_{i\in N}\rangle$	$\rightarrow$	$\langle N, (S_i)_{i\in N}, (\succsim_i^*)\rangle$

where S_i is set of strategies of player i in extensive game and $\succsim_i^*$ are i's preferences over strategy profiles induced by $\succsim_i$

Example of strategic form



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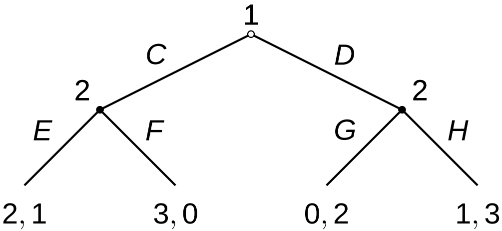
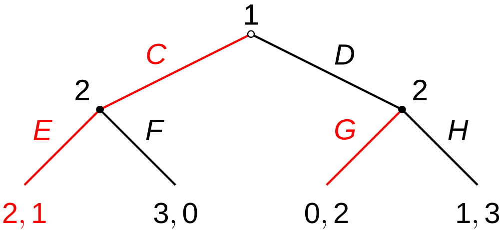


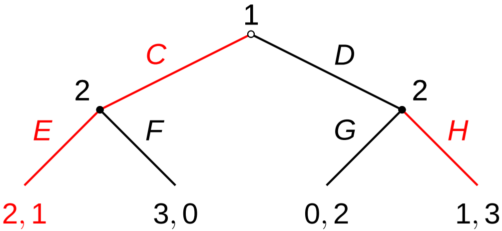
	EG	*EH*	*FG*	*FH*
C				
D				

Example of strategic form



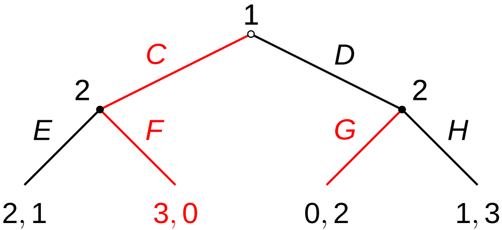
	EG	*EH*	*FG*	*FH*
C	2, 1			
D				

Example of strategic form



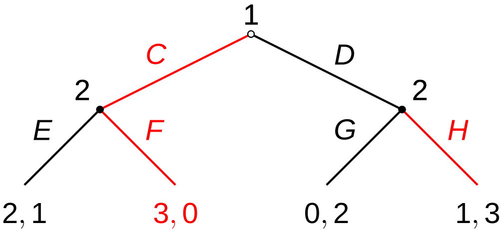
	EG	EH	FG	FH
C	2, 1	2, 1		
D				

Example of strategic form



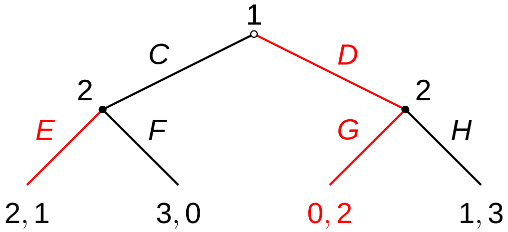
	EG	EH	FG	FH
C	2, 1	2, 1	3, 0	
D				

Example of strategic form



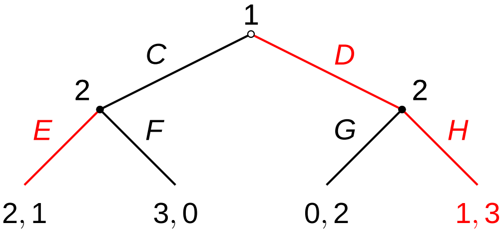
	EG	EH	FG	FH
C	2, 1	2, 1	3, 0	3, 0
D				

Example of strategic form



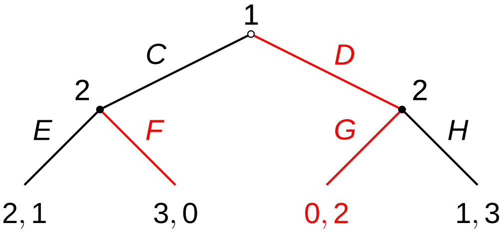
	EG	EH	FG	FH
C	2, 1	2, 1	3, 0	3, 0
D	0, 2			

Example of strategic form



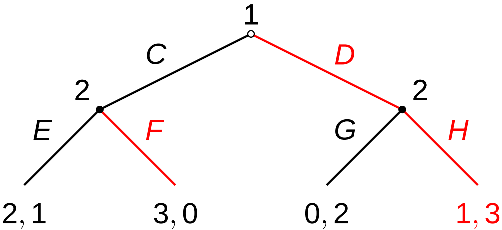
	EG	EH	FG	FH
C	2, 1	2, 1	3, 0	3, 0
D	0, 2	1, 3		

