ECO316: Applied game theory

Lecture 12

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- Extensive game with perfect information: players are perfectly informed about *past* actions

Extensive games with imperfect information

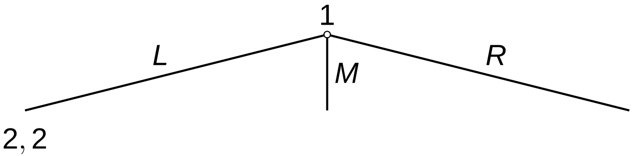
- Extensive game with perfect information: players are perfectly informed about *past* actions
- Now consider games in which players are *not* perfectly informed about past actions

Extensive games with imperfect information

- Extensive game with perfect information: players are perfectly informed about *past* actions
- Now consider games in which players are *not* perfectly informed about past actions
- Example: firm may know whether another firm has entered its industry, but may not know how much the entrant has decided to spend on R & D

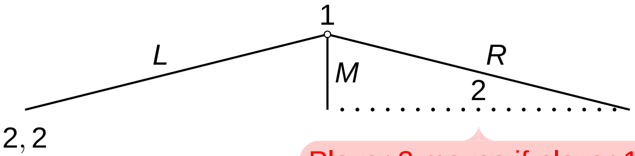
Extensive games with imperfect information

Example



Extensive games with imperfect information

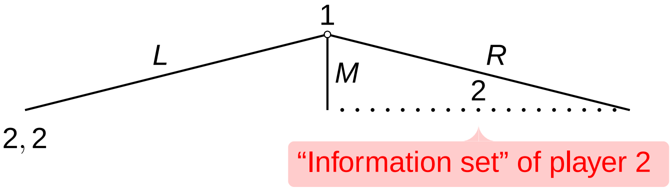
Example



Player 2 moves if player 1 chooses M or R, but does not know whether player 1 chose M or R

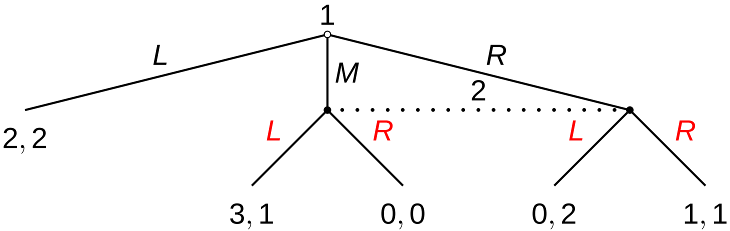
Extensive games with imperfect information

Example



Extensive games with imperfect information

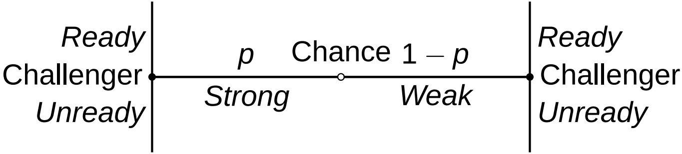
Example



Set of actions available to player 2 at each history in her information set ($\{L, R\}$) is the same (if not, she could deduce player 1's action from set of actions available to her)

Extensive games with imperfect information

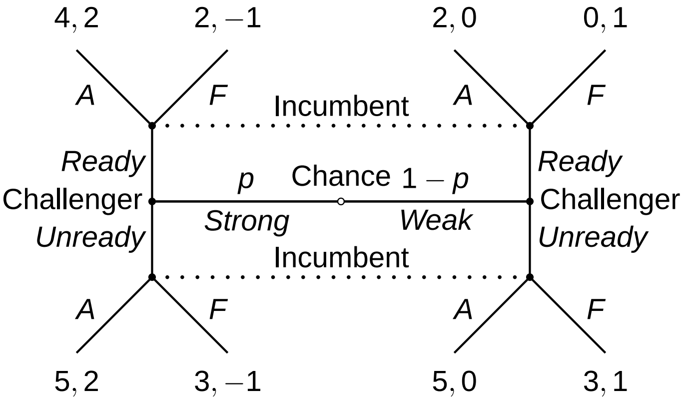
Example with chance move



Challenger is Strong with probability p and Weak with probability $1 - p$, and knows her type. In each case she has two actions, *Ready* and *Unready*.

Extensive games with imperfect information

Example with chance move



Incumbent knows Challenger's action, but not her type

Extensive games with imperfect information

Definition

An extensive game consists of

- A set N of players

Extensive games with imperfect information

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Extensive games with imperfect information

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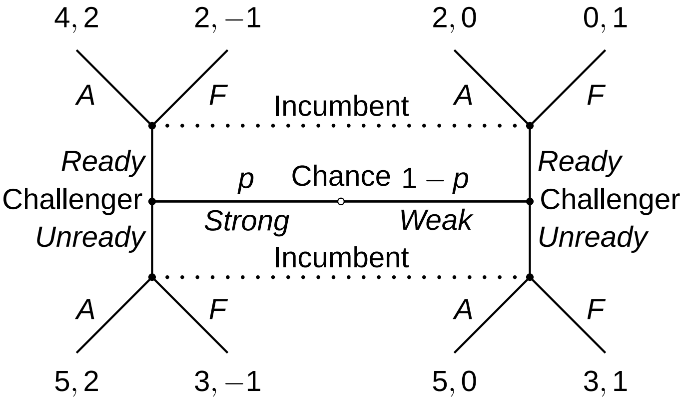
- A specification of the probabilities used by chance
- A specification of the set of histories among which each player cannot distinguish at each point at which she moves (the player's *information sets*)
- Preferences over terminal histories for each player

Extensive games with imperfect information

Example



Extensive games with imperfect information

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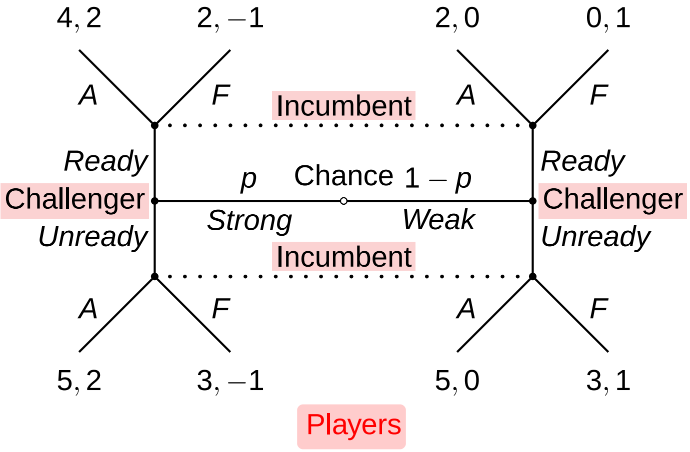

Extensive games with imperfect information

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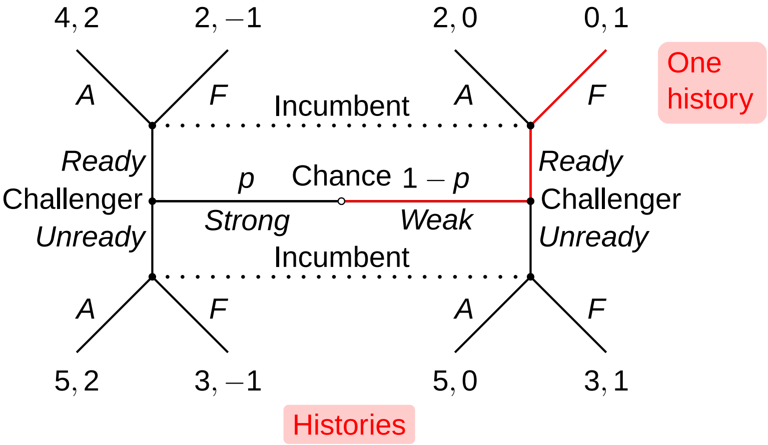
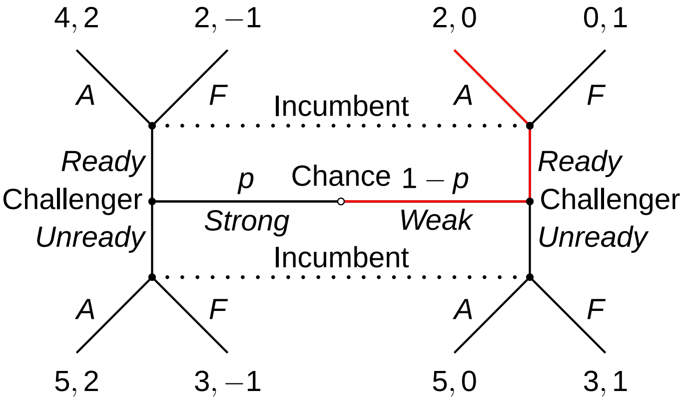


Extensive games with imperfect information

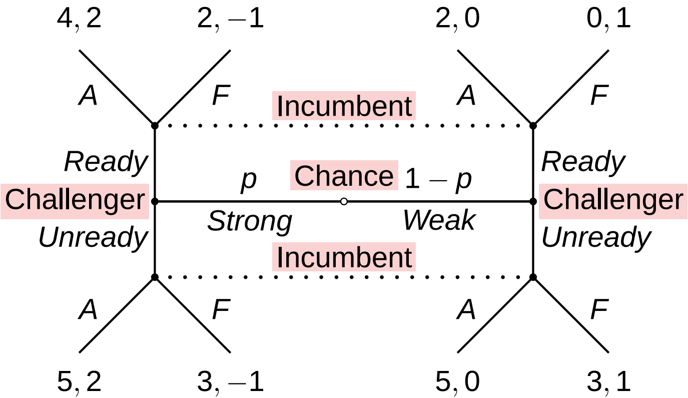
Another history

Example



Extensive games with imperfect information

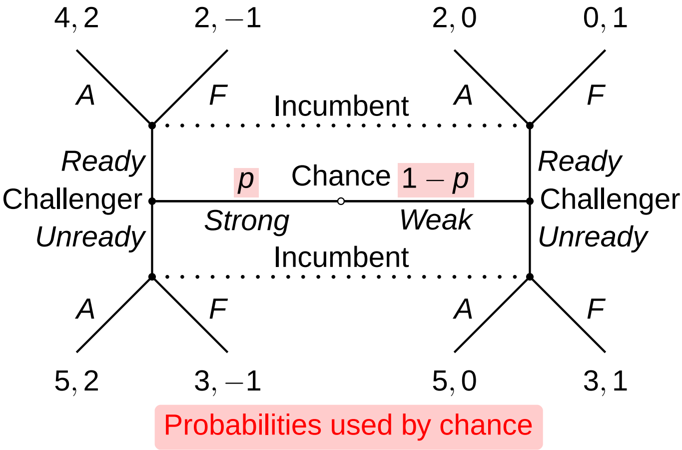
Example



Player function—assigns players and chance to histories

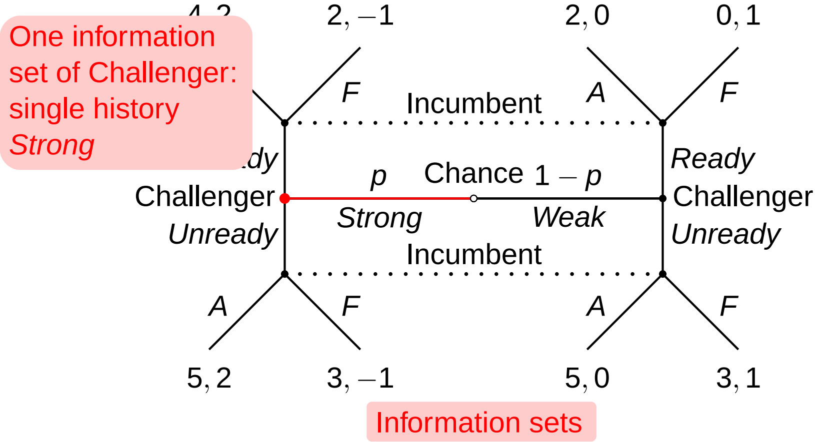
Extensive games with imperfect information

Example



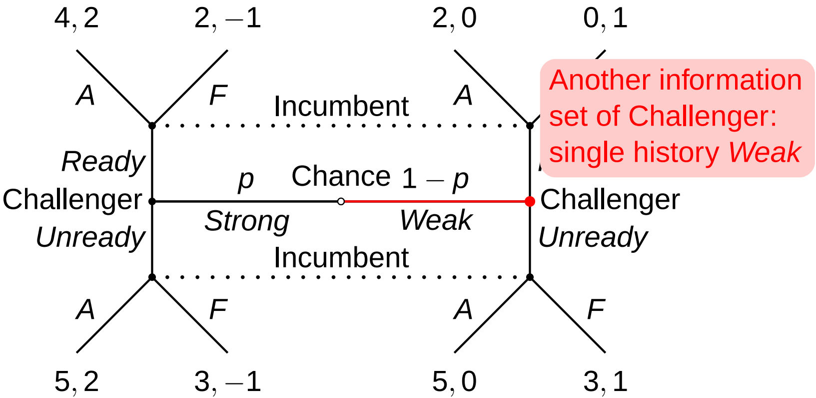
Extensive games with imperfect information

Example



Extensive games with imperfect information

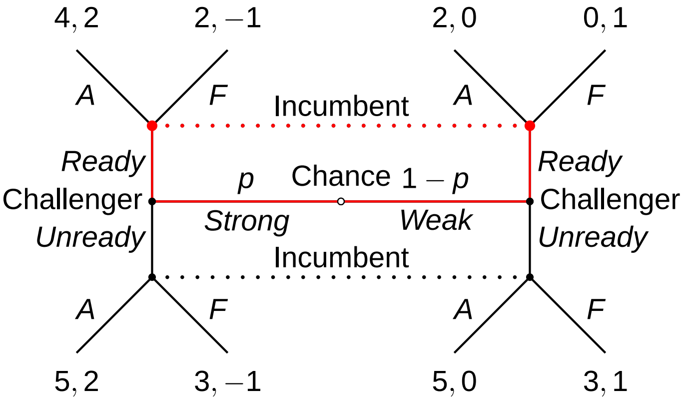
Example



Extensive games with imperfect information

One information set of Incumbent: histories (*Strong*, *Ready*) and (*Weak*, *Ready*)

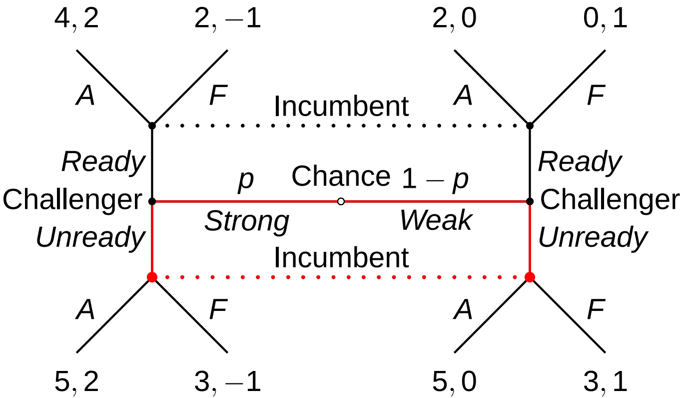
Example



Extensive games with imperfect information

Another information set of Incumbent: histories (*Strong*, *Unready*) and (*Weak*, *Unready*)

Example



Extensive games with imperfect information

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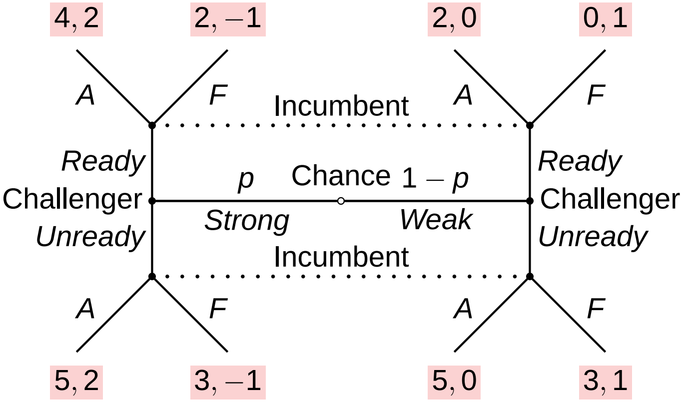
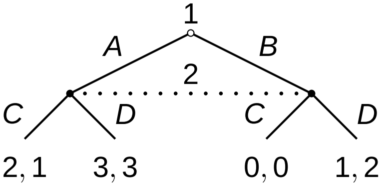


Payoffs representing preferences

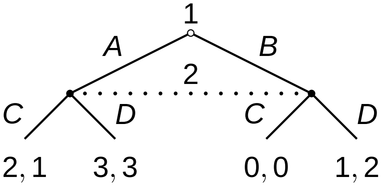
Extensive games with imperfect information

