Economics 2030

Fall 2018 Martin J. Osborne

Problem Set 2

1. Consider a variant of the model of an auction in Problem 2 in Problem Set 1 in which the payment that the winner makes is the highest losing bid (not the price the winner bids). Such a model is known as a *second price auction*. (If every player submits a different bid then the highest losing bid is the second highest bid.) Show that in such a model the bid v_i of any player i is a *weakly dominant* action: player i's payoff when she bids v_i is at least as high as her payoff when she submits any other bid, regardless of the actions of the other players. Show that nevertheless there are ("inefficient") equilibria in which the winner is not player 1.

2. Each of $n \geq 3$ people announces a number in the set $\{1, \ldots, K\}$. A prize of \$1 is split equally between all the people whose number is closest to $\frac{2}{3}$ of the average number.

   (a) Show that if $K \geq 4$ then no action for any player is strictly dominated.

   (b) Consider a variant of the game in which each player is not indifferent between action profiles in which she loses: player i prefers a to a' if a_i is closer to two-thirds of the average for a than a'_i is to two-thirds of the average for a'. Suppose each player has similar preferences among action profiles in which she ties, but still prefers every action profile in which she wins to every one in which she ties, and prefers every one in which she ties to every one in which she loses. Show that the action K is strictly dominated, and use this fact to show that the only possible Nash equilibrium of the game is the action profile in which every player announces 1.

3. Consider Cournot's oligopoly game in which there is an arbitrary number n of firms, the inverse demand function is given by $P(Q) = \alpha - Q$ for $Q \leq \alpha$ and $P(Q) = 0$ for $Q > \alpha$, each firm's cost function is given by $C_i(q_i) = cq_i$, where $0 < c < \alpha$, and each firm's set of actions is $[0, \alpha]$. Show that this game satisfies all the assumptions of Proposition 20.3 in the book.

4. Which of the games in Figure 1 is/are equivalent to the *Prisoner's Dilemma* as specified in Figure 17.1 of the book (*A course in game theory*), when the numbers are interpreted as Bernoulli payoffs? [Two games are "equivalent" if each player's preference relation over lotteries is the same in both games.]

	Don't confess	*Confess*
Don't confess	$3,3$	$0,4$
Confess	$4,0$	$2,2$

	Don't confess	*Confess*
Don't confess	$9,\ \ 2$	$0,\ \ 4$
Confess	$12,-4$	$3,-2$

Figure 1. Two strategic games with vNM preferences.

5. By finding the players' best response functions, find all the mixed strategy Nash equilibria of the games in Figure 2.

	L	*R*
T	$6,0$	$0,6$
B	$3,2$	$6,0$

	L	*R*
T	$0,1$	$0,2$
B	$2,2$	$0,1$

Figure 2. The games in Exercise 5.