Example of strategic form



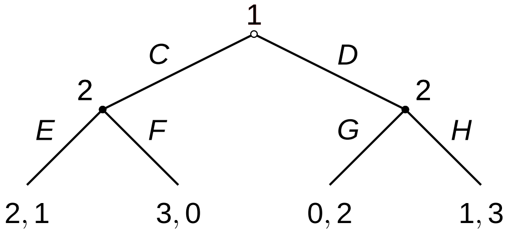
	EG	EH	FG	FH
C	2, 1	2, 1	3, 0	3, 0
D	0, 2	1, 3	0, 2	

Example of strategic form



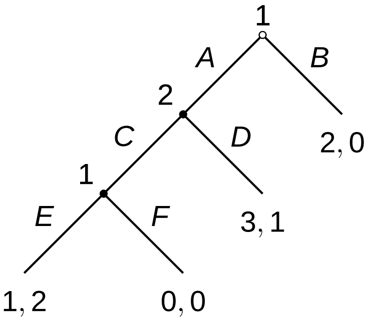
	EG	EH	FG	FH
C	2, 1	2, 1	3, 0	3, 0
D	0, 2	1, 3	0, 2	1, 3

Example of strategic form



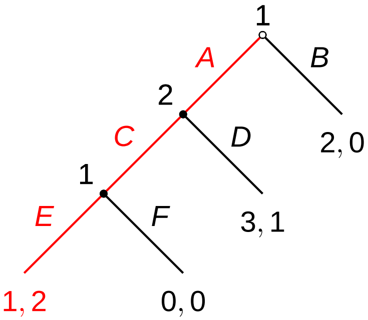
	EG	EH	FG	FH
C	2, 1	2, 1	3, 0	3, 0
D	0, 2	1, 3	0, 2	1, 3

Example of strategic form



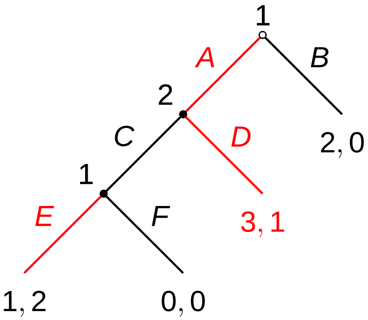
	C	D
AE		
AF		
BE		
BF		

Example of strategic form



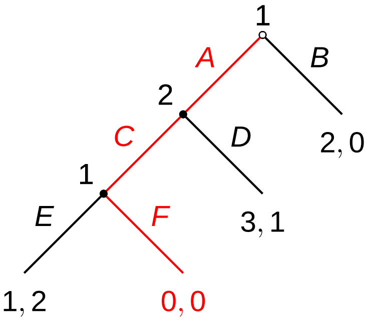
	C	D
AE	1, 2	
AF		
BE		
BF		

Example of strategic form



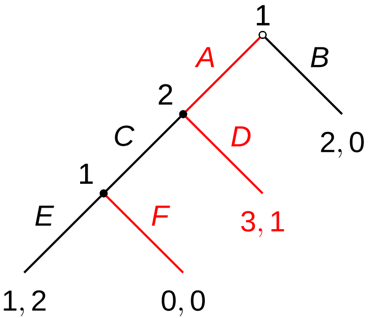
	C	D
AE	1, 2	3, 1
AF		
BE		
BF		

Example of strategic form



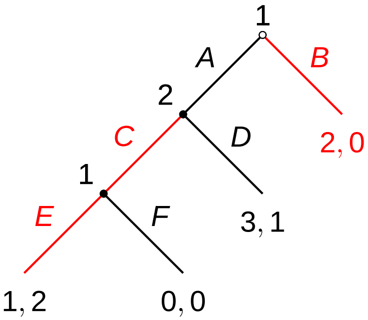
	C	D
AE	1, 2	3, 1
AF	0, 0	
BE		
BF		

Example of strategic form



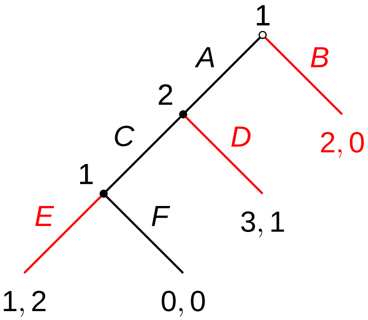
	C	D
AE	1, 2	3, 1
AF	0, 0	3, 1
BE		
BF		

Example of strategic form



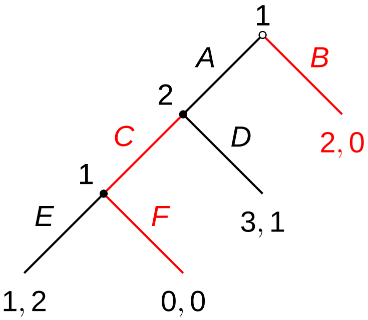
	C	D
AE	1, 2	3, 1
AF	0, 0	3, 1
BE	2, 0	
BF		