Example



Extensive games with imperfect information

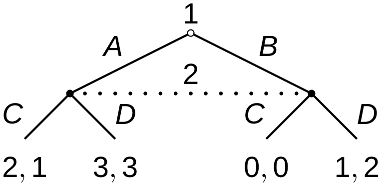
Example



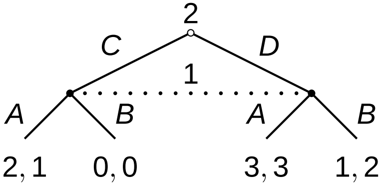
Models same situation as strategic game in which players 1 and 2 choose actions simultaneously

Extensive games with imperfect information

Example

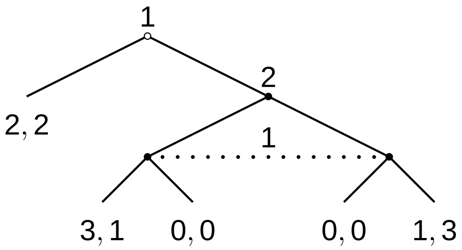


Models same situation as strategic game in which players 1 and 2 choose actions simultaneously, as does following game



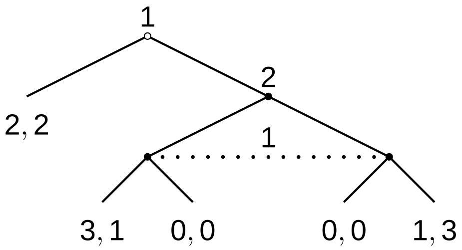
Extensive games with imperfect information

Example



Extensive games with imperfect information

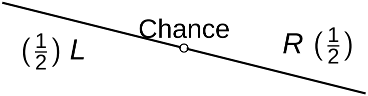
Example



Models situation in which player 1 moves and then players 1 and 2 move simultaneously

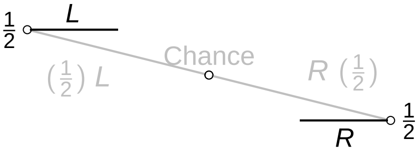
Extensive games with imperfect information

Example



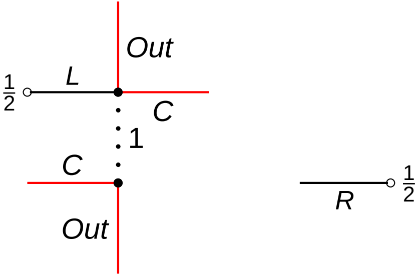
Extensive games with imperfect information

Example



Extensive games with imperfect information

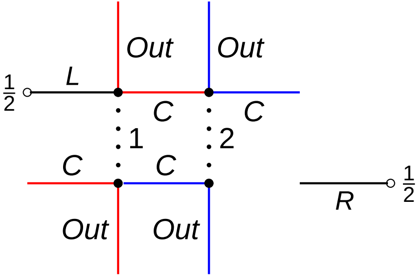
Example



- Player 1 does not know whether she is the first mover or whether she is moving after the other players have moved

Extensive games with imperfect information

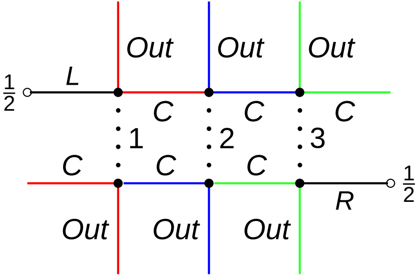
Example



- Player 1 does not know whether she is the first mover or whether she is moving after the other players have moved
- Player 2 does not know whether she is moving after player 1 and before player 3, or the other way around

Extensive games with imperfect information

Example



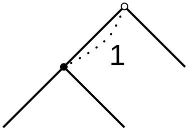
- Player 1 does not know whether she is the first mover or whether she is moving after the other players have moved
- Player 2 does not know whether she is moving after player 1 and before player 3, or the other way around
- Player 3 does not know whether she is the first mover or whether she is moving after the other players have moved

Extensive games with imperfect information

Examples

Extensive games with imperfect information

Examples



Extensive games with imperfect information

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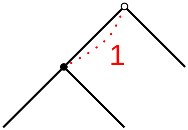

Player does not know whether she is choosing action at start of game, or whether she has already chosen an action

Extensive games with imperfect information

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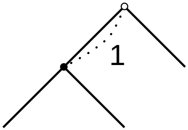

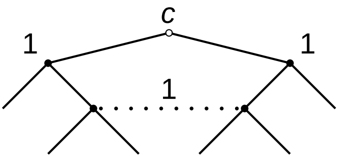

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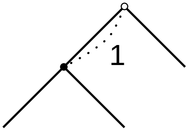

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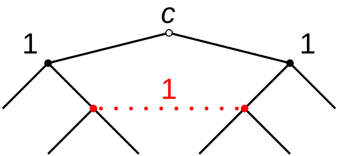
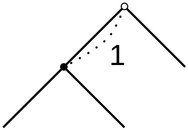


When choosing at her last information set, player does not know move of chance, which she knew at start of game

Extensive games with imperfect information

Examples



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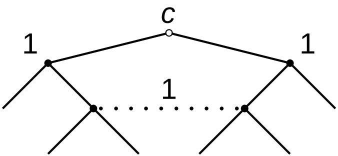

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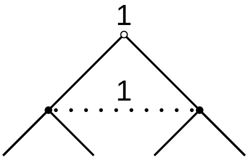

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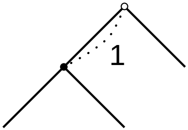

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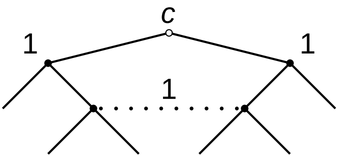

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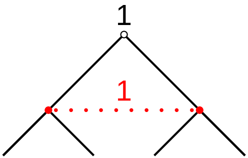
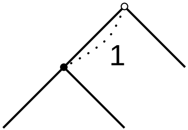


When making her second choice, player does not know action she chose at start of game

Extensive games with imperfect information

Examples



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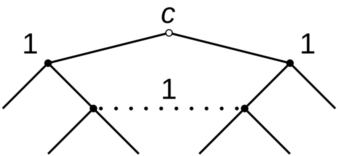

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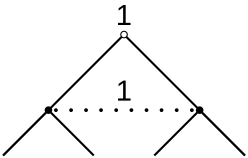

When making her second choice, player does not know action she chose at start of game

These games have *imperfect recall*

Extensive games with imperfect information

Perfect recall

- Game has perfect recall if at every point every player remembers whatever she knew in the past

Extensive games with imperfect information

Perfect recall

- Game has perfect recall if at every point every player remembers whatever she knew in the past
- Will restrict throughout to games with perfect recall

Strategies

Definition

A pure strategy of a player in an extensive game is a function that assigns an action to every information set at which the player moves

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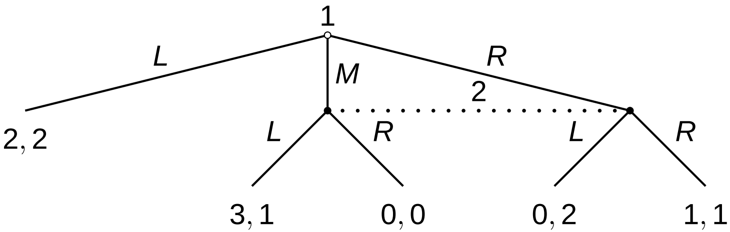
A pure strategy of a player in an extensive game is a function that assigns an action to every information set at which the player moves

Given set of strategies for each player, can define strategic form of extensive game as before

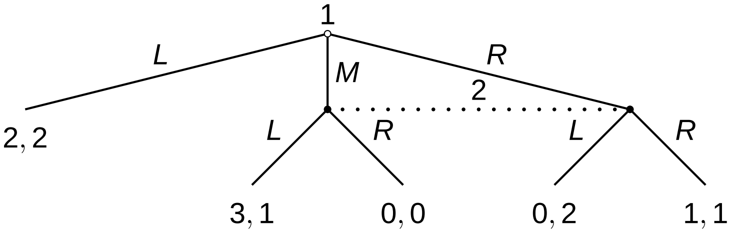
Strategic form

Example



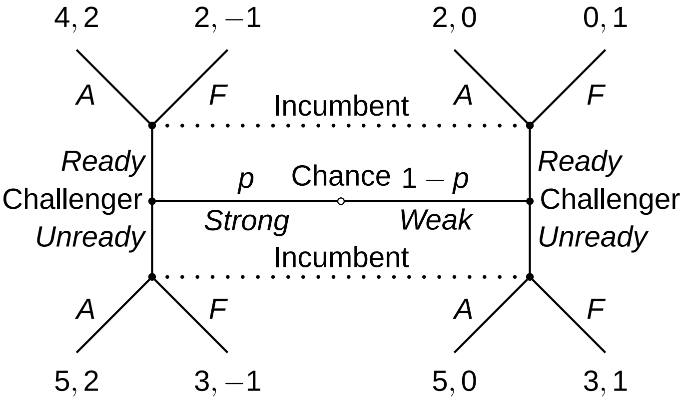
Strategic form

Example

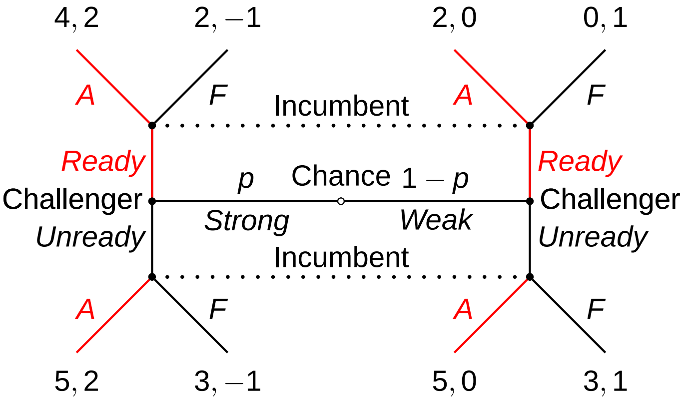


	L	R
L	2, 2	2, 2
M	3, 1	0, 0
R	0, 2	1, 1

Strategies

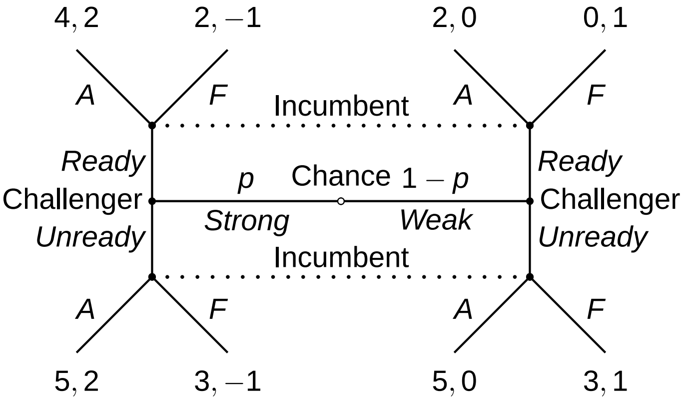

4, 2
2, −1
2, 0
0, 1
A
F
Incumbent
A
F
Ready
Ready
p
Chance
1 − p
Challenger
Challenger
Strong
Weak
Unready
Unready
Incumbent
A
F
A
F
5, 2
3, −1
5, 0
3, 1

Strategies



	AA	*AF*	*FA* ··· *FF*
RR	$4p + 2(1-p), 2p$		...
RU			...
UR			...
UU			...

Strategies



	AA	AF	$FA \cdots FF$
RR	$4p + 2(1-p), 2p$	$4p, 2p + 1 - p$	...
RU	$4p + 5(1-p), 2p$	$4p + 3(1-p), 2p + 1 - p$	...
UR	$5p + 2(1-p), 2p$	$5p, 2p + 1 - p$	...
UU	$5, 2p$	$5p + 3(1-p), 2p + 1 - p$	...

Nash equilibrium

Definition

A Nash equilibrium of an extensive game is a Nash equilibrium of the strategic form of the game

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- For any strategy profile s, let $O(s)$ be the outcome of s: the terminal history generated by s

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- For any strategy profile s, let $O(s)$ be the outcome of s: the terminal history generated by s

Definition

A Nash equilibrium of an extensive game is a strategy profile s such that for every player $i \in N$

$$O(s^*_{-i}, s^*_i) \succsim_i O(s^*_{-i}, s_i) \text{ for every strategy } s_i \text{ of player } i$$

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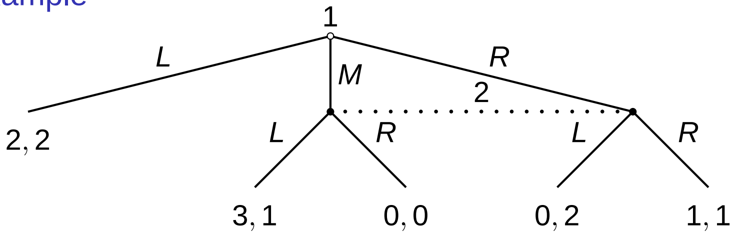
$$O(s^*_{-i}, s^*_i) \succsim_i O(s^*_{-i}, s_i) \text{ for every strategy } s_i \text{ of player } i$$

That is, no player is better off deviating from her strategy, given the other players' strategies

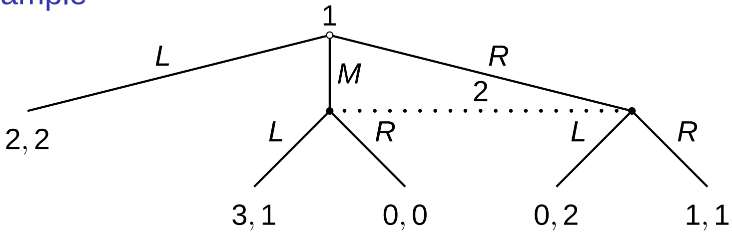
Nash equilibrium

Example



Nash equilibrium

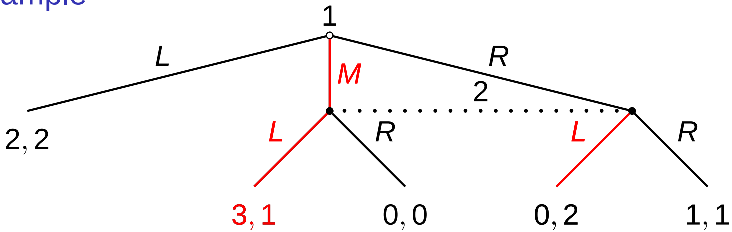
Example



- One Nash equilibrium:

Nash equilibrium

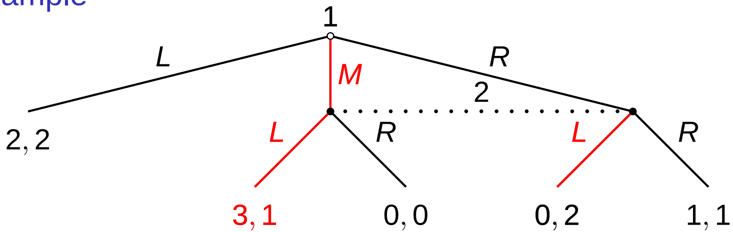
Example



- One Nash equilibrium: (M, L)

Nash equilibrium

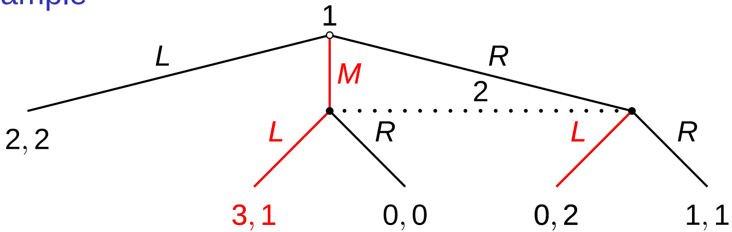
Example



- One Nash equilibrium: (M, L)
- Player 1's payoff: 3

Nash equilibrium

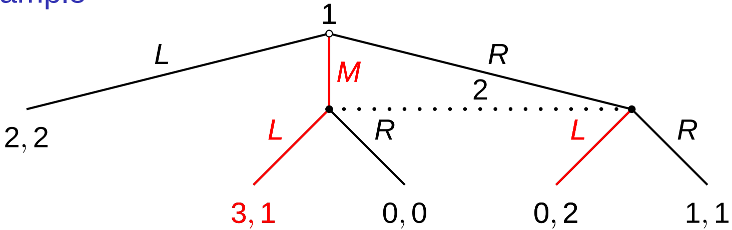
Example



- One Nash equilibrium: (M, L)
- Player 1's payoff: 3
- Player 1's payoff to deviating to L: 2

