Economics 316

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Solutions to problems for Tutorial 12

1. (a) The game has two pure strategy Nash equilibria, (L, R, L) and (R, R, R).

   (b) Consider the Nash equilibrium (L, R, L). Player 2's action is not sequentially rational—the action R generates the payoff 1 whereas the action L generates the payoff 4, given player 3's strategy. Thus the game has no weak sequential equilibrium in which the strategy profile is (L, R, L).

   Now consider the Nash equilibrium (R, R, R). The strategies of players 1 and 2 are both sequentially rational. Denote the belief at player 3's information set by $(p, 1-p)$. Player 3's information set is not reached if the players follow their strategies, so this belief is not constrained by the weak consistency condition. Given such a belief, player 3's strategy is sequentially rational if $1 - p \geq 2p$, or $p \leq \frac{1}{3}$.

   We conclude that the set of weak sequential equilibria is the set of assessments (β, μ) for which $\beta = (R, R, R)$ and $\mu(\{L, (R, L)\})(L) \leq \frac{1}{3}$.

2. The assessments $(((1,0,0),(q,1-q)),(\frac{1}{2},\frac{1}{2}))$ with $0 \leq q \leq \frac{3}{5}$ and $(((1,0,0),(0,1)),(p,1-p))$ with $0 \leq p \leq \frac{1}{2}$ are both weak sequential equilibria.

   Given that player 1 chooses L, weak consistency puts no restriction on player 2's belief that the history is M. Denote the value p.

   If $p > \frac{1}{2}$, then player 2's optimal action is L, in which case M is better for player 1 than L, so there is no weak sequential equilibrium in which player 1 chooses L and $p > \frac{1}{2}$.

   If $p = \frac{1}{2}$, then player 2 is indifferent between L and R. Denote the probability that player 2 chooses L by q. For L to be optimal for player 1 we require $1 \geq 3q - 2(1-q)$ and $1 \geq 2q - (1-q)$, or $q \leq \frac{3}{5}$.

Thus the game has a weak sequential equilibrium $(((1,0,0),(q,1-q)),(\frac{1}{2},\frac{1}{2}))$ for any q with $0 \leq q \leq \frac{3}{5}$.

If $p < \frac{1}{2}$, then player 2's optimal action is R, and hence L is optimal for player 1. Thus the game has a weak sequential equilibrium $(((1,0,0),(0,1)),(p,1-p))$ for any p with $0 \leq p < \frac{1}{2}$.

Thus an assessment $((1,0,0),(q,1-q)),(p,1-p))$ is a weak sequential equilibria in which player 1 chooses L if and only if it either (a) $0 \leq q \leq \frac{3}{5}$ and $p = \frac{1}{2}$ or (b) $q = 0$ and $0 \leq p \leq \frac{1}{2}$.