Economics 2030

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Problem Set 5

1. For each player i, find a Nash equilibrium of a second-price sealed-bid auction (with independent private valuations, as formulated in class) in which player i wins.

2. Consider auctions in which each player is risk averse. Specifically, suppose each of the n players' preferences are represented by the expected value of the Bernoulli payoff function $x^{1/m}$, where x is the player's monetary payoff and $m > 1$. Suppose also that each player's valuation is distributed uniformly between 0 and 1. Find the symmetric Nash equilibrium of the Bayesian game that models a first-price sealed-bid auction under these assumptions. Compare the auctioneer's revenue in this equilibrium with her revenue in the symmetric Nash equilibrium of a second-price sealed-bid auction in which each player bids her valuation. (Note that the equilibrium of the second-price auction does not depend on the players' payoff functions.)

3. A single invisible object is to be sold in a sealed-bid auction to one of two players. Player 1's valuation of the object is 1 and is known by player 2. Player 2's valuation is known by her but not by player 1, who believes it to be 0 with probability p and 1 with probability $1 - p$, where $0 < p < 1$. The player who bids the most obtains the object and pays either the price she bids ("first-price auction") or the highest of the remaining bids ("second-price auction"). If there is a tie for the highest bid, each player obtains the object with probability $\frac{1}{2}$, and only the player who does so makes a payment. Each player's payoff if she obtains the object is the expected value of the difference between her valuation and the amount she pays; her payoff if she does not obtain the object is 0.

   Explain how to model these situations (first-price and second-price auctions) as games and study the equilibria of each game *when each player is restricted to bid either 0 or 1* and uses a strategy that is not weakly dominated. Compare the auctioneer's revenue from the two auctions.

4. Consider an example of a second-price auction with common values in which a painting is for sale and that painting may be a fake. There are two bidders; bidder 1 is an expert and bidder 2 is not. The expert knows whether the painting is fake, but bidder 2 does not. Bidder 2 believes that the probability the painting is fake is $\frac{1}{2}$. Both bidders' valuations contain a random component, as in the case of independent private valuations, but depend also on whether the painting is fake. Specifically, before the auction two numbers, x_1 and x_2, are drawn independently from a uniform distribution on $[0, 1]$. Bidder 1's valuation is x_1 if the painting is fake and $x_1 + 10$ if the painting is authentic, and bidder 2's valuation is x_2 if the painting is fake and $x_2 + 10$ if the painting is authentic.

   (a) Suppose that each player bids her expected valuation for the painting. That is, player 1 bids x_1 if the painting is fake and $x_1 + 10$ if it is authentic, and player 2 bids $\frac{1}{2}x_2 + \frac{1}{2}(x_2 + 10) = x_2 + 5$. If the players use this strategy pair and player 2 wins, what does she learn? Show that the strategy pair is not an equilibrium of the auction.

   (b) Find an equilibrium in which player 1 bids her valuation, which is x_1 if the painting is fake and $10 + x_1$ if the painting is authentic.

5. There are two players and two wallets. Player 1 knows the amount of money in wallet 1 and player 2 knows the amount of money in wallet 2. The players submit sealed bids for the right to own **both** wallets. The player who submits the higher bid wins both wallets and pays the **lower** of the two bids. If the bids are equal, each player wins with probability $\frac{1}{2}$. For every value of the amount in wallet 1, player 1 believes that the amount in wallet 2 is drawn from the distribution F_2, which has a density f_2, and for every amount in wallet 2, player 2 believes that the amount of money in wallet 1 is drawn from the distribution F_1, which has a density f_1. (No additional restrictions are placed on F_1 and F_2.)

   (a) Formulate this situation as a Bayesian game.

   (b) Consider the strategy s_i of player i that bids k times the amount of money in wallet i. Is there any value of k for which the strategy pair (s_1, s_2) is a Nash equilibrium?

   (c) Does the game have any other Nash equilibria?