Nash equilibrium

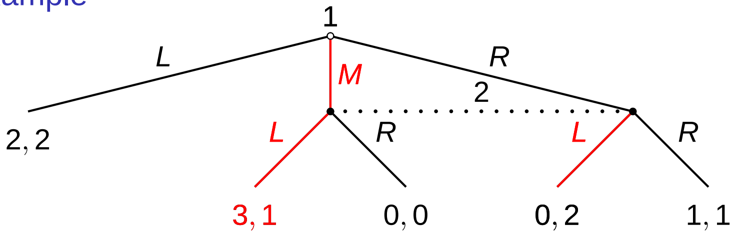
Example



- One Nash equilibrium: (M, L)
- Player 1's payoff: 3
- Player 1's payoff to deviating to L: 2
- Player 1's payoff to deviating to R: 0

Nash equilibrium

Example



- One Nash equilibrium: (M, L)
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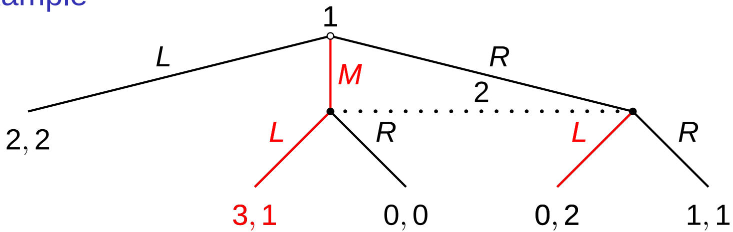
Nash equilibrium

Example



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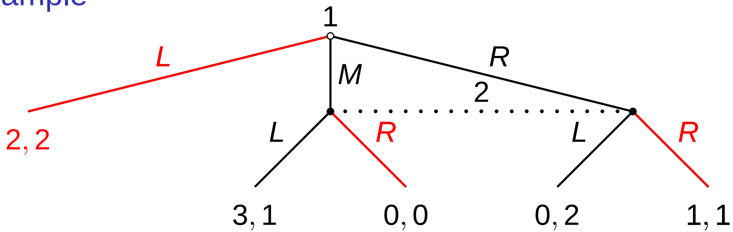
Nash equilibrium

Example



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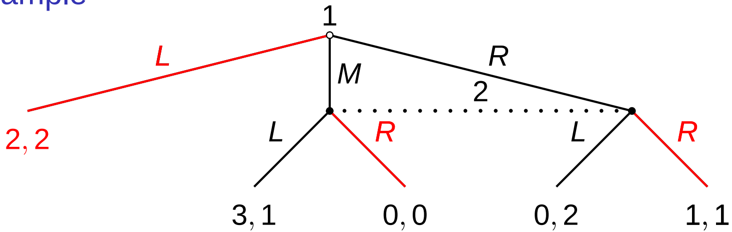
Nash equilibrium

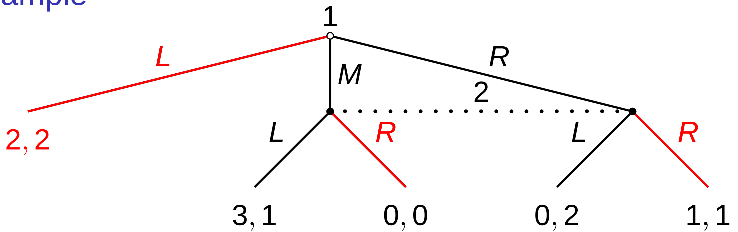
Example



- Another Nash equilibrium: (L, R)

Nash equilibrium

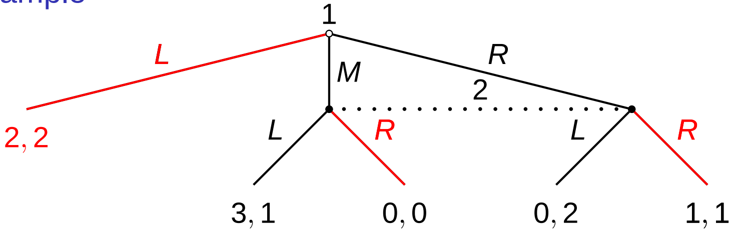
Example



- Another Nash equilibrium: (L, R)
- Player 1's payoff: 2

Nash equilibrium

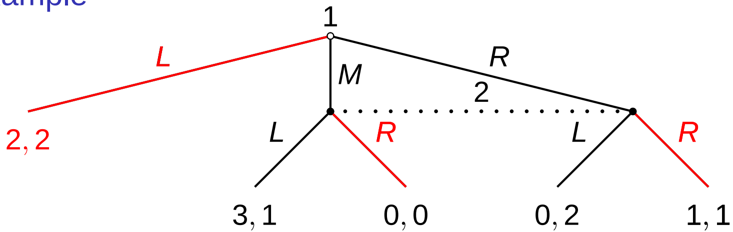
Example



- Another Nash equilibrium: (L, R)
- Player 1's payoff: 2
- Player 1's payoff to deviating to M: 0

Nash equilibrium

Example



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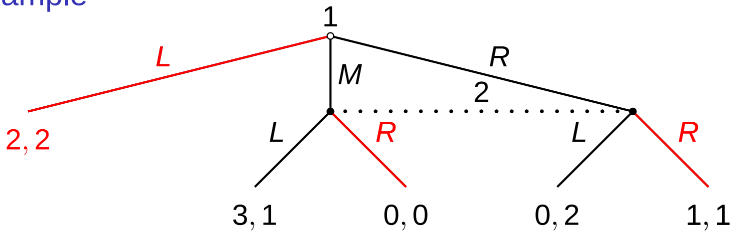
Nash equilibrium

Example



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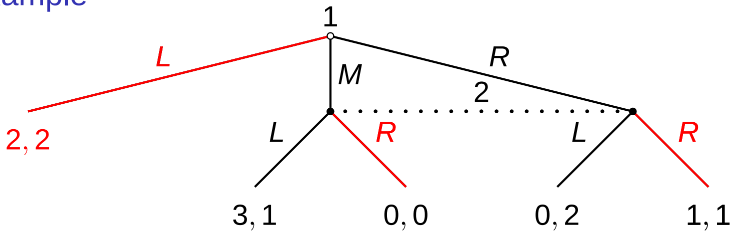
Nash equilibrium

Example



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- Player 1's payoff: 2
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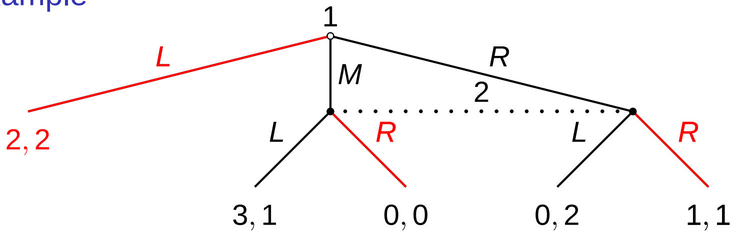
- Player 2's payoff: 2
- Player 2's payoff to deviating to L: 2

Nash equilibrium

Example



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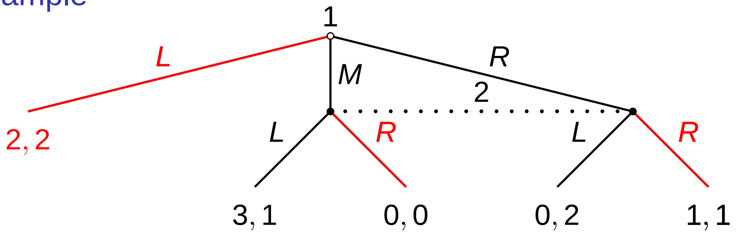
Nash equilibrium

Example



- Another Nash equilibrium: (L, R)
  - But if player 1 deviates to M or R, player 2's action L is better than R *regardless of whether she believes player 1 chose M or R*

Nash equilibrium

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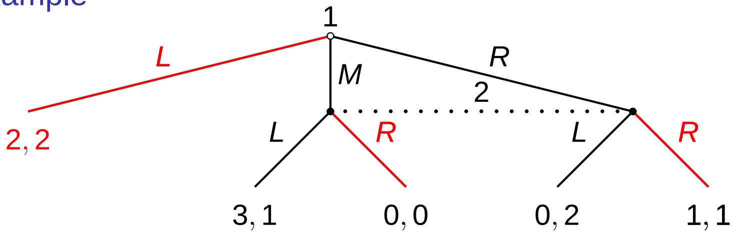

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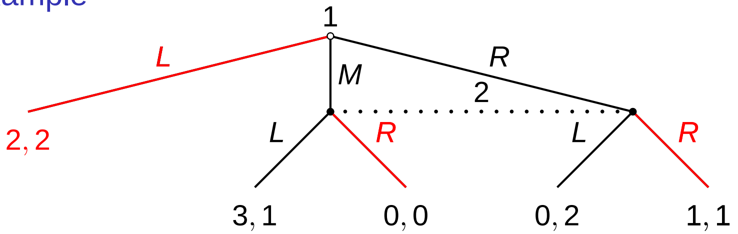
Nash equilibrium

Example



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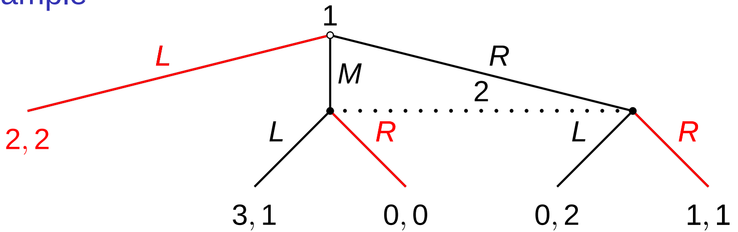
  - But this NE *is* subgame perfect, because the game has no proper subgame

Nash equilibrium

Example



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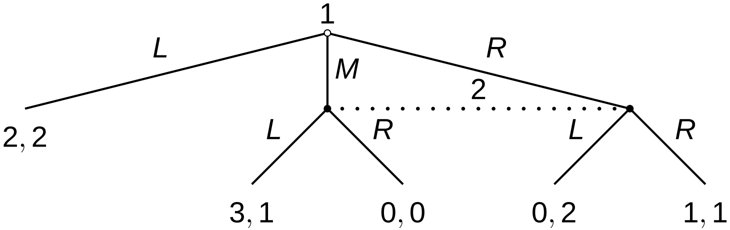
  - Like incumbent's action *Fight* in NE (*Out*, *Fight*) of Entry game, player 2's strategy R is not optimal if player 2's information set is reached
  - But this NE *is* subgame perfect, because the game has no proper subgame
  - We need new refinement of NE

Nash equilibrium

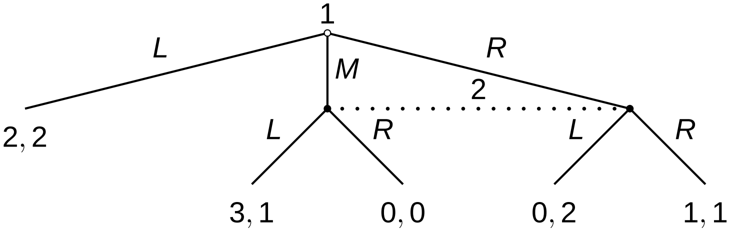
Example



- In this game, optimal action of player 2 is *L* regardless of her belief about whether player 1 chose *M* or *R*

Nash equilibrium

Example



- In this game, optimal action of player 2 is *L* regardless of her belief about whether player 1 chose *M* or *R*
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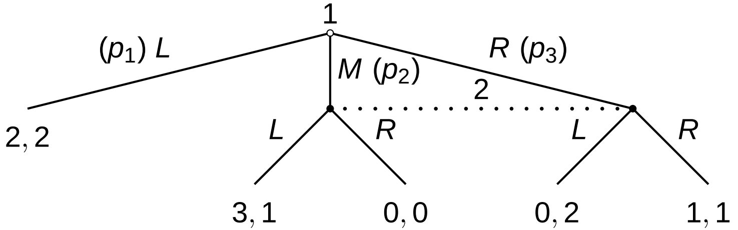
Nash equilibrium

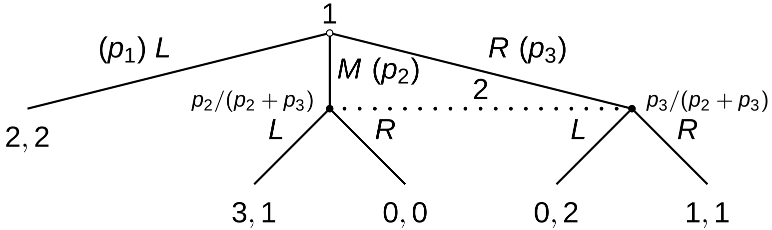
Example



- In this game, optimal action of player 2 is L regardless of her belief about whether player 1 chose M or R
- But for other payoffs, optimal action depends on her belief
- If player 1 chooses M and/or R with positive probability, player 2's belief can be derived from player 1's strategy

Nash equilibrium

Example



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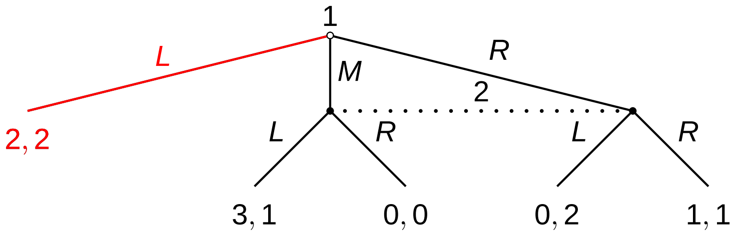

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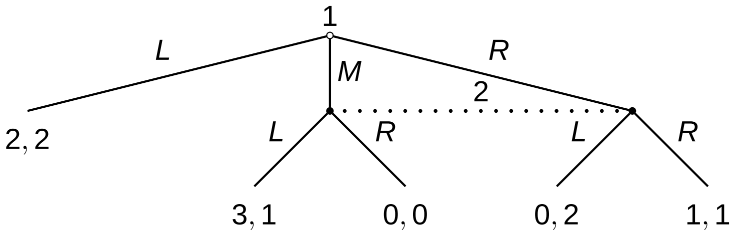
- If player 1 chooses M and/or R with positive probability, player 2's belief can be derived from player 1's strategy
- But if player 1 chooses L, player 2's belief *cannot* be derived from player 1's strategy

Nash equilibrium

Example



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- If player 1 chooses *M* and/or *R* with positive probability, player 2's belief can be derived from player 1's strategy
- But if player 1 chooses *L*, player 2's belief *cannot* be derived from player 1's strategy
  - Need to specify player 2's belief as part of equilibrium

Beliefs and assessments

A belief system for an extensive game is a function that assigns to every information set a probability distribution over the set of histories in the information set

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An assessment in an extensive game is pair (s, μ) where s is a strategy profile and μ is a belief system

Equilibrium

Sequential rationality

Each player’s strategy is optimal given her beliefs

Equilibrium

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Consistency of beliefs

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Equilibrium

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: Each player's strategy is optimal given her beliefs

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Sequential rationality

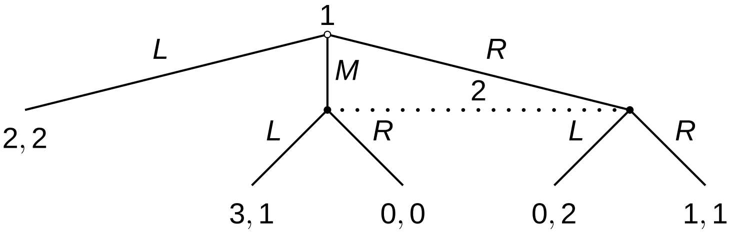
An assessment is sequentially rational if for every information set of every player, the player's strategy is a best response to the other players' strategies given the player's belief at the information set

Weak consistency

An assessment is weakly consistent if for every information set reached with positive probability given the strategy profile, the probability assigned by the beliefs to each history in the information set is derived by using Bayes' rule

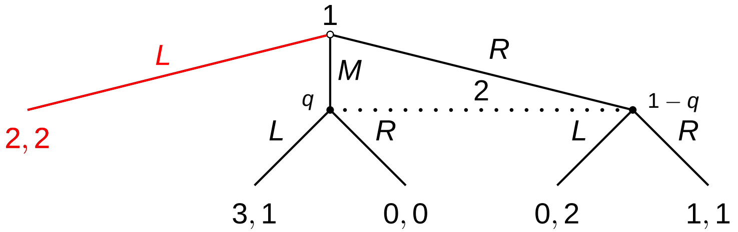
Equilibrium

Weak consistency



Equilibrium

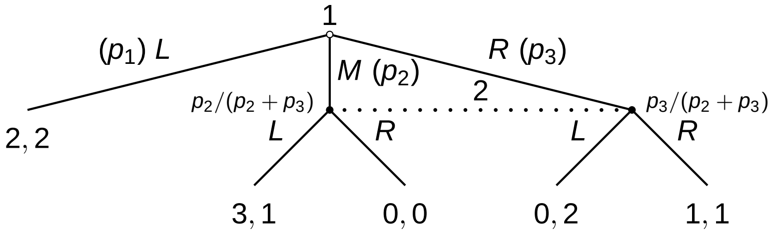
Weak consistency



- An assessment in which player 1 chooses *L* and player 2 holds *any* belief at her information set is weakly consistent: given player 1's strategy, player 2's information set is not reached

