Economics 2030

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Questions for Tutorial 2

1. Each of three people chooses a positive integer $(1, 2, 3, \ldots)$. If all three people choose the *same* integer, each person's payoff is $\frac{1}{3}$. If all three choose different integers, the payoff of the player who chooses the *smallest* integer is 1 and the payoffs of the others are 0. If two people choose the same integer and the third person chooses a different integer, the third person's payoff is 1 and the others' payoffs are 0.

   (One interpretation is that an integer is a way of dressing. Everyone wants to dress differently from everyone else, and, in situations where people dress differently, everyone wants to be as "cool" as possible (cool = small integer).)

   (a) Find all the pure strategy Nash equilibria of the strategic game that models this situation.

   (b) Does the game have a ***symmetric*** mixed strategy Nash equilibrium in which each player assigns positive probability only to 1 and 2? If so, find such an equilibrium. If not, argue why no such equilibrium exists.

2. Two firms choose locations in the set $\{0, 1/k, 2/k, \ldots, 1\}$, where k is a positive even integer. Consumers are uniformly distributed along the line segment $[0, 1]$. Each consumer buys one unit of a good from the firm located closest to her; the price is fixed at 1. If the firms choose the same location, then half of the consumers patronize each of them. Given that the price is 1, each firm's profit is the fraction of consumers it serves.

   (a) Formulate this situation as a strategic game.

   (b) Find the Nash equilibrium (equilibria?) of the game.

   (c) Find the rationalizable actions of each firm.

3. A buyer and a seller simultaneously name prices (nonnegative numbers), p_b and p_s. If $p_b \geq p_s$, trade occurs; the buyer pays p_b and the seller receives p_s. (The remainder, $p_b - p_s$, goes to a third party.) The buyer's payoff is $v - p_b$ and the seller's payoff is $p_s - c$. If trade does not occur, each player receives the payoff zero.

   (a) Find a Nash equilibrium of the strategic game that models this situation for the case $v > c \geq 0$.

   Now assume that the buyer knows only her value, not the seller's cost, and the seller knows only her cost, not the buyer's value. The buyer believes that the seller's cost is drawn from a uniform distribution over $[0, 1]$, independent of her valuation, and the seller believes that the buyer's valuation is drawn from a uniform distribution over $[0, 1]$, independent of her cost.

   (b) Formulate this situation as a Bayesian game.

   (c) Show that the Bayesian game has a Nash equilibrium in which the players' strategies are given by

   $$p_b(v) = \alpha + \beta v \quad \text{for the buyer}$$

   and

   $$p_s(c) = \gamma + \delta c \quad \text{for the seller,}$$

   where $\alpha = \frac{1}{6}$, $\beta = \frac{1}{2}$, $\gamma = \frac{1}{3}$, and $\delta = \frac{1}{2}$.

4. Two workers are deciding to which of two firms to apply for a job. The wage paid by firm 1 is 1 and the wage paid by firm 2 is 2. The productivity of each worker is either L or H, with $L < H$. Each worker can apply to only one firm and each firm hires exactly one worker. If both workers apply to the same firm, the firm chooses the one with the higher productivity or, if the applicants' productivities are the same, hires each with probability $\frac{1}{2}$. If the workers apply to different firms, each is hired by the firm to which she applies.

   Each worker knows her own productivity, but not the productivity of the other worker. Each worker believes that the productivity of the other worker is L with probability $\frac{1}{2}$ and H with probability $\frac{1}{2}$. Each worker's payoff is the wage she receives, or 0 if she does not obtain a job.

(a) Model this situation as a Bayesian game in which the players are the workers. (The firms have no decisions to make.) A diagram is sufficient.

(b) Find a Nash equilibrium of the game.