**Economics 2030**

Fall 2018 Martin J. Osborne

**Problem Set 1**

1. Consider Cournot's oligopoly game in the case of an arbitrary finite number $n$ of firms. Assume that the inverse demand function $P$ takes the form
$$P(Q) = \begin{cases} \alpha - Q & \text{if } Q \leq \alpha \\ 0 & \text{if } Q > \alpha \end{cases}$$
and the cost function of each firm $i$ is $C_i(q_i) = cq_i$ for all $q_i$, with $c < \alpha$.
   (a) Find the best response function of each firm.
   (b) Write down the conditions for $(q_1^*, \ldots, q_n^*)$ to be a Nash equilibrium, assuming that there is a Nash equilibrium in which every firm's output is positive.
   (c) Solve the equations in part (b) to find the Nash equilibrium (equilibria?). (*Note*: You cannot *assume* that all firms produce the same output in an equilibrium. You need to show that every equilibrium has this property.)
   (d) Find the price at which output is sold in a Nash equilibrium and show that this price decreases as $n$ increases, approaching $c$ as the number of firms increases without bound.

2. An object is to be assigned to a player in the set $\{1, \ldots, n\}$ in exchange for a payment. Player $i$'s valuation of the object is $v_i$, and $v_1 > v_2 > \cdots > v_n > 0$. The mechanism used to assign the object is a (sealed-bid) auction: the players simultaneously submit bids (nonnegative numbers), and the object is given to the player with the lowest index among those who submit the highest bid, in exchange for a payment. In a *first price* auction the payment that the winner makes is the price that he bids.

   Formulate a first-price auction as a strategic game and analyze its Nash equilibria. In particular, show that in all equilibria player 1 obtains the object.

3. A third-price auction with perfect information is a variant of a second-price auction with perfect information in which the price paid by the winner (the player who submits the highest bid) is the third highest of the bids submitted. [That is, $n \geq 3$ players simultaneously submit bids for a single indivisible object. Player $i$'s valuation of the object is $v_i$, where $v_1 > v_2 > \cdots > v_n$. The highest bid wins; in the event of a tie, the player whose index is smallest wins. (E.g. if players 1 and 2 tie for the highest bid, player 1 wins.)]

   Denote by $G$ the strategic game that models this situation.

   (a) Either find a (pure strategy) Nash equilibrium of $G$ in which the winner is player 1 and the price is less than $v_2$ (the second-highest valuation) or show that the game has no such equilibrium.

   (b) Either find a (pure strategy) Nash equilibrium of $G$ in which the winner is player $n$ (who has the lowest valuation) or show that the game has no such equilibrium.

4. Two people can choose how much to contribute to the provision of a public good. If person 1 contributes $c_1$ and person 2 contributes $c_2$ then the amount of the public good provided is $c_1 + c_2$ and person $i$'s payoff (for $i = 1, 2$) is

$$v_i \sqrt{c_1 + c_2} - c_i,$$

   where $v_1$ and $v_2$ are constants with $v_1 \neq v_2$. Each person can choose any nonnegative number for her contribution.

   Find the Nash equilibria of the strategic game that models this situation. (The character of the equilibria depend on the values of $v_1$ and $v_2$.)

5. Each of $n$ people chooses whether or not to become a political candidate, and if so which position to take. There is a continuum of citizens, each of whom has a favorite position; the distribution of favorite positions is given by a density function $f$ on $[0, 1]$ with $f(x) > 0$ for all $x \in [0, 1]$. A candidate attracts the votes of those citizens whose favorite positions are closer to her position than to the position of any other candidate; if $k$ candidates choose the same position then each receives the fraction $1/k$ of the votes that the position attracts. The winner of the competition is the candidate who receives the most votes. Each person prefers to be the unique winning candidate than to tie

for first place, prefers to tie for first place than to stay out of the competition, and prefers to stay out of the competition than to enter and lose.

Formulate this situation as a strategic game, find the set of Nash equilibria of the game when $n = 2$, and show that the game has no Nash equilibrium when $n = 3$.