Economics 2030

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Questions for Tutorial 3

1. In many auctions, the auctioneer sets a "reserve price"—a price below which she will not sell the object. Consider a second-price sealed-bid auction with two bidders, each of whose valuations is distributed uniformly on $[0, 1]$. Denote by r the reserve price. If both bids are greater than r, then the player who submits the higher bid wins and pays a price equal to the lower bid. If one bid is greater than r and the other is less than r, then the player who submits the bid greater than r wins and pays r. If both bids are less than r, then the object is not sold.

   Find the value of r that maximizes the expected revenue of the auctioneer, as follows.

   (a) Show that the strategy of a player in which she bids her valuation weakly dominates every other strategy.

   (b) Find the expected price paid by player i with valuation v_i who bids v_i.

   (c) Denote the expected price paid by player i with valuation v_i by $\pi(v_i)$. Find the expected revenue of the auctioneer and the value of r that maximizes this revenue.

2. Consider an independent private value auction that differs from the auctions discussed in class in that *every* bidder pays the price she bids. There are n players. The one who submits the highest bid wins the object, but every player pays the auctioneer the price she bids. Each bidder knows her own valuation, which lies in $[0, 1]$, but does not know the other bidders' valuations. Each player believes that every other player's valuation is distributed *uniformly* on $[0, 1]$, independently of the other valuations. The preferences over lotteries of a player with valuation v are represented by the expected value of a Bernoulli payoff function that assigns the payoff $v - b$ to winning and the payoff $-b$ to losing, where b is the player's bid. (Given that each player's belief about the distribution of every other player's valuations is continuous, you do not need to worry about ties.)

(a) Specify the Bayesian game that models this situation.

(b) Find a symmetric Nash equilibrium of the game in which each player's bidding function is increasing in her valuation, under the assumption that such an equilibrium exists.

3. Each of two people may work on a research project. Each person who works succeeds with probability $\lambda < 1$, independently of the other person. If *either* person succeeds, *each* person's payoff is 1. (That is, both people benefit from the success of the research, whoever succeeds.) If neither person succeeds, each person's payoff is 0.

   Each person chooses whether to work (W) or not (N). If person i works, she incurs the cost c_i. Each person knows her own cost, but not the other person's cost. Each person believes that the other person's cost is drawn from the uniform distribution on $[0, 1]$. Each person cares about her expected payoff. Thus, for example, if both people work then the payoff of person i is $1 - (1 - \lambda)^2 - c_i$ (because the probability that at least one person succeeds is $1 - (1 - \lambda)^2$ and person i obtains the payoff 1 if at least one person succeeds and the payoff 0 if neither person succeeds).

   (a) Model this situation as a Bayesian game.

   (b) Find a Nash equilibrium of the Bayesian game.

4. Suppose we define a strategy profile s^* in an extensive game with perfect information and no simultaneous moves to satisfy the "one deviation principle along the equilibrium path" if, for each subgame reached if s^* is followed, the player who moves at the start of the subgame cannot increase her payoff by choosing a different action at the start of the subgame, given the rest of her strategy and the other players' strategies. Is it true that a strategy profile is a Nash equilibrium (note: not subgame perfect equilibrium) if and only if it satisfies this property?