Equilibrium

Weak consistency



- An assessment in which player 1 chooses L and player 2 holds *any* belief at her information set is weakly consistent: given player 1's strategy, player 2's information set is not reached
- If $p_2 + p_3 > 0$, so that the information set *is* reached, then weak consistency requires that player 2's belief assign probability $p_2/(p_2 + p_3)$ to M and probability $p_3/(p_2 + p_3)$ to R

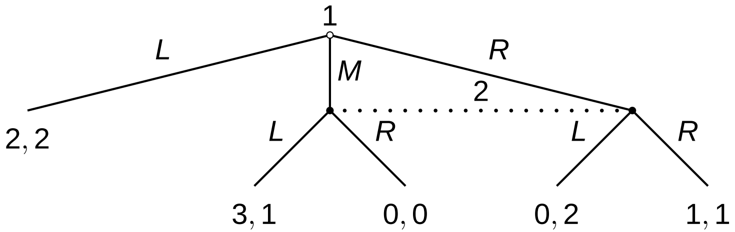
Equilibrium

Definition

An assessment is a weak sequential equilibrium of an extensive game if it is sequentially rational and weakly consistent

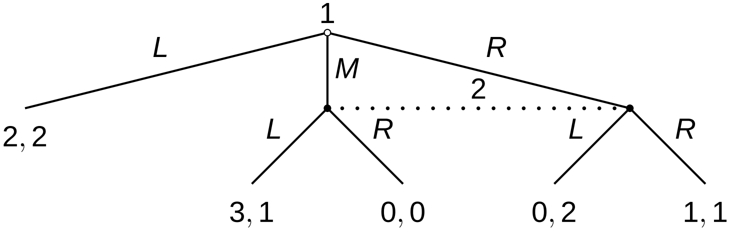
Equilibrium

Example 1



Equilibrium

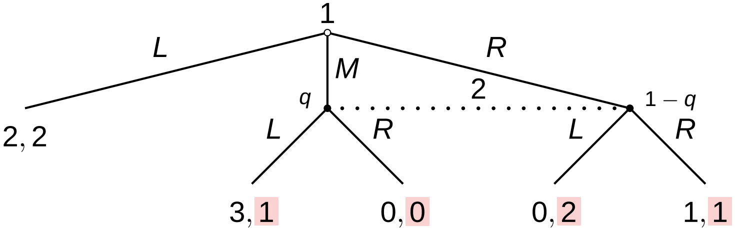
Example 1



- Start by looking at P2's choice

Equilibrium

Example 1



- Start by looking at P2's choice
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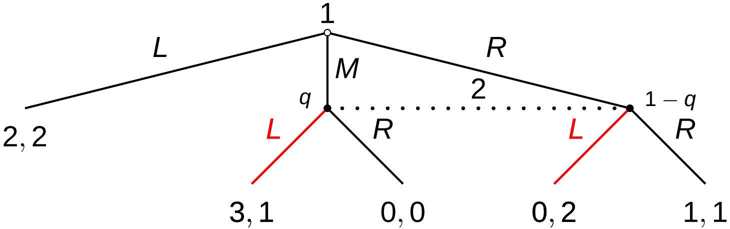
Equilibrium

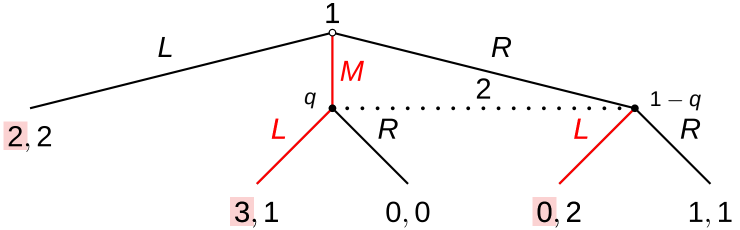
Example 1



- Start by looking at P2's choice
- For *any* belief at P2's information set, only L is optimal
- So in any WSE P2 chooses L

Equilibrium

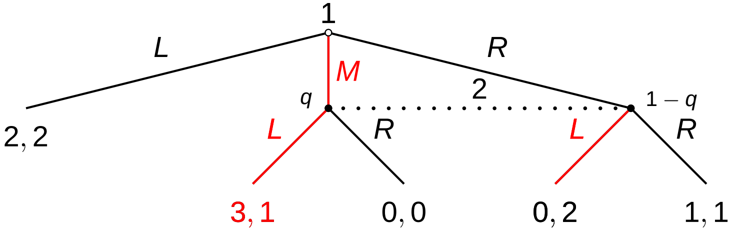
Example 1



- Start by looking at P2's choice
- For *any* belief at P2's information set, only *L* is optimal
- So in any WSE P2 chooses *L*
- Given that P2 chooses *L*, P1's optimal action is *M*

Equilibrium

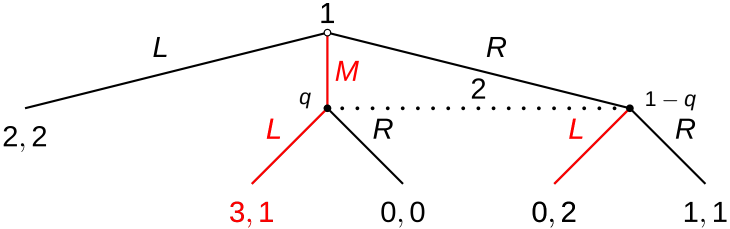
Example 1



- Start by looking at P2's choice
- For *any* belief at P2's information set, only L is optimal
- So in any WSE P2 chooses L
- Given that P2 chooses L, P1's optimal action is M
- What are P2's beliefs at her information set?

Equilibrium

Example 1



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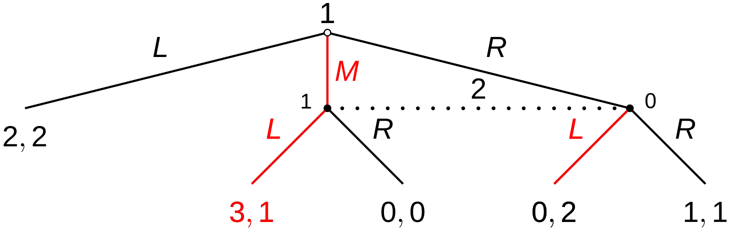
Equilibrium

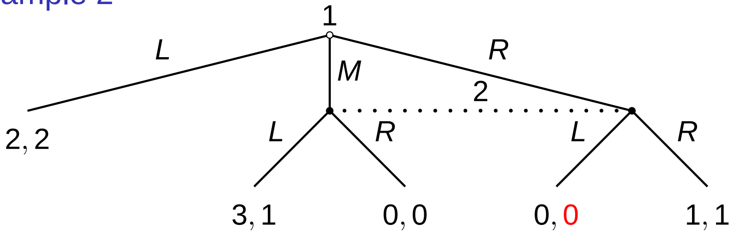
Example 1



- Start by looking at P2's choice
- For *any* belief at P2's information set, only L is optimal
- So in any WSE P2 chooses L
- Given that P2 chooses L, P1's optimal action is M
- What are P2's beliefs at her information set?
- Weak consistency $\Rightarrow q = 1$
- So unique WSE, with strategies (M, L) and beliefs $(1, 0)$

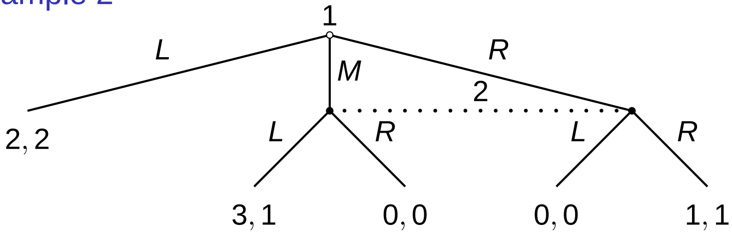
Equilibrium

Example 2



Equilibrium

Example 2



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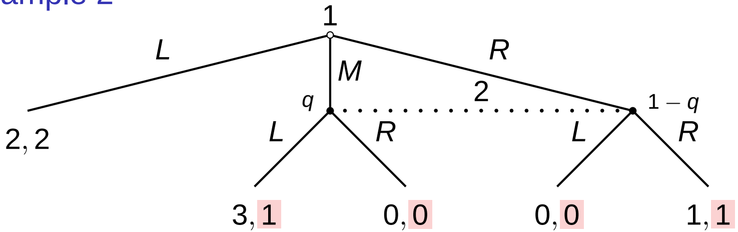
Equilibrium

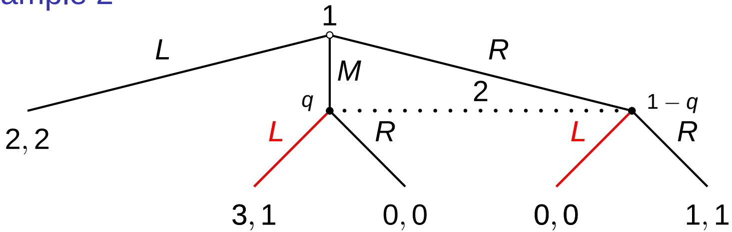
Example 2



- Start by looking at P2's choice
- If $q > \frac{1}{2}$ then L is only optimal action; if $q < \frac{1}{2}$ then R is only optimal action; if $q = \frac{1}{2}$ then both L and R are optimal

Equilibrium

Example 2



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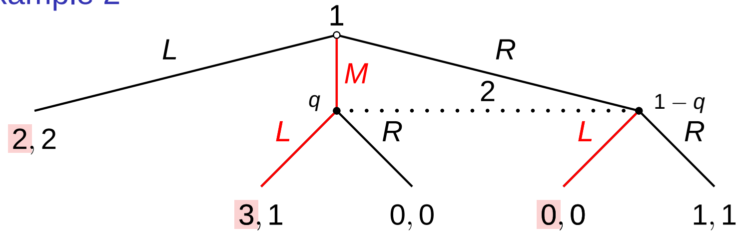
- If P2 chooses L then P1

Equilibrium

Example 2



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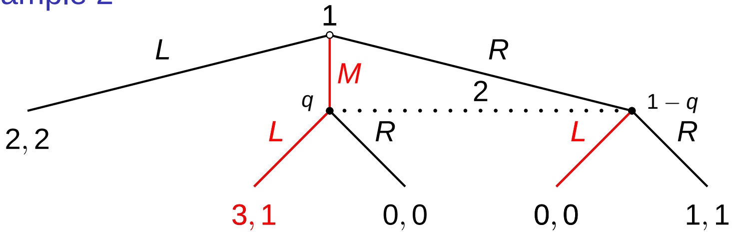
Equilibrium

Example 2



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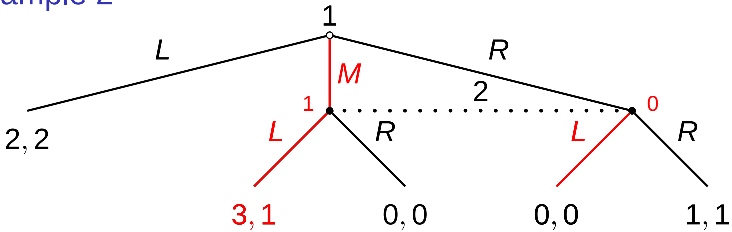
- If P2 chooses L then P1 chooses M $\Rightarrow$ beliefs

Equilibrium

Example 2



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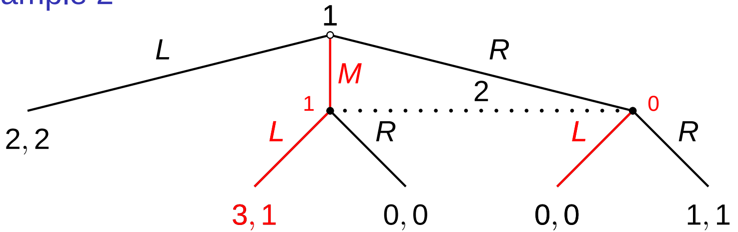
Equilibrium

Example 2



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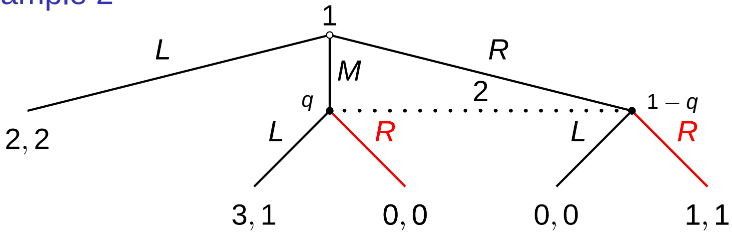
Equilibrium

Example 2



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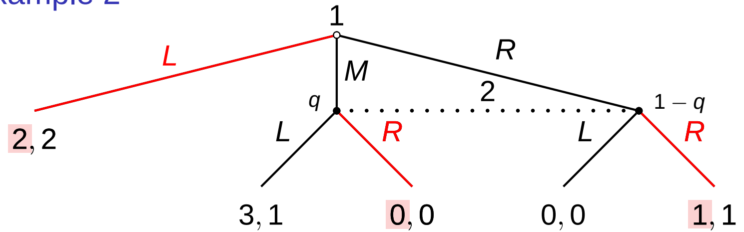
- If P2 chooses R then P1

Equilibrium

Example 2



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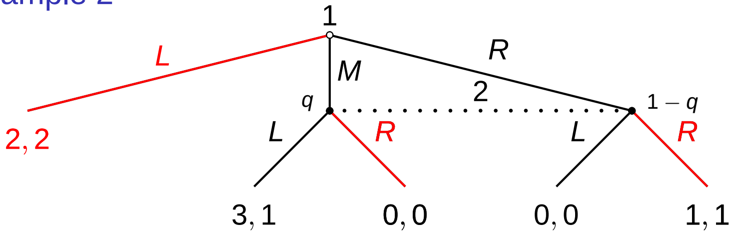
Equilibrium

Example 2



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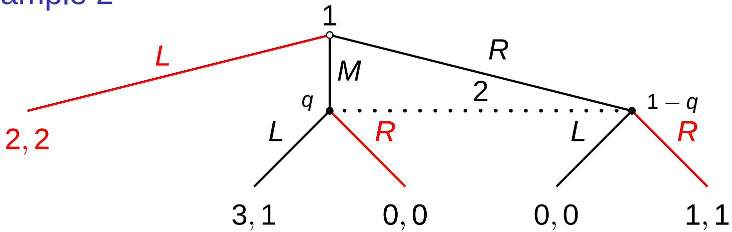
- If P2 chooses R then P1 chooses $L \Rightarrow$ beliefs

Equilibrium

Example 2



- Start by looking at P2's choice
- If $q > \frac{1}{2}$ then L is only optimal action; if $q < \frac{1}{2}$ then R is only optimal action; if $q = \frac{1}{2}$ then both L and R are optimal
- If P2 chooses L then P1 chooses $M \Rightarrow$ beliefs $(1, 0) \Rightarrow L$ is optimal $\Rightarrow$ assessment $((M, L), (1, 0))$ is WSE
- If P2 chooses R then P1 chooses $L \Rightarrow$ beliefs unrestricted by weak consistency; need $q \leq \frac{1}{2}$ for R to be optimal

Equilibrium

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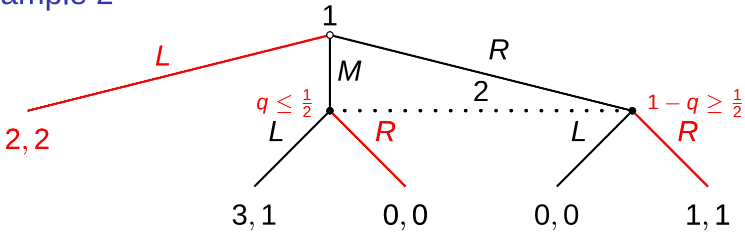

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Signaling games

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- Example:
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  - person of high ability may obtain a degree at low cost, while one of low ability may do so only at high cost
  - then the fact that a person has a degree may signal to an employer that she has high ability—not because college teaches any skills, but because only high-ability individuals find obtaining a degree worthwhile, given the cost

Two-player signaling games

- A *sender* is informed about a variable relevant to both her and a *receiver* who is uninformed

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- Extensive game in which chance move determines sender's value of variable about which sender is informed
- Receiver sees sender's action but not the value of the variable, and then herself takes an action

Signaling games: Entry

- A *challenger* contests an *incumbent*'s turf

Signaling games: Entry

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- Cost of getting ready for battle is higher for weak challenger
- Incumbent observes challenger's readiness, but not its type, and chooses whether to *fight* or *acquiesce*

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- Incumbent may *acquiesce* to (share market with) challenger or *fight* it

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So incumbent's payoff:

$$\begin{cases} 1 & \text{if } \textit{acquiesces} \\ 2 & \text{if } \textit{fights} \text{ and challenger is weak} \\ 0 & \text{if } \textit{fights} \text{ and challenger is strong} \end{cases}$$