Example of strategic form



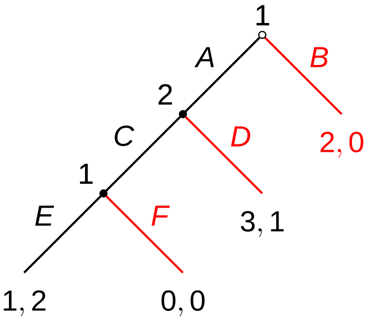
	C	D
AE	1, 2	3, 1
AF	0, 0	3, 1
BE	2, 0	2, 0
BF		

Example of strategic form



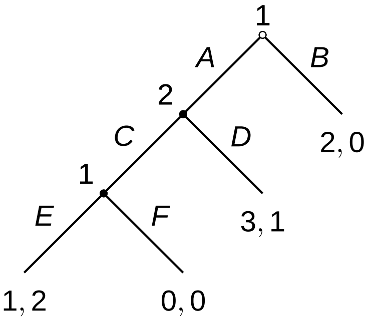
	C	D
AE	1, 2	3, 1
AF	0, 0	3, 1
BE	2, 0	2, 0
BF	2, 0	

Example of strategic form



	C	D
AE	1, 2	3, 1
AF	0, 0	3, 1
BE	2, 0	2, 0
BF	2, 0	2, 0

Example of strategic form



	C	D
AE	1, 2	3, 1
AF	0, 0	3, 1
BE	2, 0	2, 0
BF	2, 0	2, 0

Note duplicate strategies of player 1

Reduced strategic form:

	C	D
AE	1, 2	3, 1
AF	0, 0	3, 1
X	2, 0	2, 0

Nash equilibrium

Definition

A Nash equilibrium of an extensive game with perfect information is a Nash equilibrium of its strategic form

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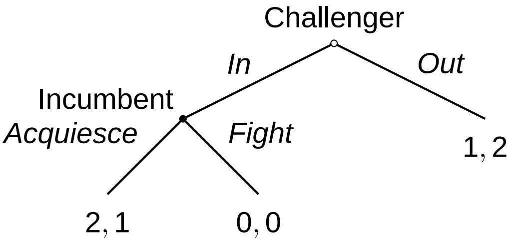
Example



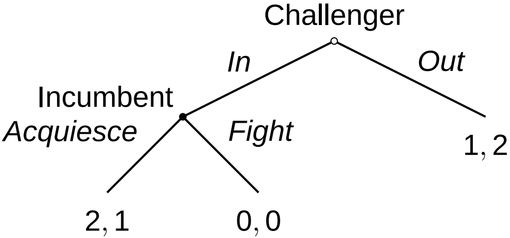
	Acquiesce	*Fight*
In	2, 1	0, 0
Out	1, 2	1, 2

Nash equilibrium

Definition

A Nash equilibrium of an extensive game with perfect information is a Nash equilibrium of its strategic form

Example



	Acquiesce	*Fight*
In	2, 1	0, 0
Out	1, 2	1, 2

Nash equilibria:

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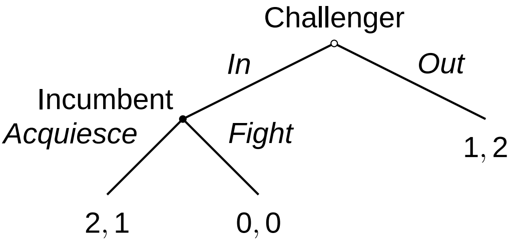
Definition

A Nash equilibrium of an extensive game with perfect information is a Nash equilibrium of its strategic form

Example



	Acquiesce	*Fight*
In	2, 1	0, 0
Out	1, 2	1, 2

Nash equilibria: (*In*, *Acquiesce*)

Nash equilibrium

Definition

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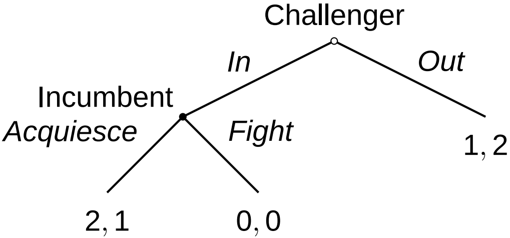
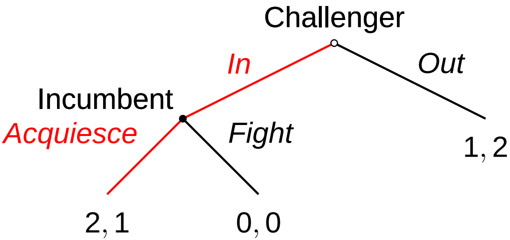
Example



	Acquiesce	*Fight*
In	2, 1	0, 0
Out	1, 2	1, 2

Nash equilibria: (*In*, *Acquiesce*) and (*Out*, *Fight*)

