Economics 2030

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Problem Set 4

1. Consider the variant of Cournot's duopoly game in which firm 1 doesn't know whether firm 2's unit cost is c_L or c_H, where $c_L < c_H$) and firm 2 knows that firm 1's cost is c. Assume that the inverse demand function is given by $P(Q) = \alpha - Q$ for $Q \leq \alpha$ and $P(Q) = 0$ for $Q > \alpha$ and that firm 1's prior belief that firm 2's cost is c_L is θ. For values of c_H and c_L close enough to each other that there is a Nash equilibrium in which all outputs are positive, find this equilibrium. Compare this equilibrium with the Nash equilibrium of the game in which firm 1 knows that firm 2's unit cost is c_L, and with the Nash equilibrium of the game in which firm 1 knows that firm 2's unit cost is c_H.

2. Each of two people can contribute either 0 or c to the provision of a public good, where $0 < c < 1$. If at least one of them contributes, the good is provided. For $i = 1, 2$, person i's payoff if the good is provided is $(\theta_i)^2 - c$ if she contributes and $(\theta_i)^2$ if she does not; her payoff if the good is not provided is 0.

   The two people choose simultaneously whether to contribute. Neither person knows the other person's value of θ_i. Each person believes that the other person's value of θ_i is distributed uniformly between 0 and 1.

   (a) Model this situation as a Bayesian game. (Carefully specify the components of the game.)

   (b) Find a Nash equilibrium of the Bayesian game. (*Note that you are asked only to find one equilibrium, not all equilibria.*)

3. Two people are involved in a dispute. Person 1 does not know whether person 2 is strong or weak; she assigns probability α to person 2's being strong. Person 2 is fully informed. Each person can either fight or yield. Each person's preferences are represented by the expected value of a Bernoulli payoff function that assigns the payoff of 0 if she yields (regardless of the other person's action) and a payoff of 1 if she

fights and her opponent yields; if both people fight then their payoffs are $(-1, 1)$ if person 2 is strong and $(1, -1)$ if person 2 is weak. Formulate this situation as a Bayesian game and find its Nash equilibria if $\alpha < \frac{1}{2}$ and if $\alpha > \frac{1}{2}$.

4. Firm A (the "acquirer") is considering taking over firm T (the "target"). It does not know firm T's value; it believes that this value, when firm T is controlled by its own management, is at least \$0 and at most \$100, and assigns equal probability to each of the 101 dollar values in this range. Firm T will be worth 50% more under firm A's management than it is under its own management. Assume that firm A submits a bid, a nonnegative number to take over firm T. Suppose that firm A's bid is y, and firm T is worth x (under its own management). Then if T accepts A's offer, A's payoff is $\frac{3}{2}x - y$ and T's payoff is y; if T rejects A's offer, A's payoff is 0 and T's payoff is x. Model this situation as a Bayesian game in which firm A chooses how much to offer and firm T decides the lowest offer to accept. Find the Nash equilibrium (equilibria?) of this game. Explain why the logic behind the equilibrium is called *adverse selection*.

5. Consider the Bayesian game in Figure 1, in which the probabilities $\frac{3}{4}$ and $\frac{1}{4}$ are the players' posterior beliefs.

   (a) In state γ, does player 1 know player 2's preferences? Does player 2 know player 1's preferences? Does player 2 know that player 1 knows player 2's preferences? Does player 1 know that player 2 knows player 1's preferences?

   (b) Find the Nash equilibria of the game.

   (c) Add to the game state δ, in which the players' payoffs are the same as they are in states β and γ and assume that player 1 can distinguish this state from each of the others but player 2 cannot distinguish the state from state γ. Assume that player 2's posterior belief that the state is γ given that she observes that it is either γ or δ is $\frac{3}{4}$. Answer the question in part (a) for state δ and find the Nash equilibria of the modified game.

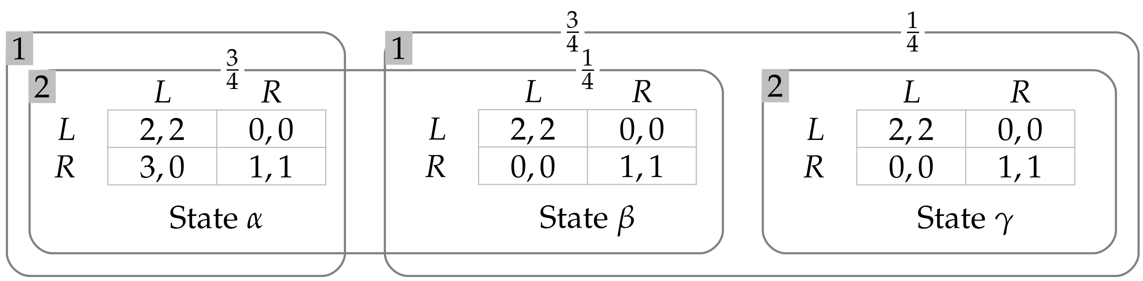

Figure 1. The Bayesian game in problem 5.