Economics 2030

Fall 2018 Martin J. Osborne

Solutions for Tutorial 1

1. Each firm's average cost function is shown in Figure 1.

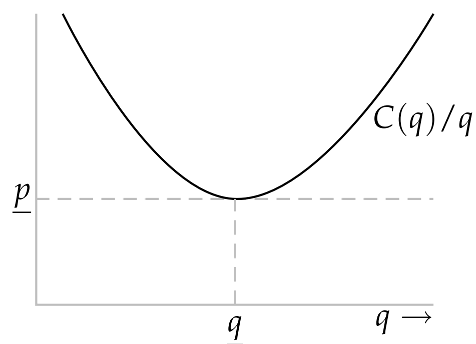

Figure 1. The average cost function of the firm in Question 1

First note that what we need to show is that if $P(Q^*) < \underline{p}$ or $P(Q^* + \underline{q}) > \underline{p}$ then there is no Nash equilibrium in which the total output of the firms is Q^*. If $P(Q^*) < \underline{p}$ then $Q^* > \overline{Q}$ and if $P(Q^* + \underline{q}) > \underline{p}$ then $Q^* < \underline{Q}$, where $P(\overline{Q}) = \underline{p}$ and $P(\underline{Q} + \underline{q}) = \underline{p}$, as shown in Figure 2. Thus we need to show that there is no Nash equilibrium in which $Q^* > \overline{Q}$ or $Q^* < \underline{Q}$. To do so, in each case we find a firm that can profitably change its output, given the other firms' outputs.

- If $P(Q^*) < \underline{p}$ then the profit of every firm producing a positive output is negative, so such a firm (of which there is at least one, given that $Q^* > 0$) can increase its profit (to 0) by deviating and producing zero.
- Suppose $P(Q^* + \underline{q}) > \underline{p}$. Let $\varepsilon = P(Q^* + \underline{q}) - \underline{p}$. Demand is finite at all prices, so Q^* is finite and hence, given that there are infinitely many firms, for every $\delta > 0$ there is a firm with output less than δ. Choose a firm whose output is less than $\delta = \varepsilon\underline{q}/(P(Q^*) - \underline{p})$. The profit of this firm is less than $\delta(P(Q^*) - \underline{p}) = \varepsilon\underline{q}$, because its unit cost is at least $\underline{p}$. If the firm deviates

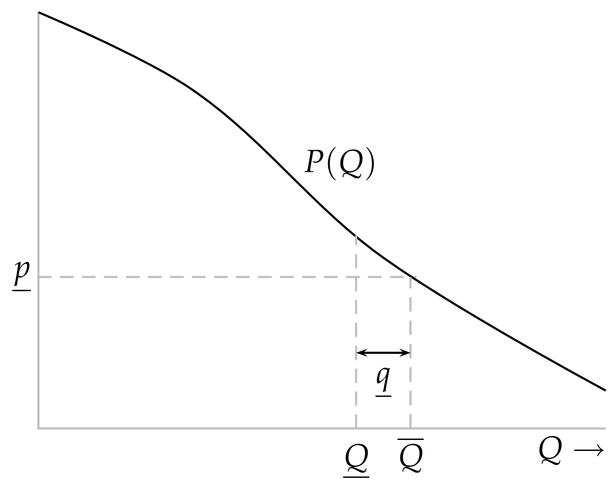

Figure 2. The range of values for Q^* in Question 1

to an output of $\underline{q}$ then the total output of the firms increases to at most $Q^* + \underline{q}$, so that the price is at least $\underline{p} + \varepsilon$. Thus the firm's profit after it deviates is at least $\varepsilon\underline{q}$. That is, the deviation increases its profit.

2. Denote by $\overline{p}$ the price p that satisfies $(p - c)(\alpha - p) = f$ and is less than the maximizer of $(p - c)(\alpha - p)$.

   The payoff function of firm i for $\overline{p} < p_j \leq p^m$ is shown in Figure 3. Arguments like those for those for the model without a fixed cost suggest that the game may have a Nash equilibrium $(\overline{p}, \overline{p})$.

   I claim that $(\overline{p}, \overline{p})$ is in fact a Nash equilibrium. At this pair of prices, both firms' profits are zero. (Firm 1 receives all the demand and obtains the profit $(\overline{p} - c)(\alpha - \overline{p}) - f = 0$, and firm 2 receives no demand.) This pair of prices is a Nash equilibrium by the following argument.

   - If either firm raises its price its profit remains zero (it receives no customers).
   - If either firm lowers its price then it receives all the demand and earns a negative profit (since f is less than the maximum of $(p - c)(\alpha - p)$).

