Economics 316

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Solutions to Problem Set 12

1. The strategic form of the game is

	A	D
NN	$0,0$	$0,0$
NC	$-50,75$	$50,0$
CN	$50,-25$	$50,0$
CC	$0,50$	$100,0$

The game has no pure strategy Nash equilibrium.

For player 1, the action NN is strictly dominated by CN and the action NC is strictly dominated by CC. The game obtained by eliminating these actions has a unique mixed strategy Nash equilibrium in which player 1 chooses CN with probability $\frac{2}{3}$ and CC with probability $\frac{1}{3}$ and player 2 chooses A with probability $\frac{1}{2}$ and D with probability $\frac{1}{2}$.

Thus the game has a unique Nash equilibrium in which player 1 chooses NN and NC with probability 0, CN with probability $\frac{2}{3}$, and CC with probability $\frac{1}{3}$, and player 2 chooses A with probability $\frac{1}{2}$ and D with probability $\frac{1}{2}$.

2. The strategic form of the game is shown in Figure 1. The players' best responses are indicated by asterisks. We see that the game has two pure strategy Nash equilibria, (D, c, L) and (C, c, R).

	c	d
C	$1\ ,1\ ,1^*$	$4^*,4^*,0$
D	$3^*,3^*,2^*$	$3\ ,3^*,2^*$

L

	c	d
C	$1^*,1^*,1^*$	$0^*,0\ ,1^*$
D	$0\ ,0^*,0$	$0^*,0^*,0$

R

Figure 1. The strategic form of the game in Exercise 2.

Consider the equilibrium (D, c, L). Player 2's action c is not sequentially rational (her action d yields her the payoff 4, given player 3's strategy), so there is no weak sequential equilibrium in which (D, c, L) is the strategy profile.

Now consider the equilibrium (C, c, R). The actions of players 1 and 2 are both optimal, given the other players' strategies. Player 3's information set is not reached, so we are free to specify any belief there. If player 3 believes that the history is D with probability at most $\frac{1}{3}$, her action R is optimal. Thus the game has weak sequential equilibria in which the strategy profile is (C, c, R), and player 3's belief assigns probability of at most $\frac{1}{3}$ to D.

3. Consider the strategy pair in which the offspring squawks if and only if it is hungry, and the parent gives it the food if and only if it squawks. The consistency condition requires the parent to believe that her offspring is hungry if and only if it squawks, so for the parent's strategy to be sequentially rational we need

$$1 + rV \geq S + r \quad \text{and} \quad S + r(1-t) \geq 1.$$

For the offspring's strategy to be sequentially rational we need

$$1 - t + rS \geq r \quad \text{and} \quad V + r \geq 1 - t + rS.$$

Combining these two conditions yields the requirements

$$\frac{1-S}{1-t} \leq r \leq \frac{1-S}{1-V} \quad \text{and} \quad \frac{1-V-t}{1-S} \leq r \leq \frac{1-t}{1-S}.$$

The condition $r < (1-V)/(1-S)$ is consistent with the left-hand side of the second inequality only if $t > 0$.

Now consider the strategy pair in which the offspring is quiet whether or not it is hungry, and the parent keeps the food whether or not the offspring squawks. The consistency condition requires the parent to believe that the offspring is hungry with probability p if it is quiet. The condition does not restrict the parent's belief if the offspring squawks; suppose that in this case the parent believes the offspring is not hungry. Then for the parent's strategy to be sequentially rational we need

$$p + (1-p)(1+rV) \geq S + r \quad \text{and} \quad 1 + rV(1-t) \geq S + r(1-t). \quad (1)$$

The parent's behavior does not depend on the offspring's action, so given that squawking is costly, the offspring's payoff when it is quiet is at least as high as its payoff when it squawks. Thus the strategy pair is a weak sequential equilibrium if the conditions in (1) are satisfied. These conditions are equivalent to

$$r \leq \frac{1-S}{1-(1-p)V} \quad \text{and} \quad r \leq \frac{1-S}{(1-t)(1-V)}.$$

If $r < (1-S)/(1-(1-p)V)$ then both these conditions are satisfied (because $1-(1-p)V > (1-t)(1-V)$ given $V < 1$).

4. Consider an assessment in which both types of worker choose the education level e^*. The consistency condition requires that a firm that observes e^* believe that the worker is type H with probability π and type L with probability $1-\pi$. Thus the firms' equilibrium wage offers after observing e^* are both equal to $\pi H + (1-\pi)L$, yielding a worker of ability K the payoff $\pi H + (1-\pi)L - e^*/K$.

   For the assessment to be a weak sequential equilibrium, neither type of worker must be able to increase her payoff by choosing a different value of e. The wage optimally offered by the firms to such a worker of course depends on the firms' beliefs. The belief that makes a profitable deviation by a worker least likely (and hence supports the widest range of equilibrium values of e^*) is that in which each firm believes that a worker who chooses $e \neq e^*$ has ability L. In response to this belief, each firm offers the wage L, yielding a worker of ability K the payoff $L - e/K$. If $e^* = 0$ then certainly neither type of worker can gain by deviating. If $e^* > 0$ then, given that the value of e that maximizes this payoff is 0, for equilibrium we need

$$\pi H + (1-\pi)L - e^*/K \geq L \text{ for } K = L, H.$$

   The value of the left-hand side is lower for $K = L$ than it is for $K = H$, so we need

$$\pi H + (1-\pi)L - e^*/L \geq L,$$

   or

$$e^* \leq \pi L(H-L).$$

   In summary, for any $e^* \leq \pi L(H-L)$ the game has a pooling equilibrium in which both types of worker obtain the education level e^*.

   Given $\pi \leq 1$, the education levels possible in a pooling equilibrium are all less than those possible in any separating equilibrium.