Signaling games

Strong challenger's payoff:

$$\left\{ \begin{array}{l} \\ \\ \end{array} \right.$$

Signaling games

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More costly for a weak challenger that for a strong challenger to prepare for a fight:

$$c_S < c_W$$

Signaling games

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Assume $\pi = 5$, $c_s = 1$, $c_w = 3$, and $d = 2$, so that:

Signaling games

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Strong challenger's payoff:

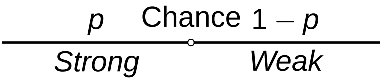
$$\begin{cases} 5 & \text{if incumbent } \textit{acquiesces} \text{ and challenger is } \textit{unready} \\ 4 & \text{if incumbent } \textit{acquiesces} \text{ and challenger is } \textit{ready} \\ 3 & \text{if incumbent } \textit{fights} \text{ and challenger is } \textit{unready} \\ 2 & \text{if incumbent } \textit{fights} \text{ and challenger is } \textit{ready} \end{cases}$$

Weak challenger's payoff:

$$\begin{cases} 5 & \text{if incumbent } \textit{acquiesces} \text{ and challenger is } \textit{unready} \\ 2 & \text{if incumbent } \textit{acquiesces} \text{ and challenger is } \textit{ready} \\ 3 & \text{if incumbent } \textit{fights} \text{ and challenger is } \textit{unready} \\ 0 & \text{if incumbent } \textit{fights} \text{ and challenger is } \textit{ready} \end{cases}$$

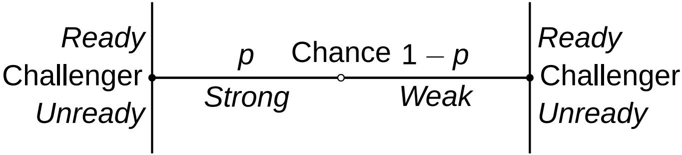
Signaling games: Entry

Challenger is *strong* with probability p and weak with probability $1 - p$



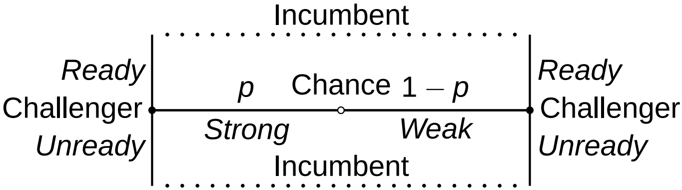
Signaling games: Entry

Challenger knows her type; each type has two actions, *Ready* and *Unready*



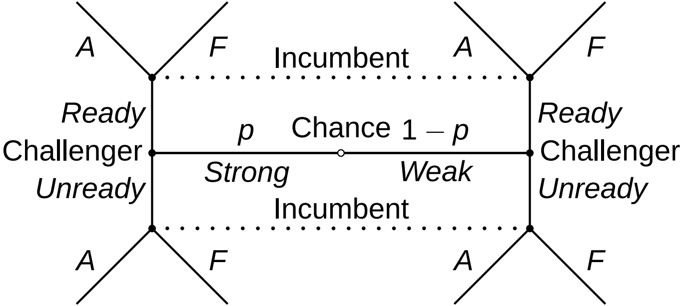
Signaling games: Entry

Incumbent observes only challenger's action, not her type



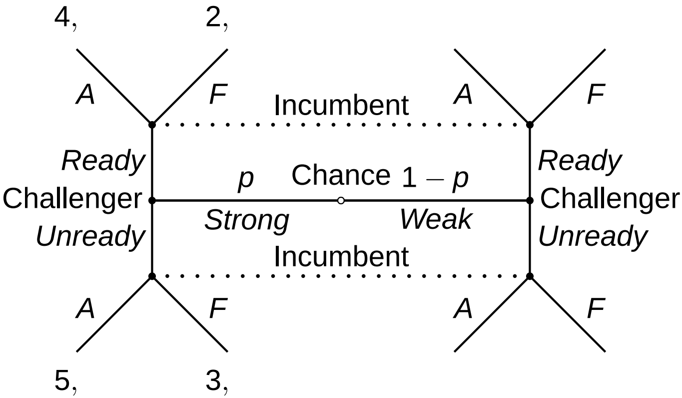
Signaling games: Entry

Incumbent has two actions, *acquiesce* and *fight*



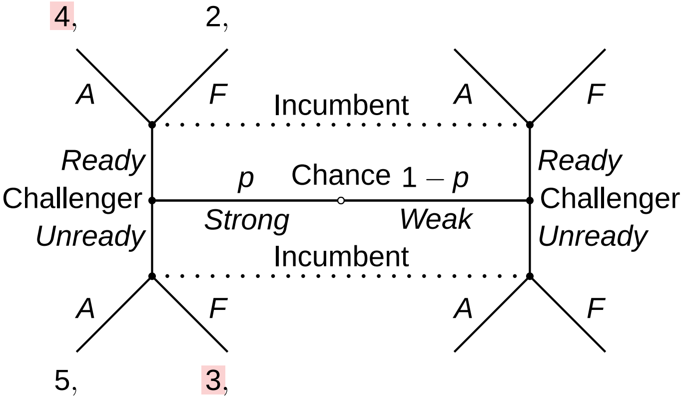
Signaling games: Entry

For strong challenger, cost of becoming *ready* is 1



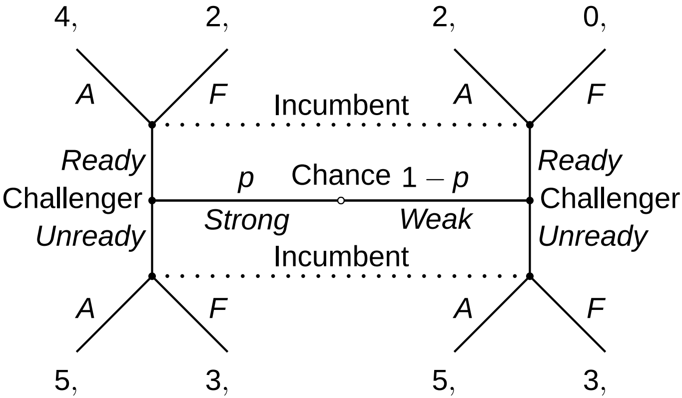
Signaling games: Entry

For strong challenger, cost of becoming *ready* is 1 and (*Ready*, *Acquiesce*) is better than (*Unready*, *Fight*)



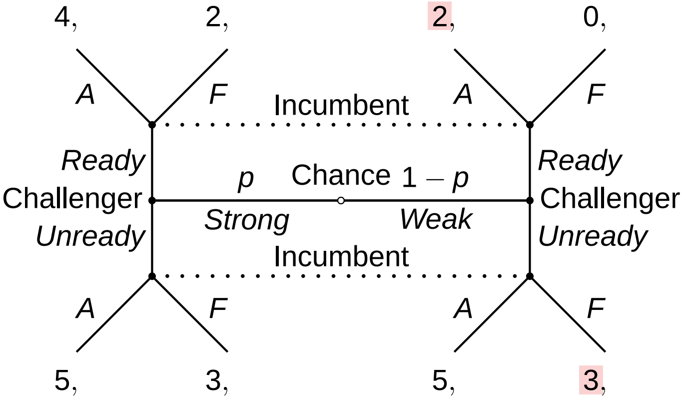
Signaling games: Entry

For weak challenger, cost of becoming *ready* is 3



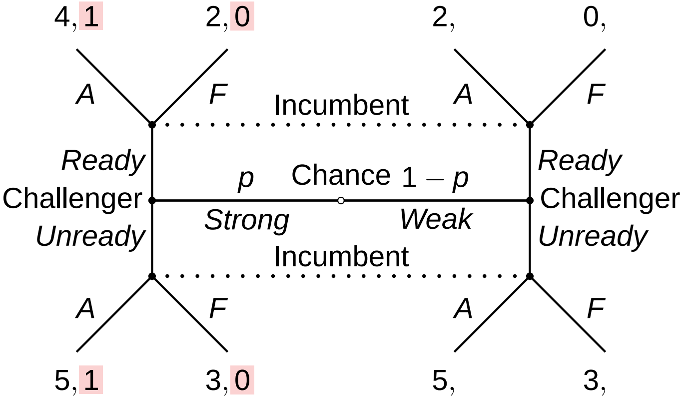
Signaling games: Entry

For weak challenger, cost of becoming *ready* is 3 and $(Ready, Acquiesce)$ is worse than $(Unready, Fight)$



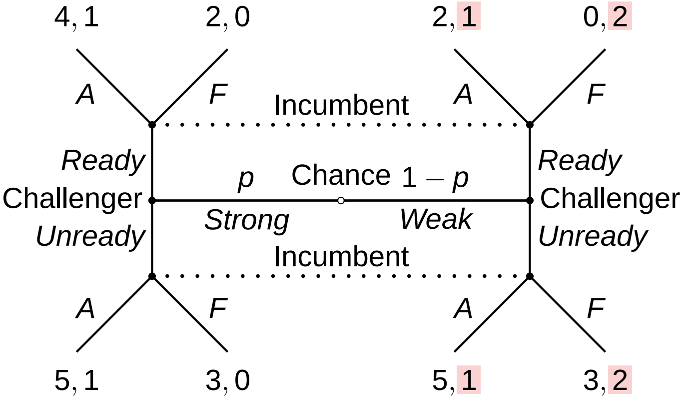
Signaling games: Entry

Incumbent is better off *acquiescing* than *fighting* if challenger is strong



Signaling games: Entry

But incumbent is better off *fighting* than *acquiescing* if challenger is weak



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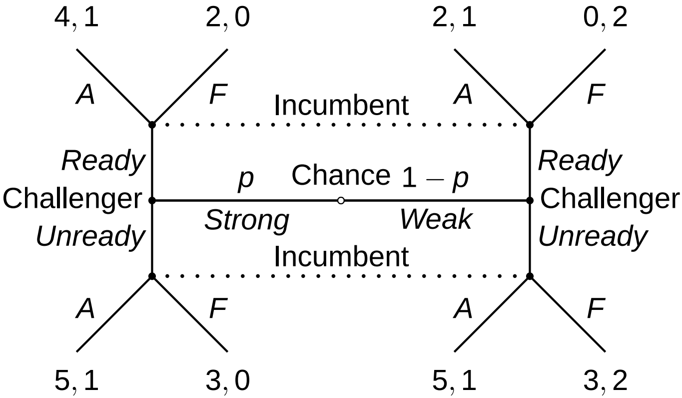

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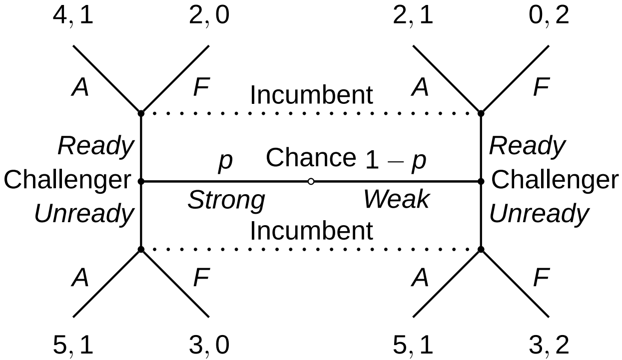

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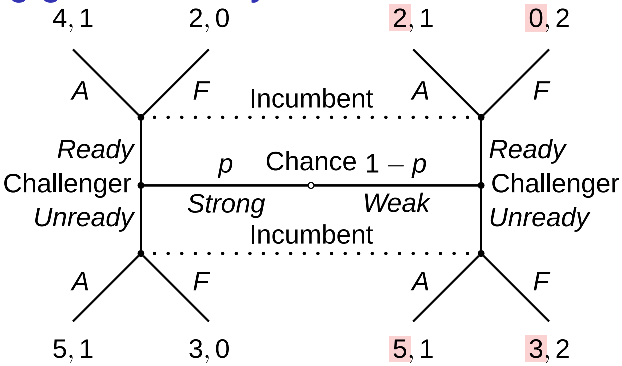
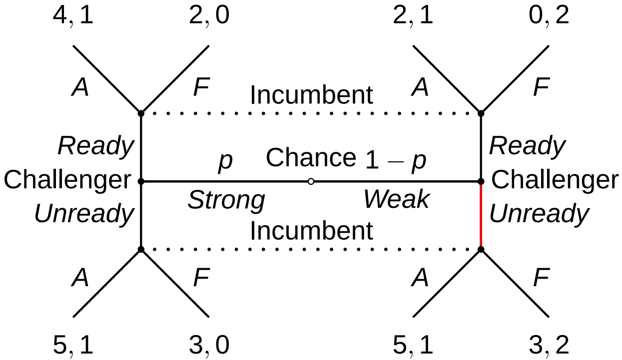


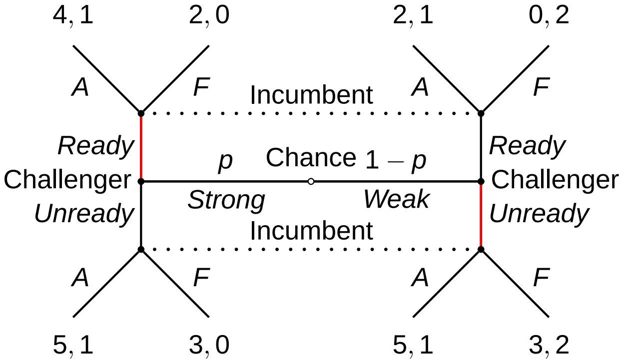
- Weak challenger prefers *Unready* to *Ready* regardless of incumbent's action

Signaling games: Entry



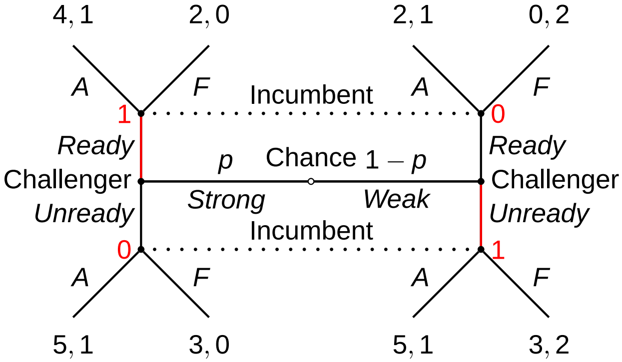
- Weak challenger prefers *Unready* to *Ready* regardless of incumbent's action
- So in any weak sequential equilibrium, weak challenger chooses *Unready*

Signaling games: Entry



- If strong challenger chooses *Ready*, incumbent must believe *Ready* $\Rightarrow$ *Strong* and *Unready* $\Rightarrow$ *Weak*

Signaling games: Entry



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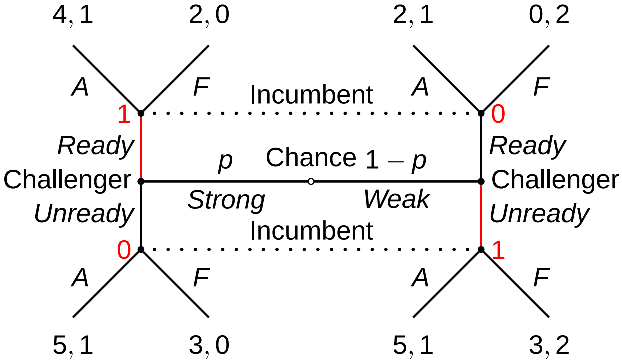
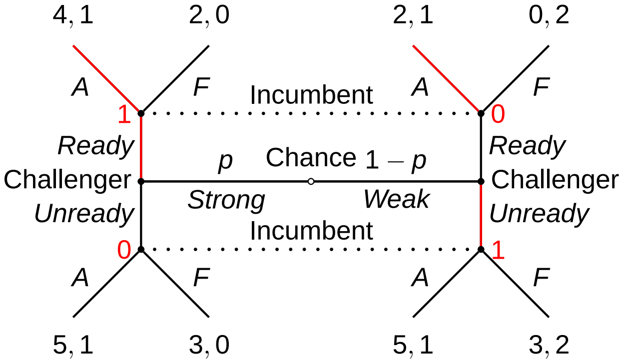


- If strong challenger chooses *Ready*, incumbent must believe *Ready* ⇒ *Strong* and *Unready* ⇒ *Weak* and hence must choose *A* in response to *Ready*