3. (a) Suppose that citizen i prefers candidate A; fix the votes of all citizens other than i. If citizen i switches from voting for B to voting for A then, depending on the other citizens' votes, either the outcome does not change, or A wins rather than B; such a switch

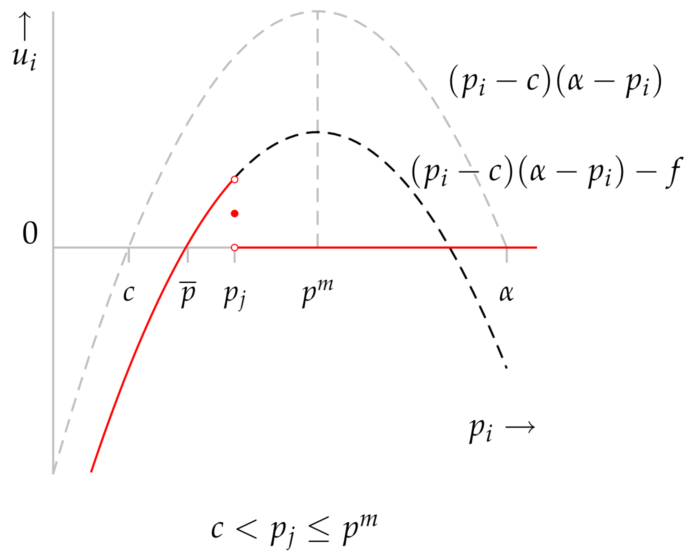

Figure 3. The payoff function of firm i for $\overline{p} < p_j \leq p^m$ in Question 2.

cannot cause the winner to change from A to B. That is, citizen i's switching from voting for B to voting for A either has no effect on the outcome, or makes her better off; it cannot make her worse off.

(b) First consider an action profile in which the winner receives one more vote than the loser and at least one citizen who votes for the winner prefers the loser to the winner. Any citizen who votes for the winner and prefers the loser to the winner can, by switching her vote, cause her favorite candidate to win rather than lose. Thus no such action profile is a Nash equilibrium.

Next consider an action profile in which the winner receives one more vote than the loser and all citizens who vote for the winner prefer the winner to the loser. Because a majority of citizens prefer A to B, the winner in any such case must be A. No citizen who prefers A to B can induce a better outcome by changing her vote, since her favorite candidate wins. Now consider a citizen who prefers B to A. By assumption, every such citizen votes for B; a change in her vote has no effect on the outcome (A still wins). Thus every such action profile is a Nash equilibrium.

Finally consider an action profile in which the winner receives at least three more votes than the loser. In this case no change in any citizen's vote has any effect on the outcome. Thus every such profile is a Nash equilibrium.

In summary, the Nash equilibria are: any action profile in which A receives one more vote than B and all the citizens who vote for A prefer A to B, and any action profile in which the winner receives at least three more votes than the loser.

The only equilibrium in which no citizen uses a weakly dominated action is that in which every citizen votes for her favorite candidate.

(c) Fix some citizen, say i; suppose she prefers A to B to C. Then citizen i's voting for C is weakly dominated by her voting for A by the argument for the two-candidate game. (Note that i's voting for C is not weakly dominated by her voting for B: if the votes of the other citizens are equally divided between A and B, with C in third place by two or more votes, then a vote for B will lead to B to win whereas a vote for C will lead to a tie between A and B, which i prefers.)

Her voting for B is clearly not weakly dominated by her voting for C. I now argue that her voting for B is not weakly dominated by her voting for A. Suppose that the other citizens' votes are equally divided between B and C; no one votes for A. Then if citizen i votes for A the outcome is a tie between B and C, while if she votes for B the outcome is that B wins. Thus for this configuration of the other citizens' votes, citizen i is better off voting for B than she is voting for A. Thus her voting for B is not weakly dominated by her voting for A.

Now fix some citizen, say i, and consider the candidate she ranks in the middle, say candidate B. The action profile in which all citizens vote for B is a Nash equilibrium. (No citizen's changing her vote affects the outcome.) In this equilibrium, citizen i does not vote for her favorite candidate, but the action she takes is not weakly dominated. (Other Nash equilibria also satisfy the conditions in the exercise.)

4. Player 1's action B is strictly dominated, so the Nash equilibria of the

game are the same as the Nash equilibria of the game

	X	Y	Z
T	$1,3$	$4,2$	$3,1$
M	$2,2$	$1,3$	$0,2$

In this game player 2's action Z is strictly dominated, so the Nash equilibria are the same as the Nash equilibria of the game

	X	Y
T	$1,3$	$4,2$
M	$2,2$	$1,3$

The players' best response functions in this game are shown in Figure 4.

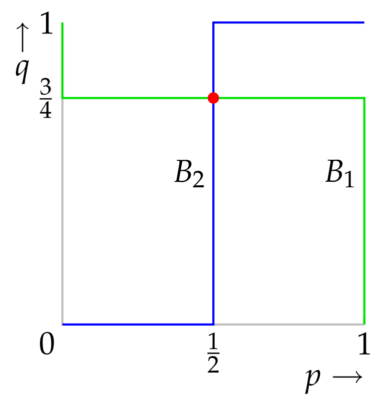

Figure 4. The players' best response functions in the 2×2 game to which the game in Question 4 is reduced after eliminating strictly dominated actions. The probability that player 1 assigns to T is p and the probability that player 2 assigns to X is q. The disk indicates the Nash equilibrium.

This game has a unique Nash equilibrium, in mixed strategies: $((\frac{1}{2}, \frac{1}{2}), (\frac{3}{4}, \frac{1}{4}))$.

Thus the unique Nash equilibrium of the original game is $((\frac{1}{2}, \frac{1}{2}, 0), (\frac{3}{4}, \frac{1}{4}, 0))$.