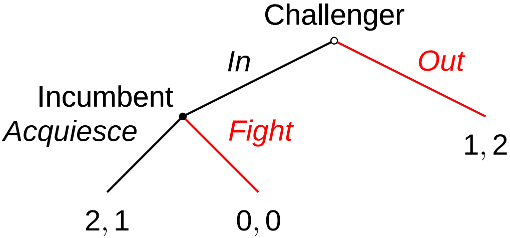
Nash equilibrium: example



Nash equilibria

$(In, Acquiesce)$ Both actions played in equilibrium; each is optimal when played

Nash equilibrium: example



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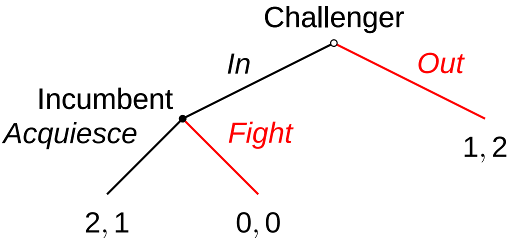

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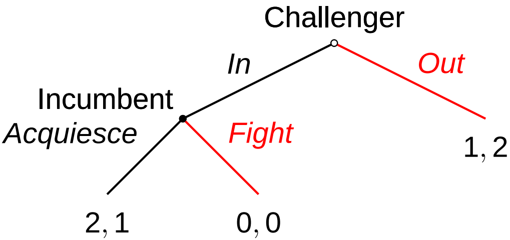

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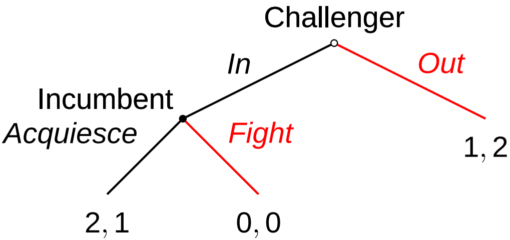

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  - *Fight* can be interpreted as *non-credible threat*

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- But a player's Nash equilibrium strategy may not be optimal in subgames not reached if players follow their strategies
- Notion of *subgame perfect equilibrium* requires that each player's strategy be optimal after *every* history, even histories that do not occur if every player follows her strategy

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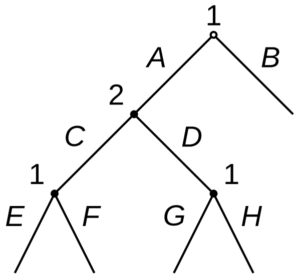

Subgame following $\varnothing$ (whole game)

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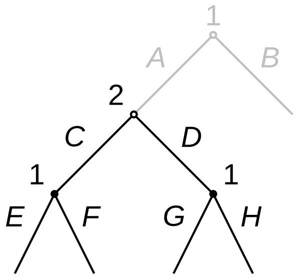

Subgame following A

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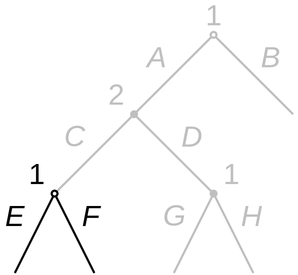

Subgame following (A, C)

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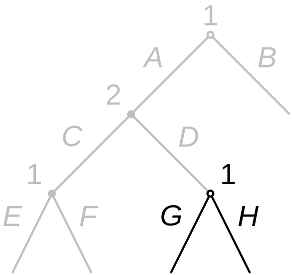

Subgame following (A, D)

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- Every subgame perfect equilibrium is a Nash equilibrium
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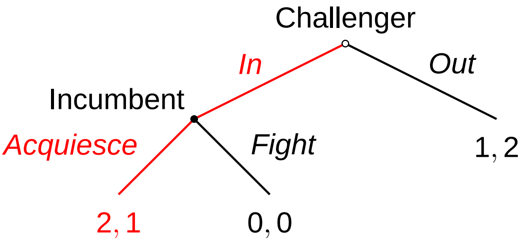
Example: entry game



$(In, Acquiesce)$ Subgame perfect equilibrium:

- *In* optimal at start of game, given Incumbent's strategy

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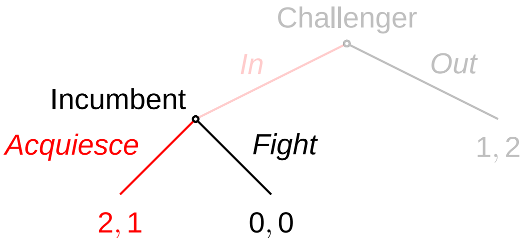

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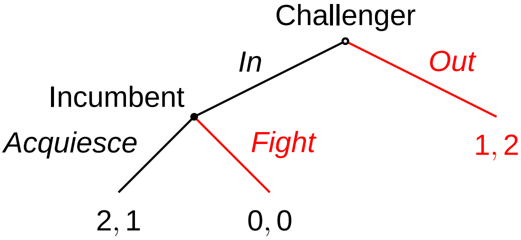