Signaling games: Entry



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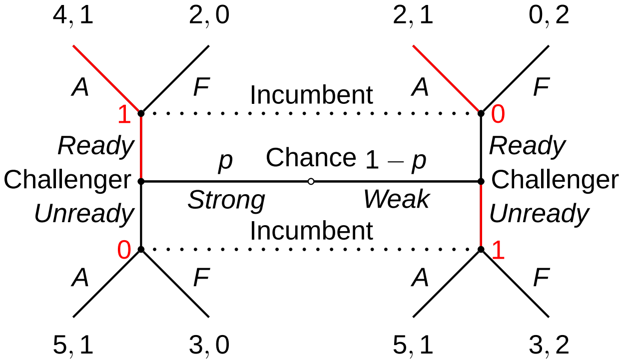
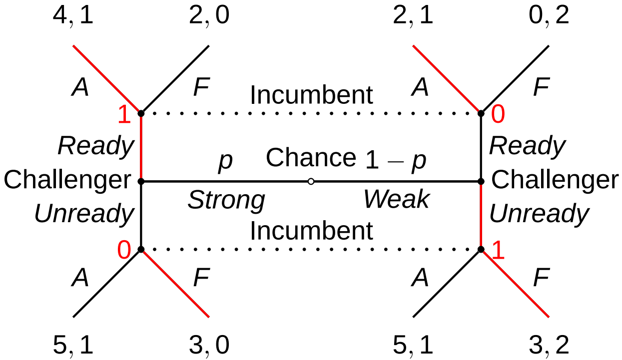


- If strong challenger chooses *Ready*, incumbent must believe *Ready* $\Rightarrow$ *Strong* and *Unready* $\Rightarrow$ *Weak* and hence must choose *A* in response to *Ready* and *F* in response to *Unready*

Signaling games: Entry



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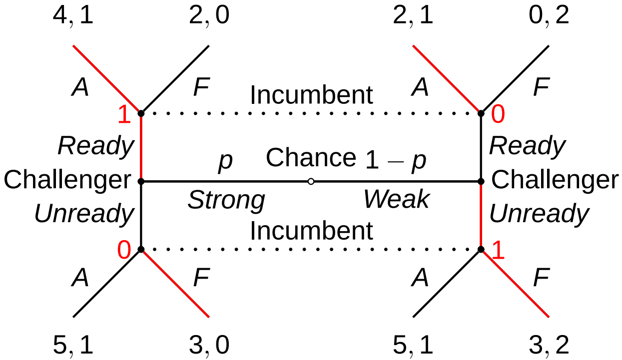
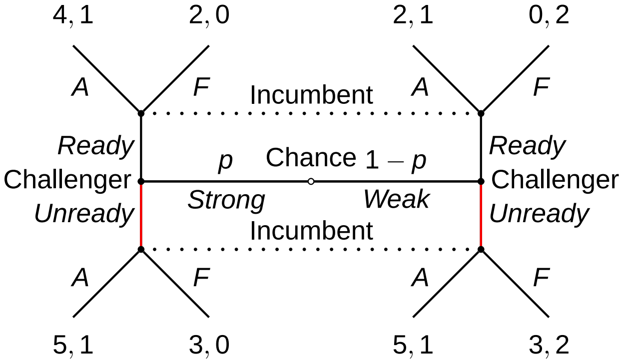


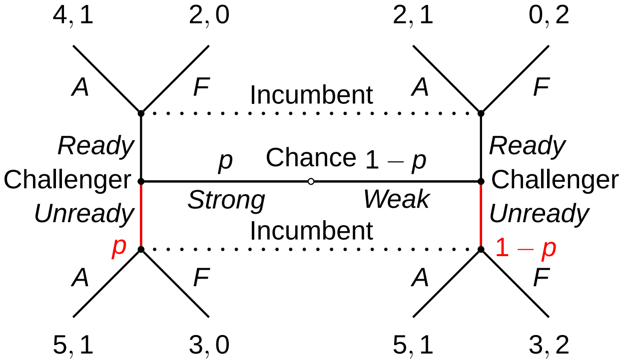
- This assessment is a weak sequential (“separating”) equilibrium: only player remaining to check is strong challenger, who is worse off deviating to *Unready*

Signaling games: Entry



- If strong challenger chooses *Unready*, incumbent must believe *Unready* ⇒ *Strong* with probability p and *Weak* with probability $1 - p$

Signaling games: Entry



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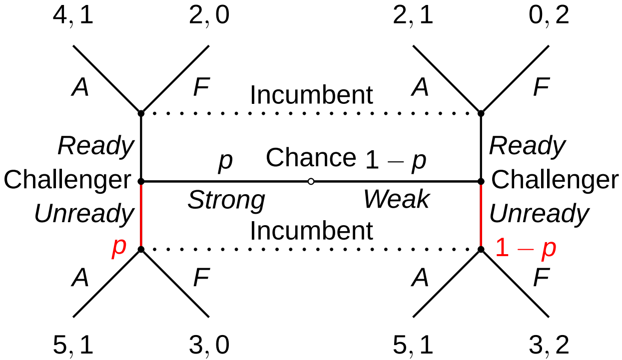
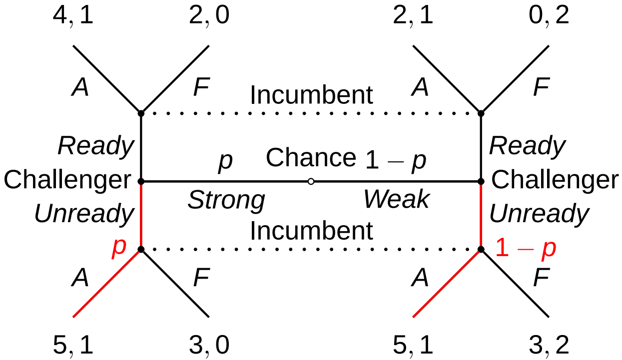


- If strong challenger chooses *Unready*, incumbent must believe *Unready* ⇒ *Strong* with probability p and *Weak* with probability $1 - p$, so incumbent's payoffs:

Signaling games: Entry



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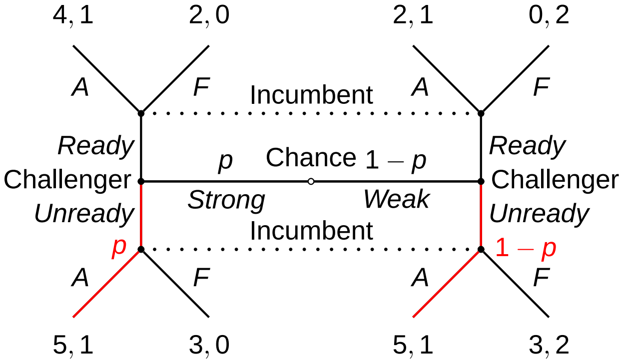

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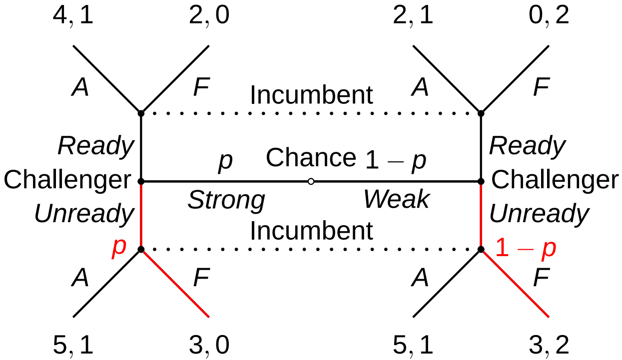
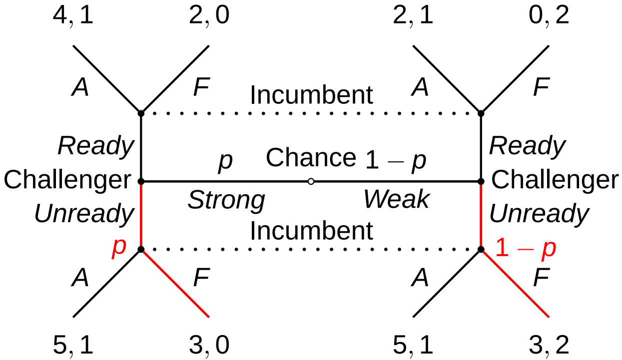
Signaling games: Entry



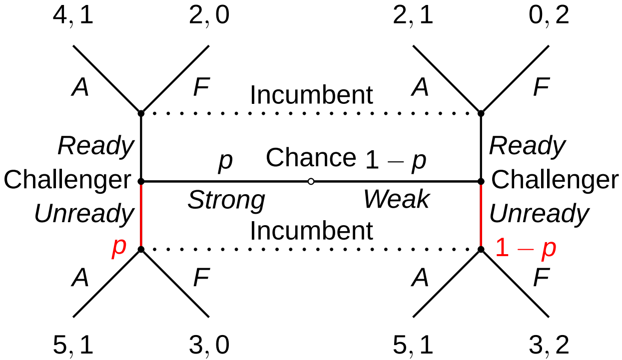
- If strong challenger chooses *Unready*, incumbent must believe *Unready* $\Rightarrow$ *Strong* with probability p and *Weak* with probability $1 - p$, so incumbent's payoffs: $A \Rightarrow 1$ and F $\Rightarrow$

Signaling games: Entry



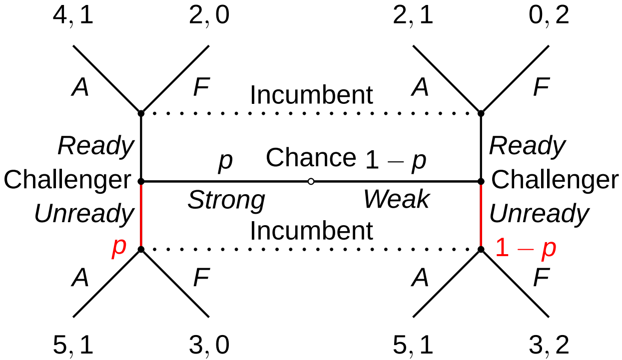
- If strong challenger chooses *Unready*, incumbent must believe *Unready* ⇒ *Strong* with probability p and *Weak* with probability $1 - p$, so incumbent's payoffs: $A \Rightarrow 1$ and $F \Rightarrow 2(1 - p)$

Signaling games: Entry



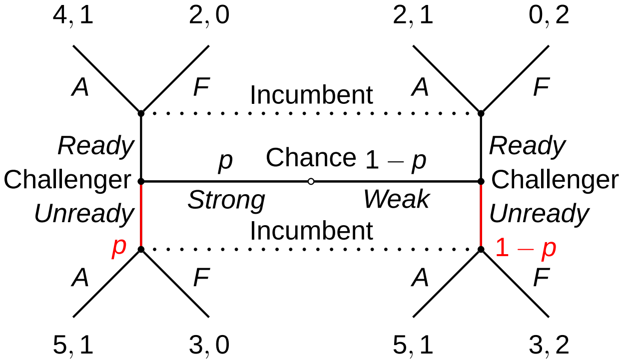
- If strong challenger chooses *Unready*, incumbent must believe *Unready* ⇒ *Strong* with probability p and *Weak* with probability $1 - p$, so incumbent's payoffs: $A \Rightarrow 1$ and $F \Rightarrow 2(1 - p)$
- Hence A optimal if $p \geq \frac{1}{2}$ and F optimal if $p \leq \frac{1}{2}$

Signaling games: Entry



- Given this behavior of incumbent, is *Unready* optimal for strong challenger?

Signaling games: Entry



- Given this behavior of incumbent, is *Unready* optimal for strong challenger?
- $p > \frac{1}{2}$: incumbent chooses A

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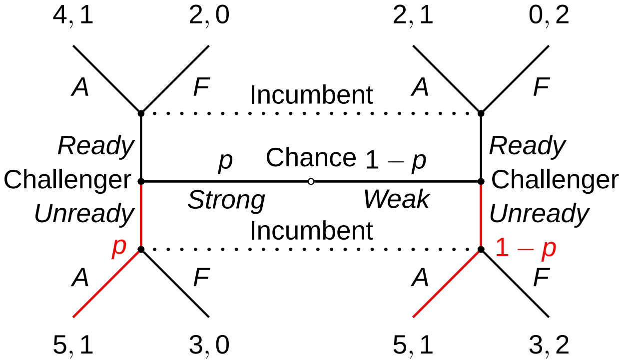

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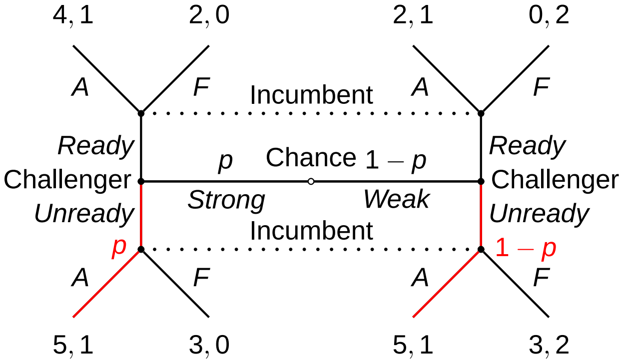
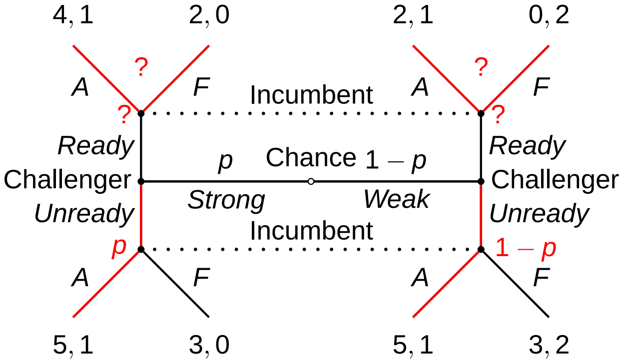


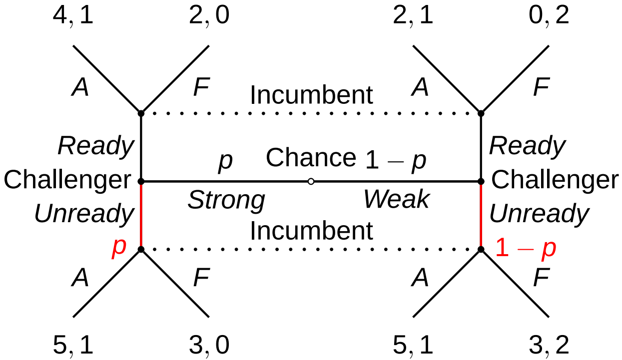
- Given this behavior of incumbent, is *Unready* optimal for strong challenger?
- $p > \frac{1}{2}$: incumbent chooses A, strong challenger who deviates to *Ready* gets at most 4, instead of 5

Signaling games: Entry



- So for $p \geq \frac{1}{2}$, any such assessment is a ("pooling") equilibrium (with any belief of incumbent after *Ready*)

Signaling games: Entry



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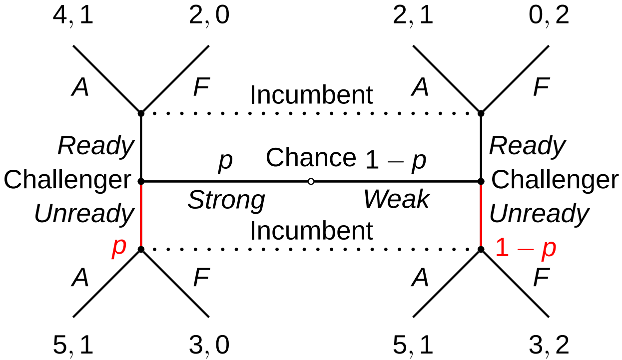
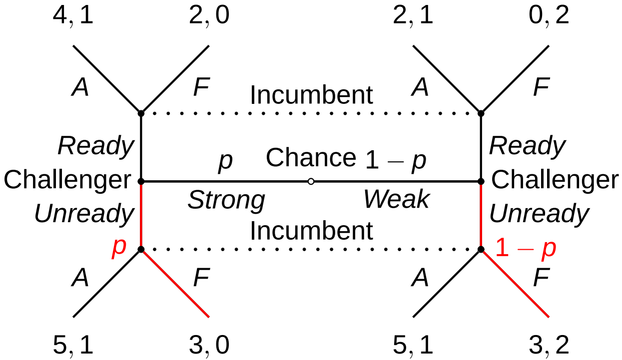


- Given this behavior of incumbent, is *Unready* optimal for strong challenger?
- $p < \frac{1}{2}$: incumbent chooses F

Signaling games: Entry



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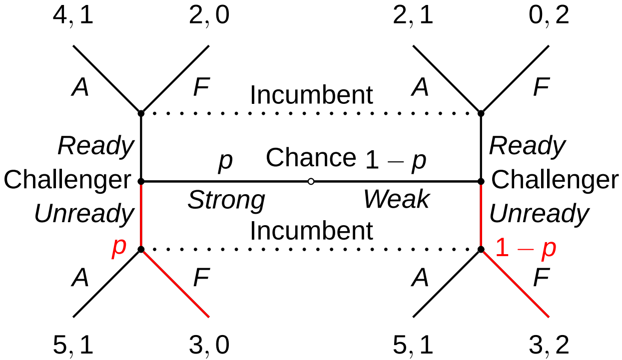
Signaling games: Entry



