Economics 2030

Fall 2018 Martin J. Osborne

Problem Set 3

1. Find all the Nash equilibria, in pure and mixed strategies, of the following strategic game. (Start by checking for dominated actions. Remember that an action may be dominated by a mixed strategy even though it is not dominated by any pure strategy.)

	L	M	R
T	$3,1$	$1,0$	$4,0$
M	$1,0$	$3,4$	$4,2$
B	$2,2$	$2,1$	$4,0$

2. Consider the strategic game in Figure 1. Assume that $0 < \pi < 1$ and $v_A > \pi v_B > 0$.

	A	B
A	$0,0$	$v_A, -v_A$
B	$v_B, -v_B$	$\pi v_B, -\pi v_B$

Figure 1. The game in Question 2.

   (a) Find the mixed strategy Nash equilibria of the game.

   (b) Suppose that player 1 has an additional action, C, that yields her the payoff h and yields player 2 the payoff 0, regardless of player 2's action. For each value of h with $0 \leq h < v_B$ find a mixed strategy Nash equilibrium of this new game. [*Note that you are asked only to find one equilibrium for each value of h, not all equilibria.*]

3. Determine whether each of the following statements is true or false. (*a*) A mixed strategy that assigns positive probability to a strictly dominated action is strictly dominated. (*b*) A mixed strategy that assigns positive probability only to actions that are not strictly dominated is not strictly dominated.

4. Two candidates, A and B, compete in an election. Of the n citizens, k support candidate A and m ($= n - k$) support candidate B. Each citizen decides whether to vote, at a cost, for the candidate she supports, or to abstain. A citizen who abstains receives the Bernoulli payoff of 2 if the candidate she supports wins, 1 if this candidate ties for first place, and 0 if this candidate loses. A citizen who votes receives the payoffs $2 - c$, $1 - c$, and $-c$ in these three cases, where $0 < c < 1$.

   Assume that $k \leq m$. Show that there is a value of p between 0 and 1 such that the game has a mixed strategy Nash equilibrium in which every supporter of candidate A votes with probability p, k supporters of candidate B vote with certainty, and the remaining $m - k$ supporters of candidate B abstain. How do the probability p that a supporter of candidate A votes and the expected number of voters ("turnout") depend upon c? (Note that if every supporter of candidate A votes with probability p then the probability that exactly $k - 1$ of them vote is $kp^{k-1}(1 - p)$.)

5. Consider Cournot's oligopoly game with two firms, linear inverse demand function P given by $P(Q) = \alpha - Q$, and constant unit cost $c > 0$ for both firms.

   (a) Find the set of actions of a firm that are strictly dominated. Use the diagram of cross-sections of firm 1's payoff function π_1 in Figure 2.

   (b) After the set of strictly dominated actions for each player are deleted, are any remaining actions strictly dominated? Again, refer to the figure.

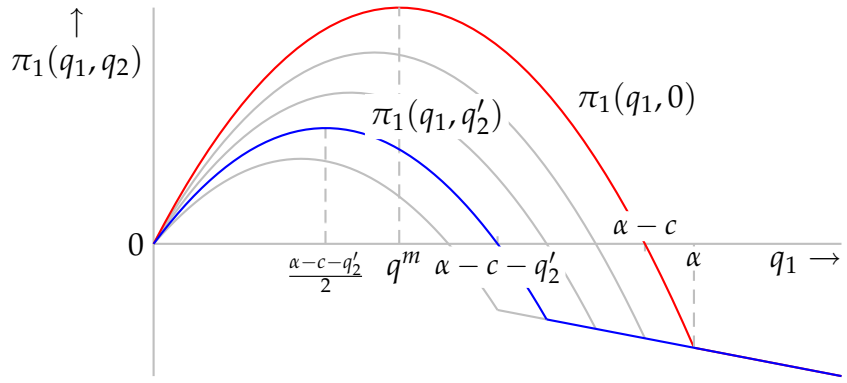

Figure 2. Some cross-sections of the payoff function π_1 of firm 1 in Cournot's duopoly game, for Question 5.