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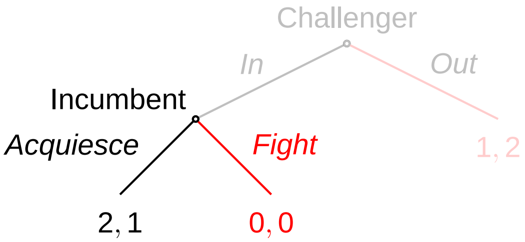

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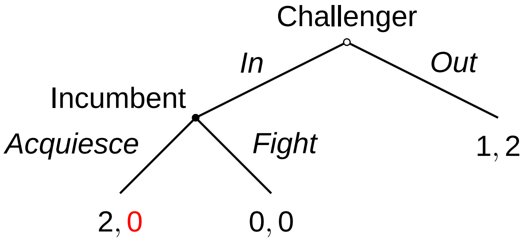

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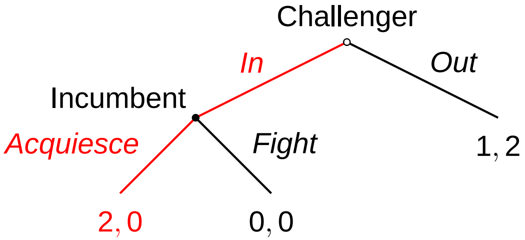

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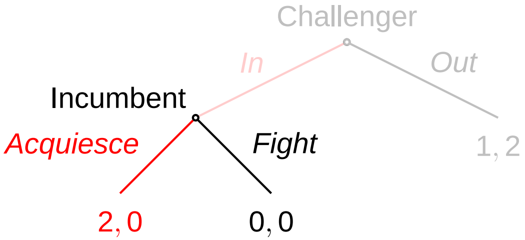

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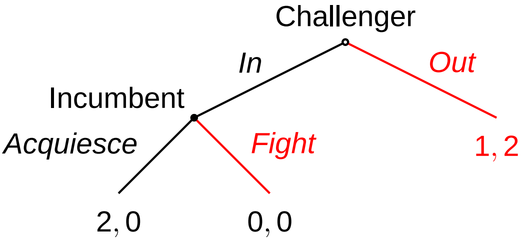

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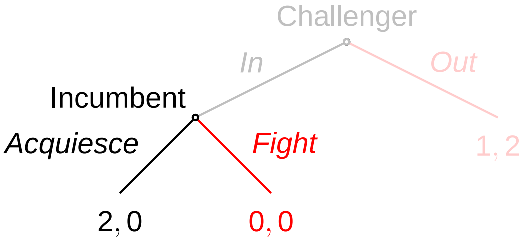

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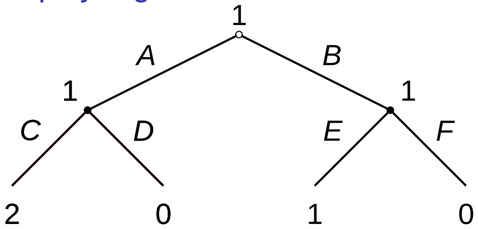
Checking strategy profile is SPE

Example: one-player game



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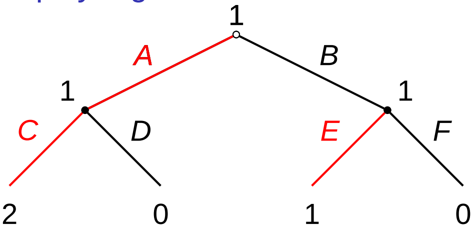

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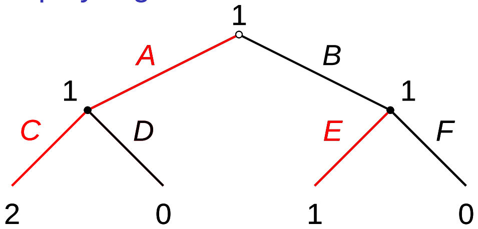

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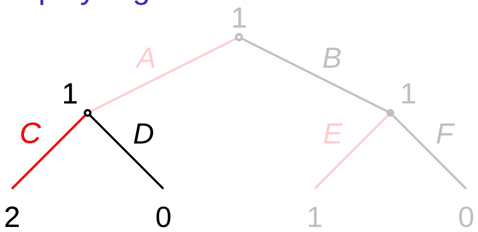

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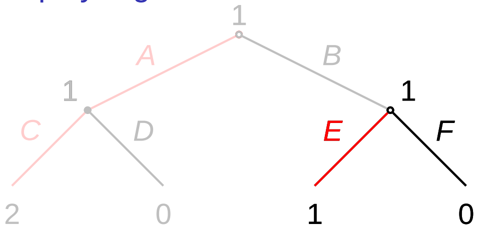