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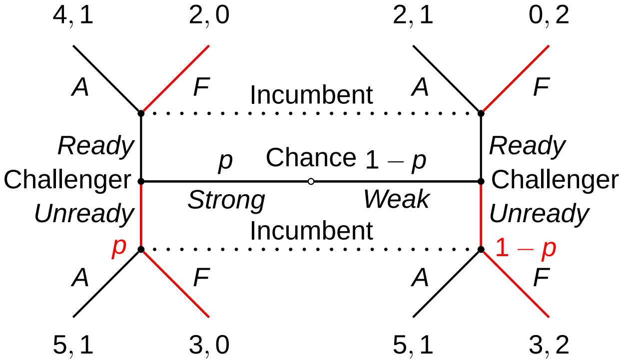
- $p < \frac{1}{2}$: incumbent chooses F, so must choose F also in response to *Ready* (otherwise strong challenger prefers *Ready*)

Signaling games: Entry



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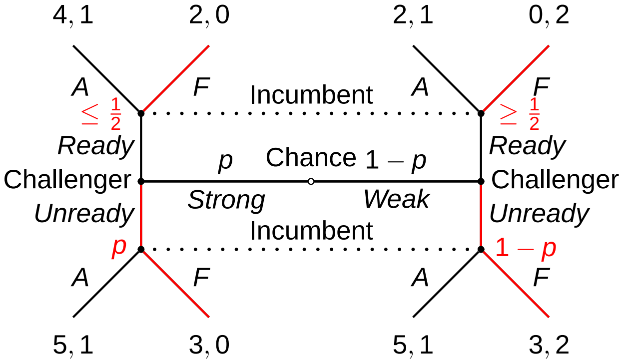
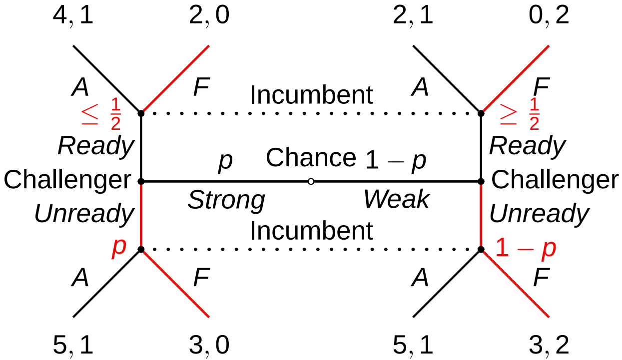


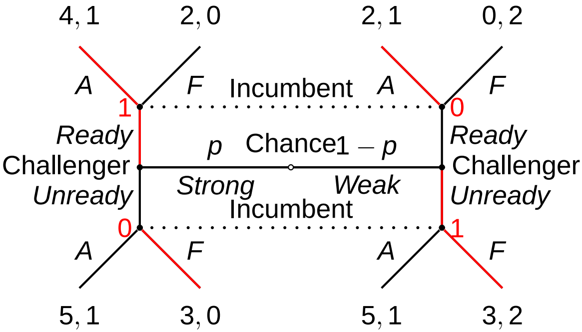
- Given this behavior of incumbent, is *Unready* optimal for strong challenger?
- $p < \frac{1}{2}$: incumbent chooses F, so must choose F also in response to *Ready* (otherwise strong challenger prefers *Ready*), which is optimal if incumbent believes *Ready* challenger is weak with probability $\geq \frac{1}{2}$

Signaling games: Entry



- So if $p < \frac{1}{2}$, any such assessment is a (“pooling”) weak sequential equilibrium

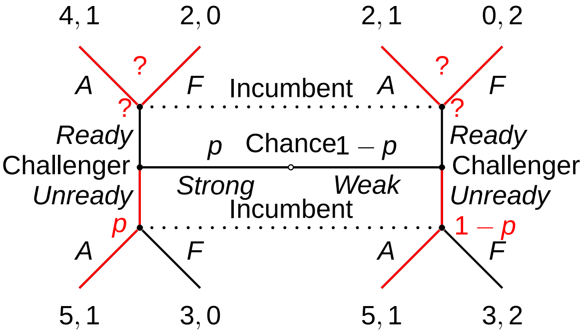
Signaling games: Entry



Separating equilibrium

- Strong challenger chooses *Ready*
- Weak challenger chooses *Unready*
- Incumbent believes *Ready* challenger is strong, *Unready* challenger is weak
- Incumbent fights *Unready* challenger and acquiesces to *Ready* challenger

Signaling games: Entry



Pooling equilibria

If $p > \frac{1}{2}$,

- both challengers chooses *Unready*
- incumbent believes *Unready* challenger is strong with probability p
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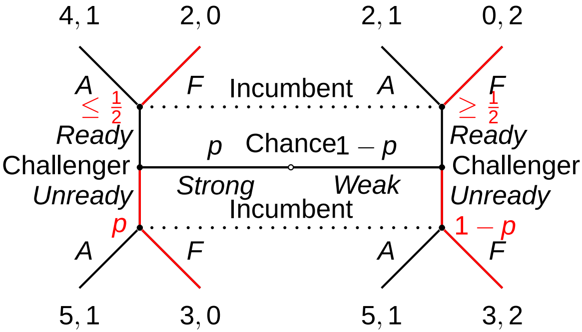
Signaling games: Entry



Pooling equilibria

If $p < \frac{1}{2}$,

- both challengers choose *Unready*
- incumbent believes *Unready* challenger strong with prob. p and *Ready* challenger is strong with prob. $\leq \frac{1}{2}$
- incumbent fights all challengers

Education as a signal of ability

- Why are you obtaining a college degree?

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- Perhaps nothing you learn in college has any bearing on the job you expect to take, but you need to get a degree to prove to potential employers that your ability is high
- How does your obtaining a degree prove this point?
- Because the cost to persons of low ability of obtaining the degree is much higher than it is for you (they will take longer, and find the process painful), so that such persons cannot profitably imitate you

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- But it is not, so you are in college

Education as a signal of ability

Model: structure

- Worker's ability: H or $L < H$

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Worker chooses amount e of education

$\downarrow$

Firms observe e and simultaneously offer wages w_1 and w_2

$\downarrow$

Worker chooses w_1 or w_2

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Model: payoffs

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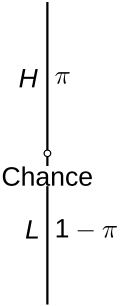
- So payoff of worker of ability K who obtains e units of education and gets wage w is $w - e/K$
- Payoff of firm that employs worker of ability K at wage w is $K - w$

Education as a signal of ability

Model: diagrammatic outline

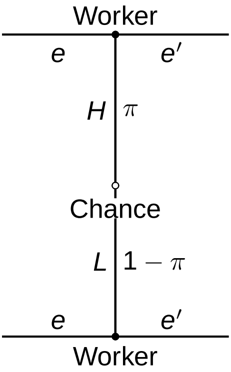
Worker has high ability for firm's job with probability π and low ability with probability $1 - \pi$



Education as a signal of ability

Model: dia

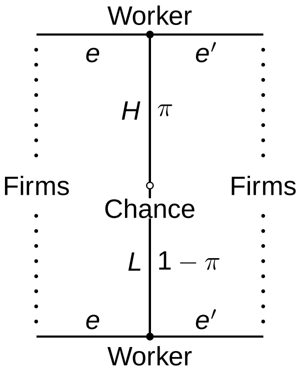
Only *two* possible actions (out of infinitely many possible actions) of worker shown



Education as a signal of ability

Model: diagrammatic outline

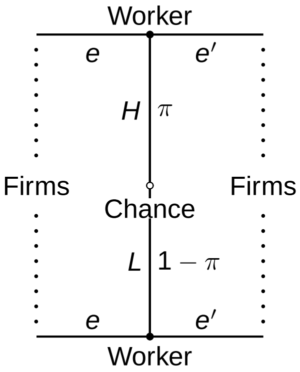
Firms observe worker's education choice, but not her ability



Education as a signal of ability

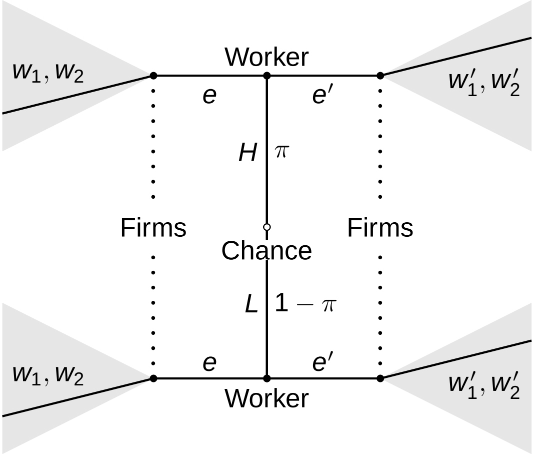
Model: diagrammatic outline

Firms observe worker's education choice, but not her ability
Firms move simultaneously; figure does not show firms separately



Education as a signal of ability

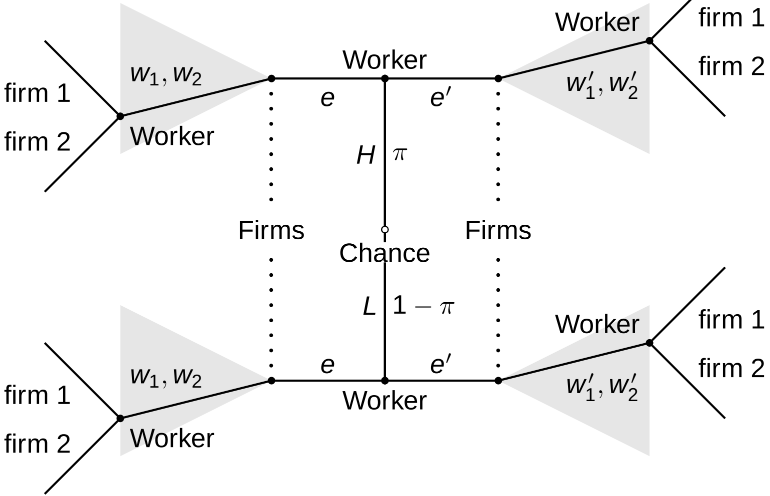
Firms offer wages (depending on worker's education, but not her ability)



Education as a signal of ability

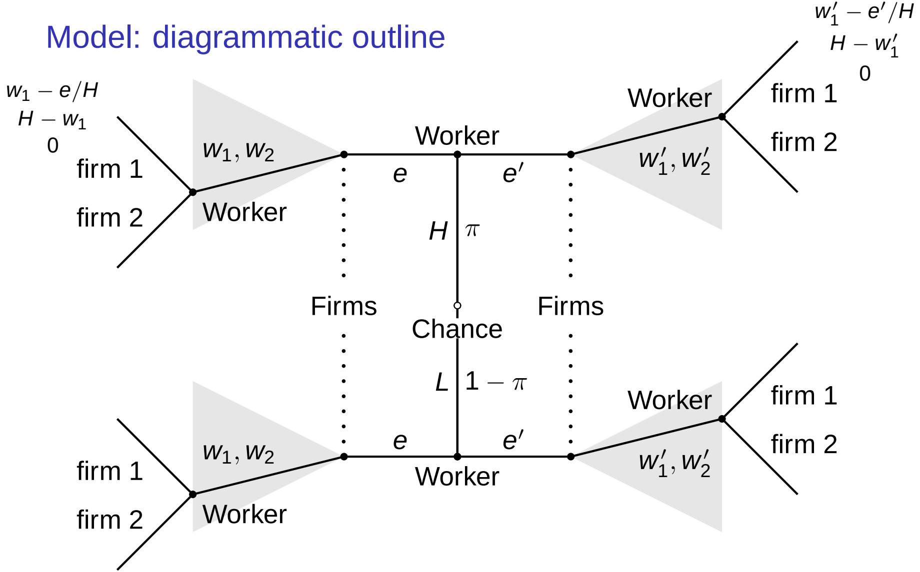
Model: diagra

Worker chooses which offer to accept



Education as a signal of ability

Model: diagrammatic outline



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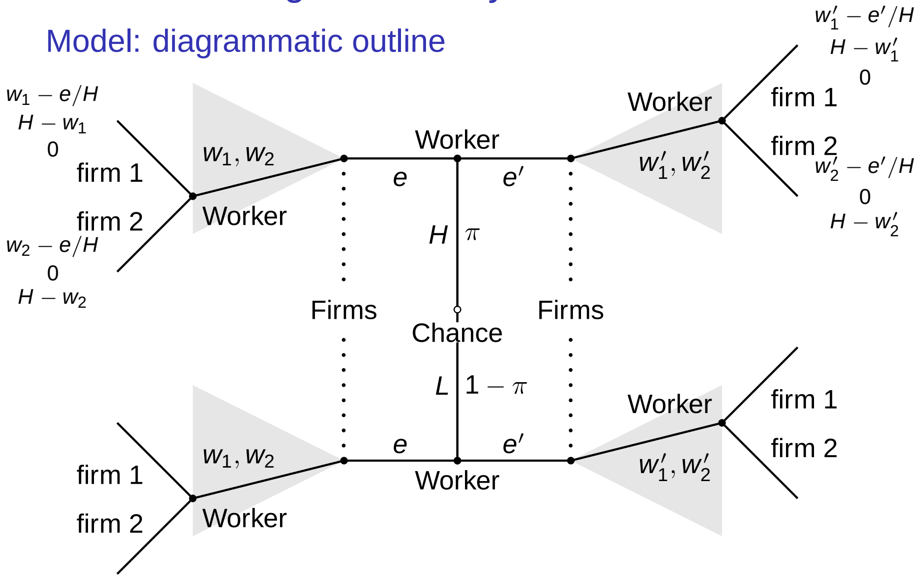

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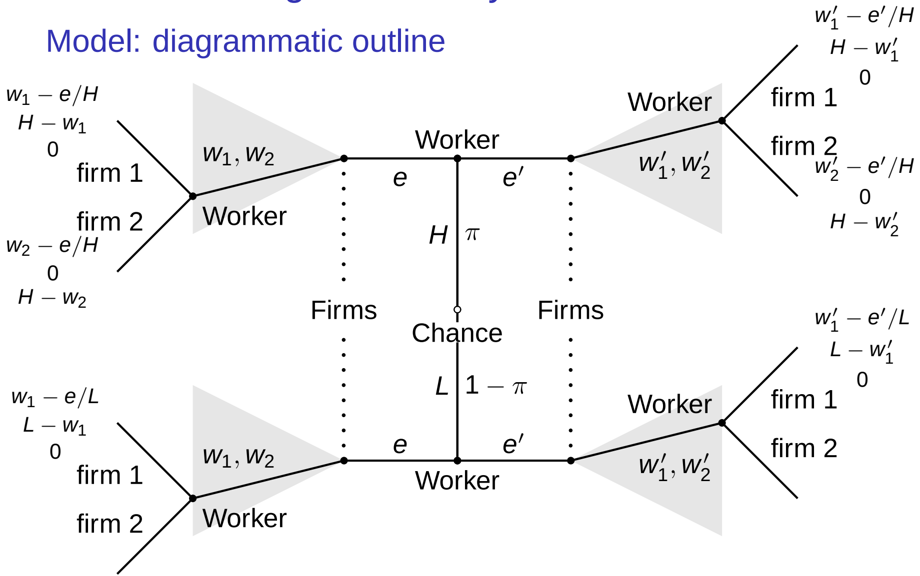

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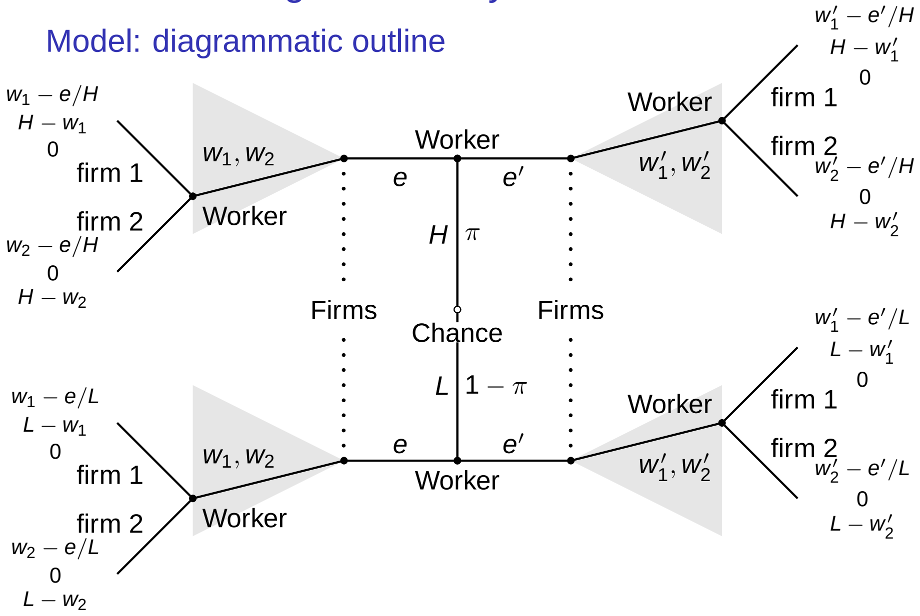

Education as a signal of ability

Fix a number $e^* > 0$ and consider the following assessment

Worker's strategy Type H chooses $e = e^*$ and type L chooses $e = 0$; after observing the firms' wage offers, both types choose the highest offer if they differ, and that of firm 1 if they are the same.

