Economics 2030

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Solutions for Tutorial 2

1. (a) An action profile is a Nash equilibrium if and only if either (i) exactly one player chooses 1 or (ii) exactly two players choose 1.

   (b) No, the game does not have such an equilibrium. Suppose that two of the players assign probability p to 1 and probability $1-p$ to 2. Then if the remaining player chooses 2 her payoff is 1 with probability p^2, $\frac{1}{3}$ with probability $(1-p)^2$, and 0 with the remaining probability. If she chooses 3 then her payoff is 1 with probability $p^2 + (1-p)^2$ and 0 with the remaining probability. Thus her expected payoff to 3 exceeds her expected payoff to 2, so that any best response to the other players' strategies assigns probability 0 to 2.

   (There *is* a value of p (namely $\frac{1}{2}$) such that the payoffs of a player to the actions 1 and 2 are equal when each of the other players chooses 1 with probability p and 2 with probability $1-p$, but for this value of p (and in fact for any value of p) the player's expected payoff to the action 3 exceeds her expected payoff to the action 2.)

2. (a) The players are the firms, the set of actions of each firm is the set $\{0, 1/k, 2/k, \ldots, 1\}$, and for any pair of actions the payoff of a firm is the fraction of consumers who are located closer to the firm than to the other firm unless the firms' locations are the same, in which case each firm's payoff is $\frac{1}{2}$.

   (b) The unique Nash equilibrium is that both firms choose the location $\frac{1}{2}$.

   (c) The action 0 is strictly dominated by $1/k$ and the action 1 is strictly dominated by $(k-1)/k$. When these actions are eliminated, the actions $1/k$ and $(k-1)/k$ are strictly dominated. Continuing in the same way, we find that the only rationalizable action for each firm is $\frac{1}{2}$.

3. (a) The set of all Nash equilibria is $\{(p_b, p_s) : c \leq p_b = p_s \leq v\} \cup \{(p_b, p_s) : p_b \leq c \text{ and } p_s \geq v\}$.

(b) A Bayesian game that models the situation is defined as follow.

Players The buyer and the seller.

States The set of pairs (v, c), where $v \in [0,1]$ and $c \in [0,1]$.

Actions The set of actions of each player is the set of possible prices, the set of nonnegative numbers.

Signals The set of signals each player may receive is $[0,1]$. The buyer's signal function is defined by $\tau_b(v,c) = v$ and the seller's signal function is defined by $\tau_s(v,c) = c$.

Beliefs Each player's prior belief is that v and c are independent draws from a uniform distribution over $[0,1]$.

Payoffs The buyer's payoff to $((p_b, p_s), (v, c))$ is $v - p_b$ if $p_b \geq p_s$ and 0 otherwise; the seller's payoff is $p_s - c$ if $p_b \geq p_s$ and 0 otherwise.

(c) Consider a buyer of type v. Her expected payoff if she chooses p is

$$\Pr(\gamma + \delta c \leq p)(v - p) = \Pr(c \leq (p - \gamma)/\delta)(v - p).$$

We have

$$\Pr(c \leq (p-\gamma)/\delta) = \begin{cases} 0 & \text{if } p < \gamma \\ (p-\gamma)/\delta & \text{if } \gamma \leq p \leq \gamma + \delta \\ 1 & \text{if } p > \gamma + \delta, \end{cases}$$

so the expected payoff of a buyer of type v who chooses p is

$$\begin{cases} 0 & \text{if } p < \frac{1}{3} \\ \frac{2}{3}(3p-1)(v-p) & \text{if } \frac{1}{3} \leq p \leq \frac{5}{6} \\ v - p & \text{if } p > \frac{5}{6}. \end{cases}$$

If $v \in [0, \frac{1}{3}]$, any value of p in $[0, \frac{1}{3}]$ maximizes this expected payoff (and the maximized value is 0). If $v \in [\frac{1}{3}, 1]$, the unique maximizer is $\frac{1}{6} + \frac{1}{2}v$.