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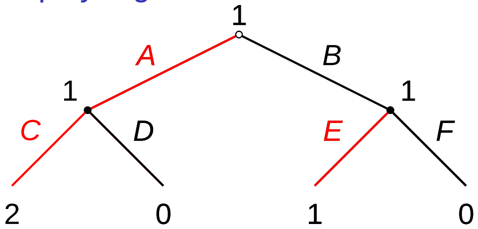

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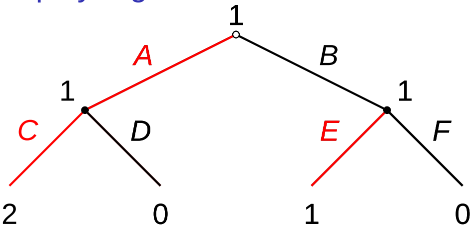

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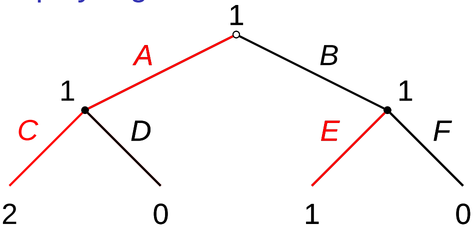

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First two steps ⇒

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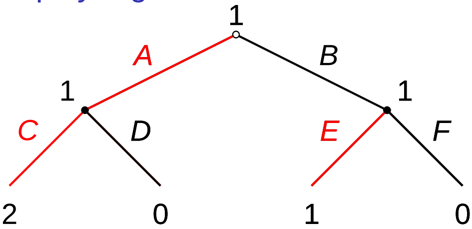

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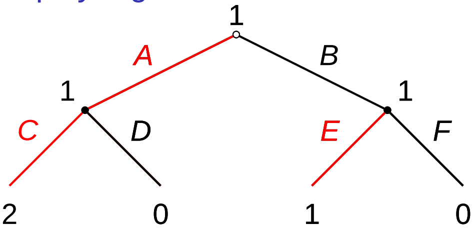

Is strategy *ACE* optimal?

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So when considering whole game, need to compare only the strategies *ACE* and *BCE*

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Proposition (*One-deviation property*)

A strategy profile in a finite horizon extensive game with perfect information is a subgame perfect equilibrium if and only if it satisfies the one-deviation property

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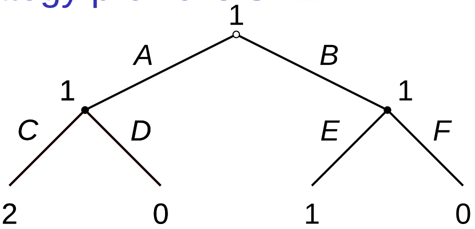

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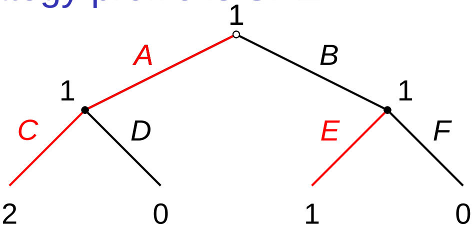

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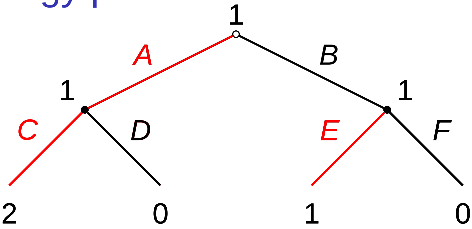
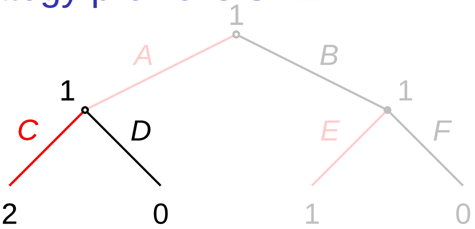


Is strategy *ACE* optimal? Use one-deviation property:

Checking strategy profile is SPE



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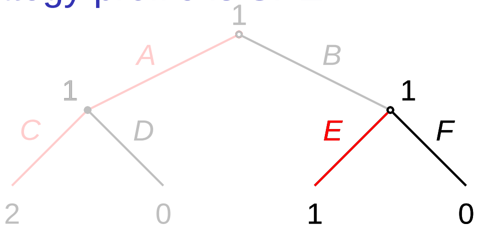

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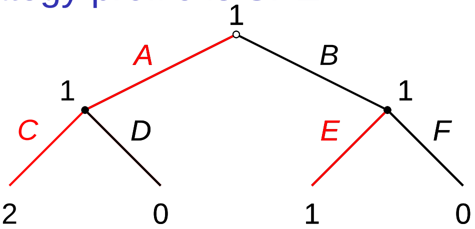
Subgame following *B* Payoff to $E >$ payoff to $F \Rightarrow$ player cannot increase payoff by changing action at start of subgame

