Economics 316

Fall 2017 Martin J. Osborne

Problem Set 12

1. Find the strategic form and Nash equilibria of the extensive game with imperfect information in Figure 1. When looking for Nash equilibria, use the strategic form and first eliminate any strictly dominated actions.

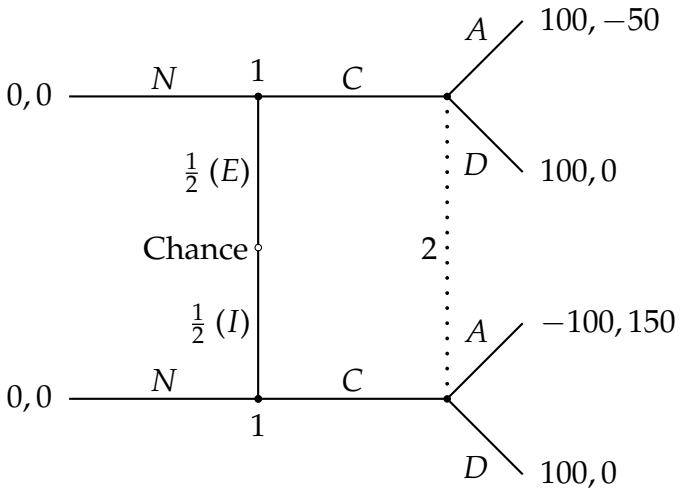

Figure 1. The game in Problem 1.

2. Find the weak sequential equilibria of the game in Figure 2 in which each player's strategy is pure. (Find the pure strategy Nash equilibria, then determine which of these equilibria are part of a weak sequential equilibrium. The game is known as "Selten's horse", after the person who first studied it and its shape.)

3. Some young animals expend energy begging for food from their parents—they squawk and bleat and scream, sometimes extravagantly. Can we expect these demands to signal their needs accurately? Consider the following signaling game.

   A hungry parent has a piece of food that it may give to its offspring, or keep for itself. It does not detect whether its offspring is hungry. In

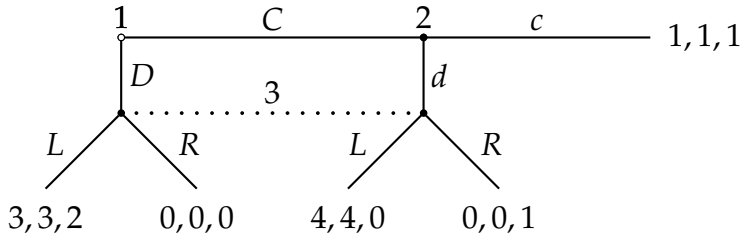

Figure 2. The game in Exercise 2 (Selten's horse).

either case, the offspring may signal that it is hungry to its parent (by squawking, for example). An animal is stronger and thus produces more offspring (i.e. has a higher "biological fitness") if it gets the food than if it does not. Normalize the parent's strength if it keeps the food to be 1, and denote its strength if it gives the food to its offspring by $S < 1$. If the offspring does not squawk, its strength is 1 if it gets the food, $V < 1$ if it is not hungry and does not get the food, and 0 if it is hungry and does not get the food. If the offspring squawks, its strength is multiplied by the factor $1 - t$, where $0 \leq t \leq 1$ (i.e. squawking may be costly). Denote the degree to which the parent and offspring are related by r, and take each player's payoff to be its strength plus r times the other player's strength. Evolutionary pressure will lead to behavior for each player that maximizes that player's payoff, given the other player's behavior. The game is shown in Figure 3.

Find the conditions on r, in terms of S, V, and t, under which the game has a separating equilibrium in which the offspring squawks if and only if it is hungry and the parent gives it the food if and only if it squawks. Show that if the offspring's payoff from obtaining the food when it is quiet exceeds its payoff from not obtaining it, whether or not it is hungry (which means that $r < (1 - V)/(1 - S)$), then the game has such an equilibrium only if $t > 0$. That is, in this case an equilibrium exists in which the signal is accurate only if the signal is costly. Show that if $r < (1 - S)/(1 - (1 - p)V)$, then the game has a pooling equilibrium in which the offspring is always quiet and the parent always keeps the food. (For some other parameter values, the game has a pooling equilibrium in which the offspring is always quiet and the parent always gives the food.)

4. For the game studied in class in which education is a signal of ability, find the range of education levels e for which there is a weak sequen-

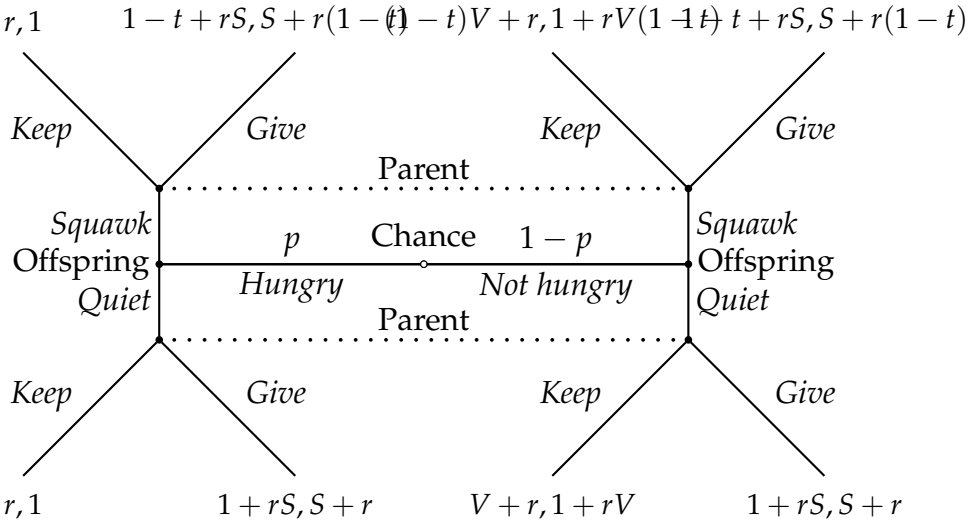

Figure 3. The Sir Philip Sydney game (Exercise 3). The empty history is in the center of the diagram. The offspring's payoff is listed first, the parent's second.

tial equilibrium in which both types of worker choose the education level e. Compare these levels with those possible in a separating equilibrium.