The expected payoff of a seller of type c who chooses p is

$$\Pr(\alpha + \beta v \geq p)(p - c) = \Pr(v \geq (p - \alpha)/\beta)(p - c).$$

We have

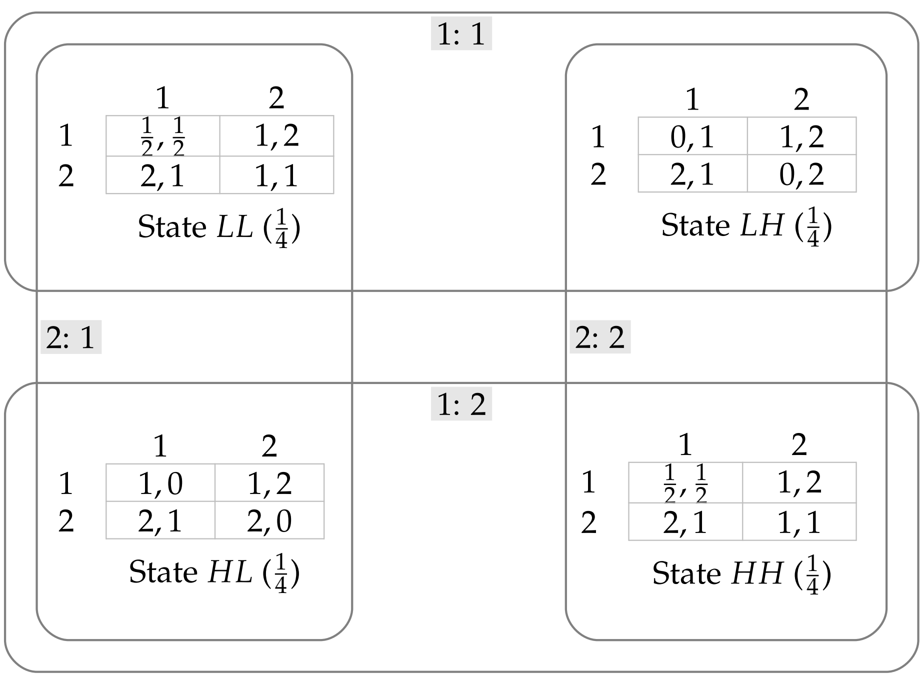
$$\Pr(v \geq (p-\alpha)/\beta) = \begin{cases} 1 & \text{if } p < \alpha \\ 1-(p-\alpha)/\beta & \text{if } \alpha \leq p \leq \alpha+\beta \\ 0 & \text{if } p > \alpha+\beta, \end{cases}$$

so the expected payoff of a seller of type c who chooses p is

$$\begin{cases} p-c & \text{if } p < \frac{1}{6} \\ \frac{2}{3}(2-3p)(p-c) & \text{if } \frac{1}{6} \leq p \leq \frac{2}{3} \\ 0 & \text{if } p > \frac{2}{3}. \end{cases}$$

If $c \in [0, \frac{2}{3}]$, the unique maximizer is $\frac{1}{3} + \frac{1}{2}c$. If $c \in [\frac{2}{3}, 1]$, any value of p in $[\frac{2}{3}, 1]$ is a maximizer (and the maximized value is 1). Thus if the seller uses the strategy $p = \frac{1}{3} + \frac{1}{2}c$ then the strategy $p = \frac{1}{6} + \frac{1}{2}v$ of the buyer is optimal, and vice versa.

4. (a) The game is given in the following figure.



(b) Look for an equilibrium in which each worker uses the same strategy.

For type H the action 2 weakly dominates the action L, so it is reasonable to look for an equilibrium in which type H chooses 2 with probability 1.

Denote the probability with which a worker of type L applies to firm 1 by p. Then the expected payoff of a worker of type L who applies to firm 1 is

$$\tfrac{1}{2}(\tfrac{1}{2}p + 1 - p) + \tfrac{1}{2}(1)$$

and the expected payoff of such a worker who applies to firm 2 is

$$\tfrac{1}{2}(2p + 1 - p) + \tfrac{1}{2}(0).$$

For these two expected payoffs to be the same, we need

$$\tfrac{1}{2}(\tfrac{1}{2}p + 1 - p) + \tfrac{1}{2} = \tfrac{1}{2}(2p + 1 - p)$$

or

$$p = \tfrac{2}{3}.$$

Now consider a worker of type H. Her expected payoff if she applies to firm 1 is

$$\tfrac{1}{2}(1) + \tfrac{1}{2}(1) = 1$$

and her expected payoff if she applies to firm 2 is

$$\tfrac{1}{2}(2) + \tfrac{1}{2}(1) = \tfrac{3}{2}.$$

Thus such a worker prefers to apply to firm 2.

We conclude that the Bayesian game has a Nash equilibrium in which each worker applies to firm 1 with probability $\frac{2}{3}$ if her type is L and to firm 2 with probability 1 if her type is H.

The game also has an asymmetric equilibrium in which both types of one player choose firm 2 and type L of the other player chooses firm 1 whereas type H of the other firm chooses firm 2.