Checking strategy profile is SPE



Is strategy *ACE* optimal? Use one-deviation property:

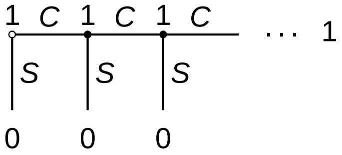
Subgame following *A* Payoff to *C* $>$ payoff to *D* $\Rightarrow$ player cannot increase payoff by changing action at start of subgame

Subgame following *B* Payoff to *E* $>$ payoff to *F* $\Rightarrow$ player cannot increase payoff by changing action at start of subgame

Whole game *A* $\Rightarrow$ payoff 2 and *B* $\Rightarrow$ payoff 1, *given rest of strategy*, so player cannot increase payoff by changing her action at start of subgame

Example

Consider following *infinite horizon* game



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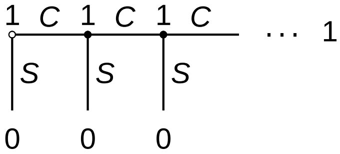

- One player

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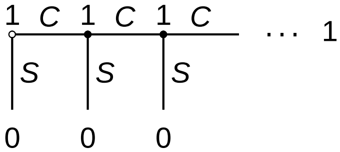

- One player
- Terminal histories: S, (C, S), (C, C, S), (C, C, C, S), $\ldots$, and infinite sequence $(C, C, \ldots)$

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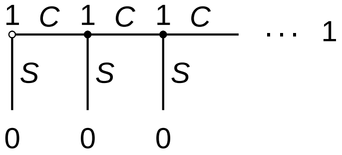

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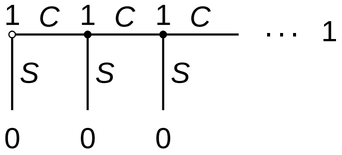

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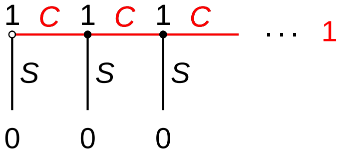

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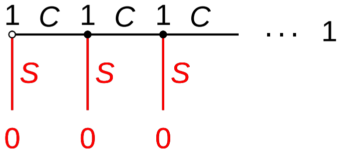

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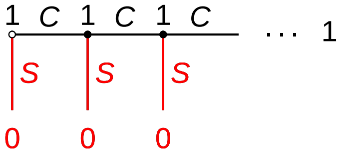

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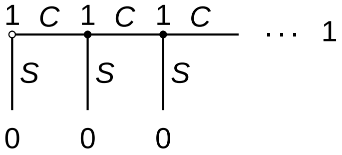
- Payoffs: 0 to every terminal history except $(C, C, \ldots)$
- Subgame perfect equilibrium: $(C, C, \ldots) \Rightarrow$ payoff 1
- Consider strategy $(S, S, \ldots)$: does it satisfy one-deviation property?
  - Yes: player cannot increase her payoff by deviating from S to C at start of any subgame, *given rest of strategy*

Example

Consider following *infinite horizon* game



- Example shows that the assumption of finite horizon cannot be removed from result

Proposition (*One-deviation property*)

A strategy profile in a finite horizon extensive game with perfect information is a subgame perfect equilibrium if and only if it satisfies the one-deviation property

Backward induction in finite horizon games

- If strategy profile in finite horizon game satisfies one-deviation property, it is subgame perfect equilibrium

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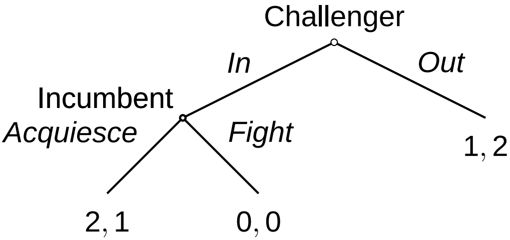
- Continue to work backwards to start of game
- In finite horizon game, strategy profile constructed satisfies one-deviation property and hence is subgame perfect equilibrium

Example: entry game



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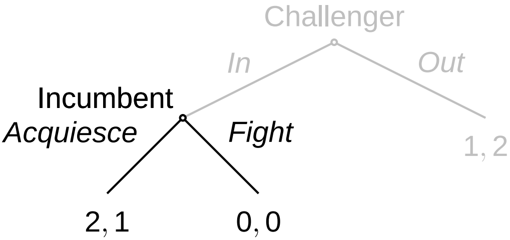

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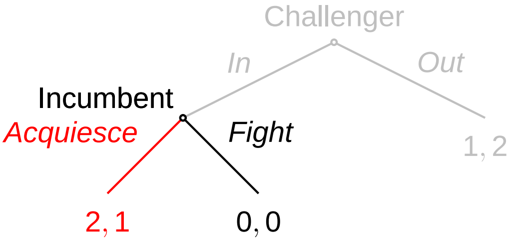

