Economics 2030

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Questions for Tutorial 1

1. Suppose that in Cournot's model there are infinitely many firms, the inverse demand function P is a bounded decreasing function, each firm's cost function is the same, denoted by C with $C(0) = 0$, and there is a single positive output, say $\underline{q}$, at which the average cost of production $C(q)/q$ is minimal. Denote the minimum of $C(q)/q$ by $\underline{p}$. Under these assumptions, any given total output is produced most efficiently by each firm's producing $\underline{q}$, and the lowest price compatible with the firms' not making losses is the minimal value of the average cost. Show that in any Nash equilibrium the firms' total output Q^* satisfies
$$P(Q^* + \underline{q}) \leq \underline{p} \leq P(Q^*).$$
(That is, the price is at least the minimal value $\underline{p}$ of the average cost, but is close enough to this minimum that increasing the total output of the firms by $\underline{q}$ would reduce the price to at most $\underline{p}$.) Note that some firms may produce no output in an equilibrium.

2. Consider Bertrand's duopoly game in the case that the demand function is given by $D(p) = \alpha - p$ if $p \leq \alpha$, and $D(p) = 0$ for $p > \alpha$, and the cost function of each firm i is given by $C_i(q_i) = f + cq_i$ for $q_i > 0$, and $C_i(0) = 0$, where f is positive and less than $\max_p(p - c)(\alpha - p)$. Assume that firm 1 gets all the demand when both firms charge the same price. Either find a Nash equilibrium of the game or show that the game has no Nash equilibrium.

3. Two candidates, A and B, vie for office. Each of an odd number of citizens may vote for either candidate. (Abstention is not possible.) The candidate who obtains the most votes wins. (Because the number of citizens is odd, a tie is impossible.) A majority of citizens prefer A to win than B to win. The following strategic game models the citizens' voting decisions.

   Players The citizens.

Actions Each player's set of actions consists of voting for A and voting for B.

Preferences All players are indifferent between all action profiles in which a majority of players vote for A and between all action profiles in which a majority of players vote for B. Some players (a majority) prefer an action profile of the first type to one of the second type, and the others have the reverse preference.

(a) Show that a citizen's voting for her less preferred candidate is weakly dominated.

(b) Find all Nash equilibria of the game. (First consider action profiles in which the winner obtains one more vote than the loser and at least one citizen who votes for the winner prefers the loser to the winner, then profiles in which the winner obtains one more vote than the loser and all citizens who vote for the winner prefer the winner to the loser, and finally profiles in which the winner obtains three or more votes than the loser.)

(c) Consider the variant of the game in which there are three candidates, A, B, and C. A tie for first place is possible in this case; assume that a citizen who prefers a win by x to a win by y ranks a tie between x and y between an outright win for x and an outright win for y. Show that a citizen's only weakly dominated action is a vote for her least preferred candidate. Find a Nash equilibrium in which some citizen does not vote for her favorite candidate, but the action she takes is not weakly dominated.

4. Find all the Nash equilibria, in pure and mixed strategies, of the following strategic game.

	X	Y	Z
T	$1,3$	$4,2$	$3,1$
M	$2,2$	$1,3$	$0,2$
B	$0,0$	$1,1$	$2,4$