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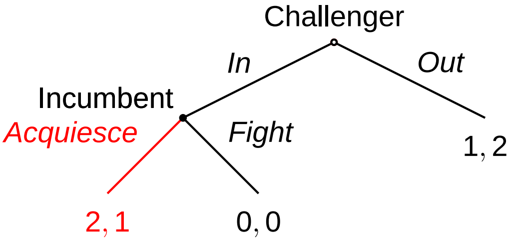

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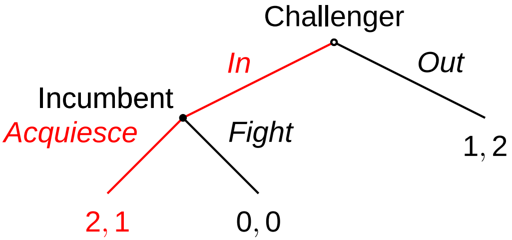

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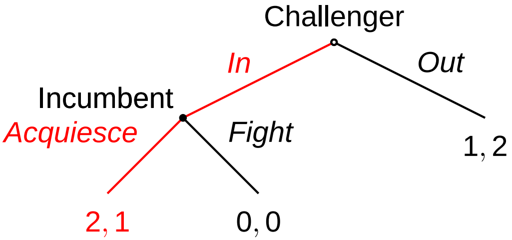

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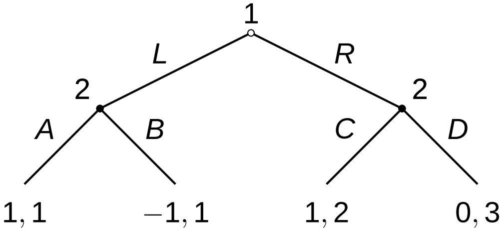
- One subgame of length 2 (whole game): optimal action (of Challenger), given outcome in subgame of length 1, is *In*
- Thus game has unique subgame perfect equilibrium, $(In, Acquiesce)$

Example: game with indifference between outcomes



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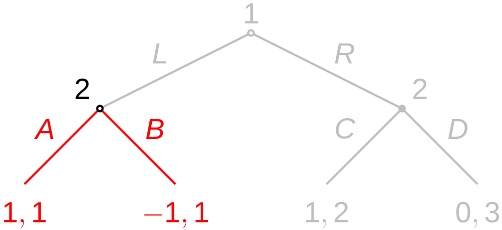

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  - following L: A and B are both optimal

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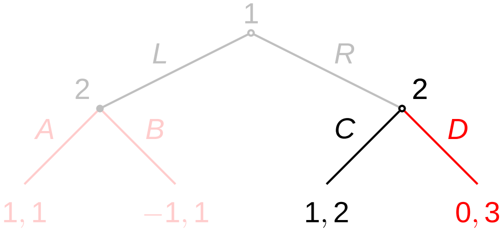

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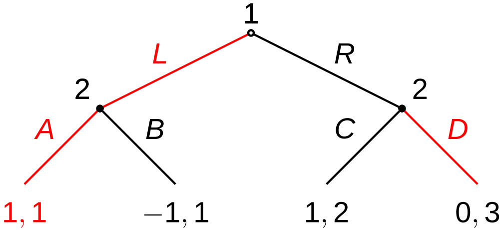

- Subgames of length one:
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- Subgame of length two (whole game): Need to consider separately each collection of optimal actions in subgames of length one:
  - *AD*: *L* is optimal

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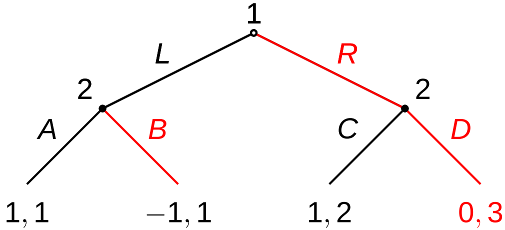

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  - BD: R is optimal

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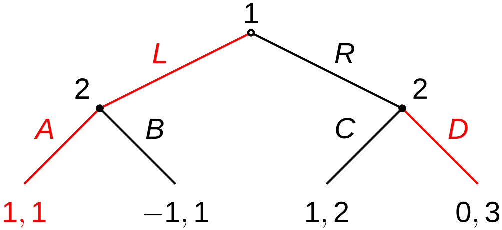

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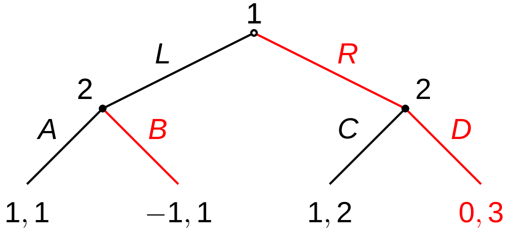

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Proposition

Every finite extensive game with perfect information has a subgame perfect equilibrium.