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Firms' belief Each firm believes that a worker is type H if she chooses e^* and type L otherwise.

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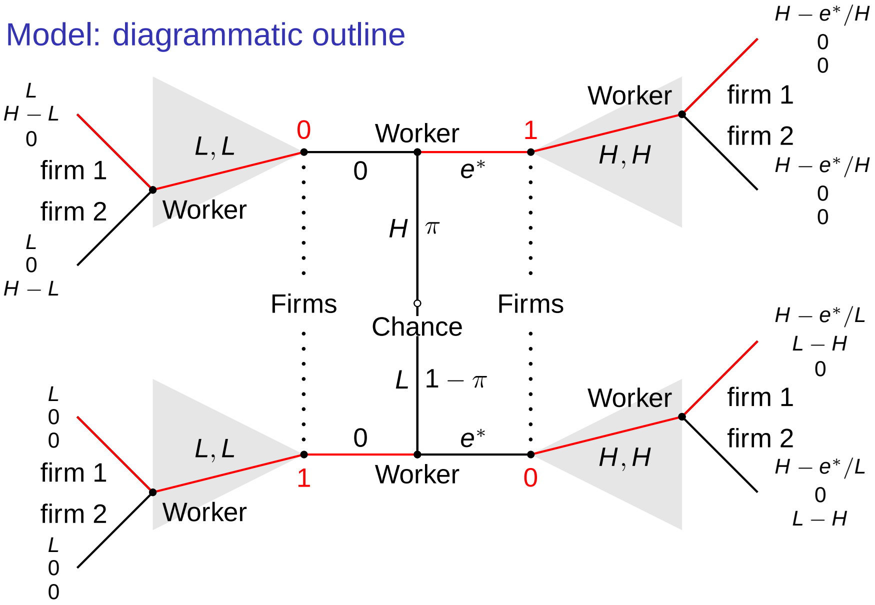
Firms' strategies Each firm offers the wage H to a worker who chooses e^* and the wage L to a worker who chooses *any* other value of e.

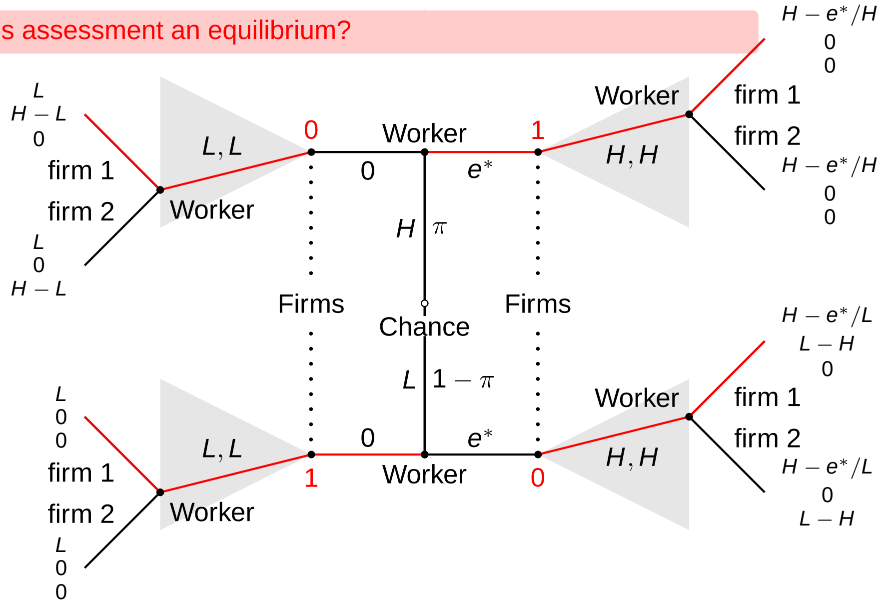
Education as a signal of ability

Model: diagrammatic outline



Education as a signal of ability

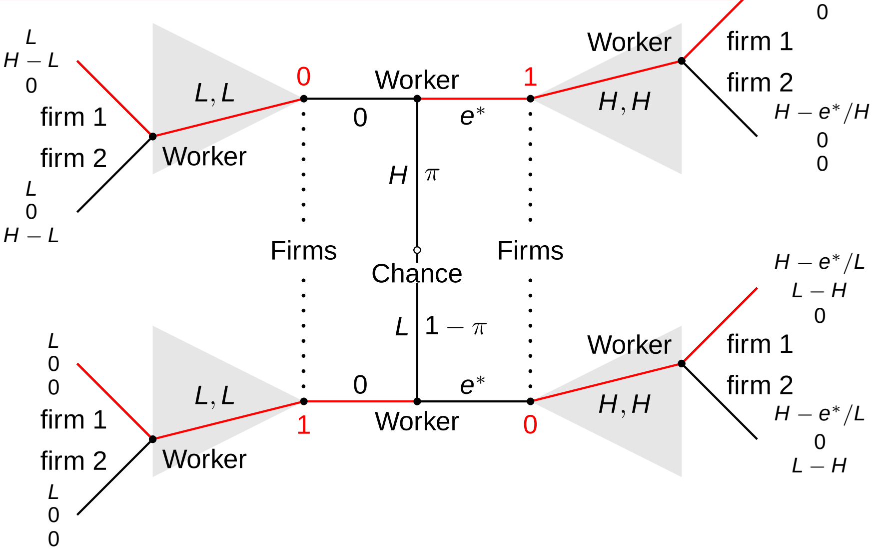
Is assessment an equilibrium?



E

Is assessment an equilibrium?

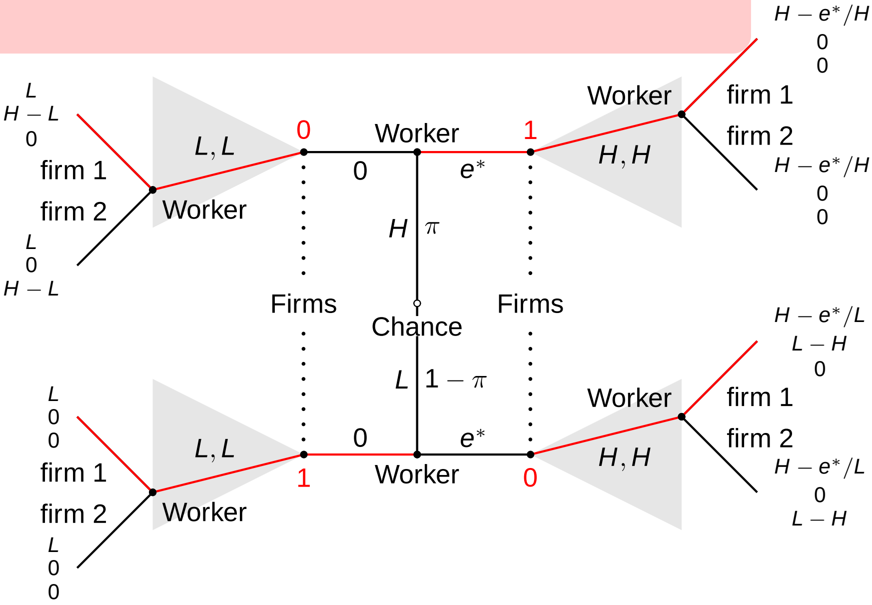
- Firm's beliefs are consistent with worker's strategy



E…

Is assessment an equilibrium?

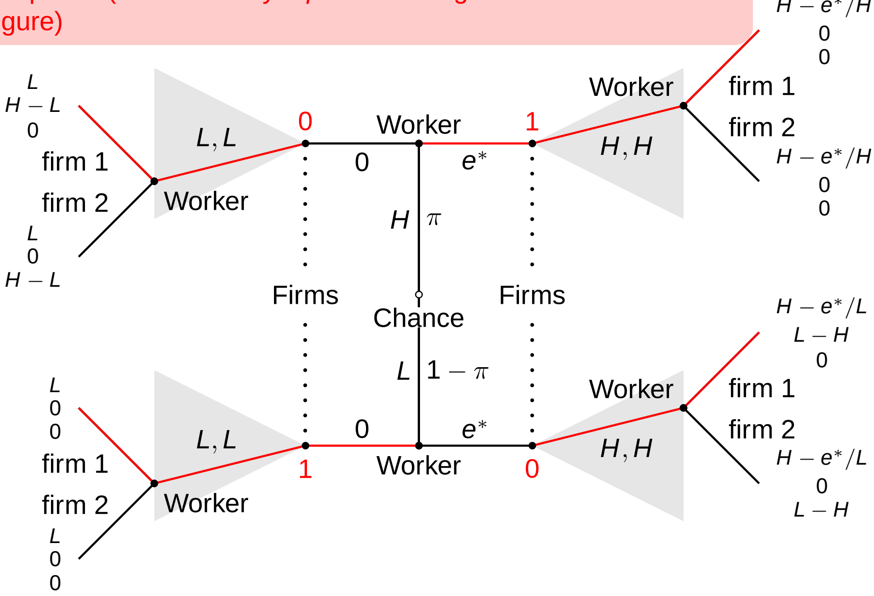
- Worker's wage-acceptance behavior:



E

Is assessment an equilibrium?

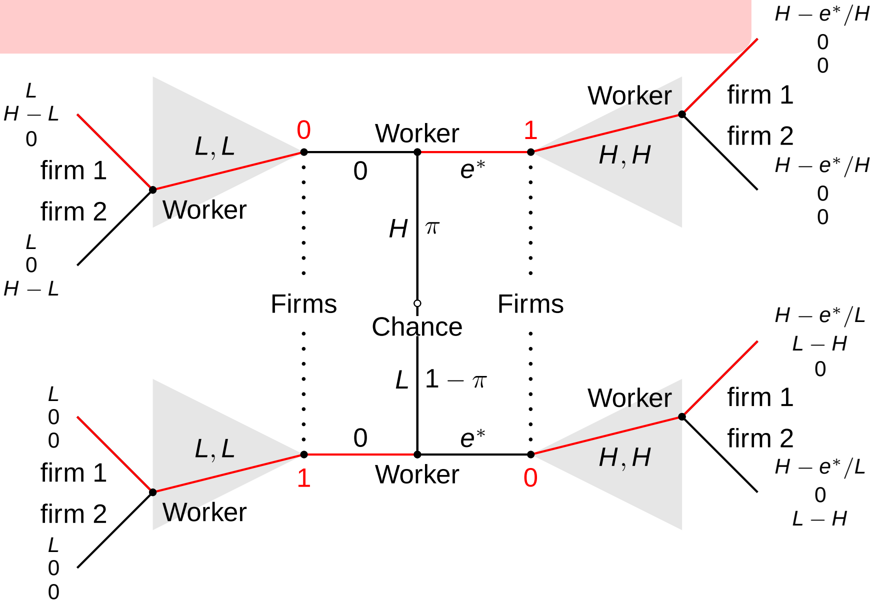
- Worker's wage-acceptance behavior: accepting highest wage is optimal (note that only *equilibrium* wage offers are show in figure)



E

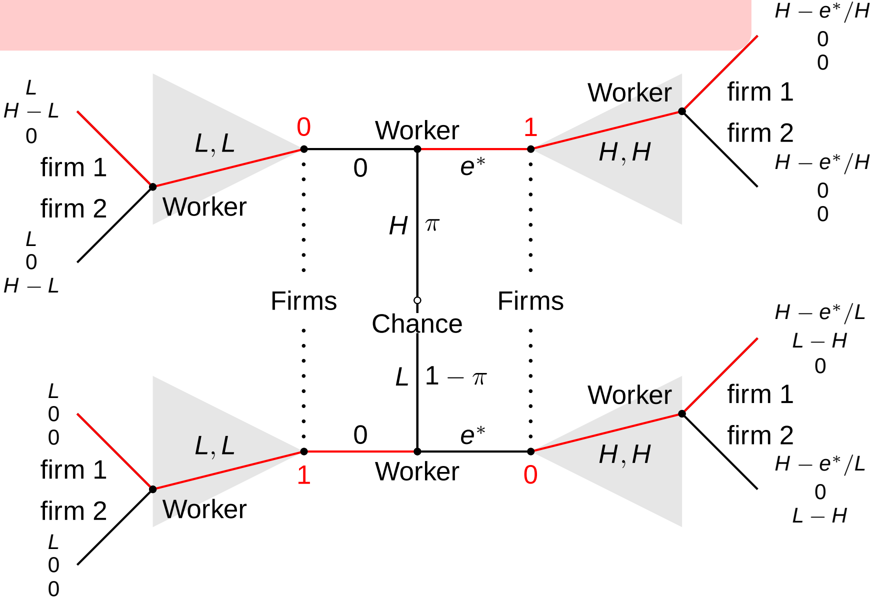
Is assessment an equilibrium?

- Firm:



Is assessment an equilibrium?

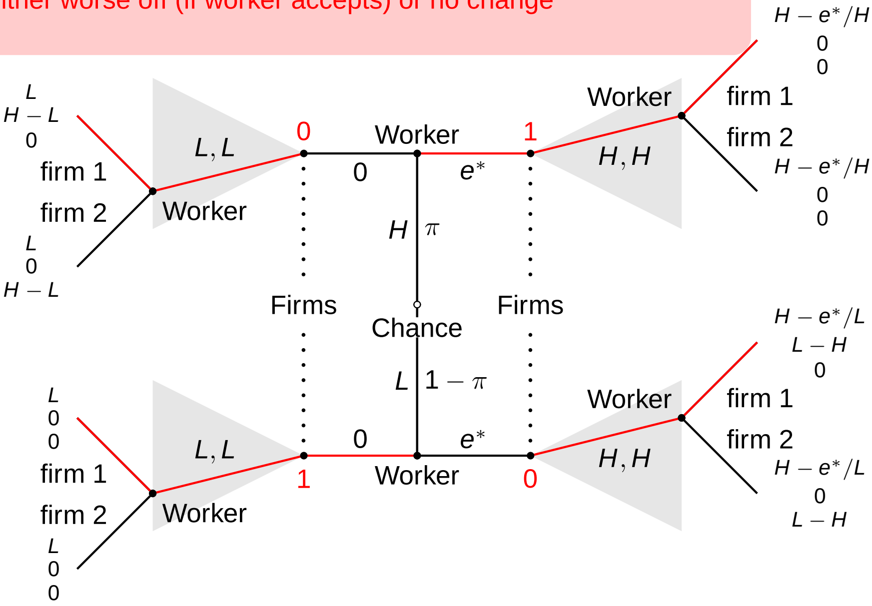
- Firm: Given belief and strategy of other firm, raise wage ⇒



Is assessment an equilibrium?

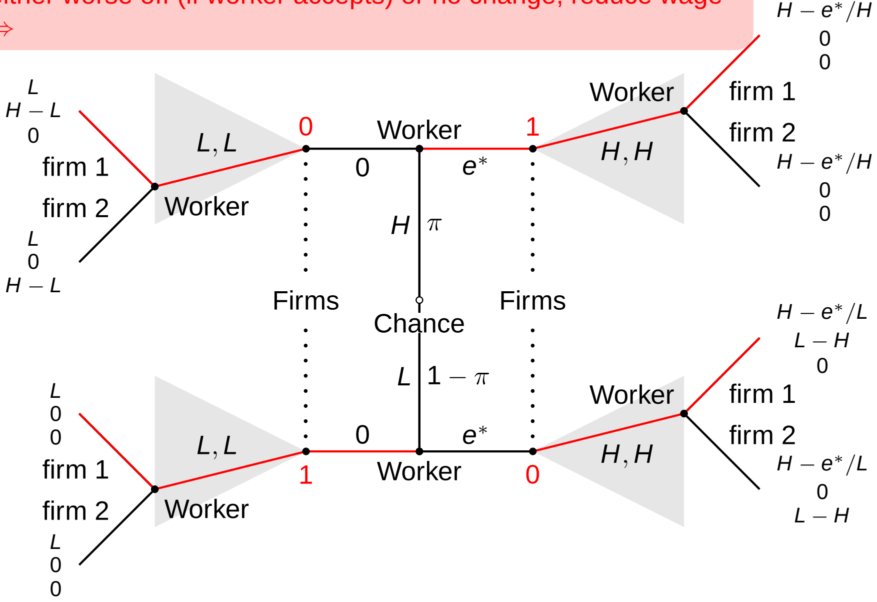
- Firm: Given belief and strategy of other firm, raise wage ⇒ either worse off (if worker accepts) or no change

E



Is assessment an equilibrium?

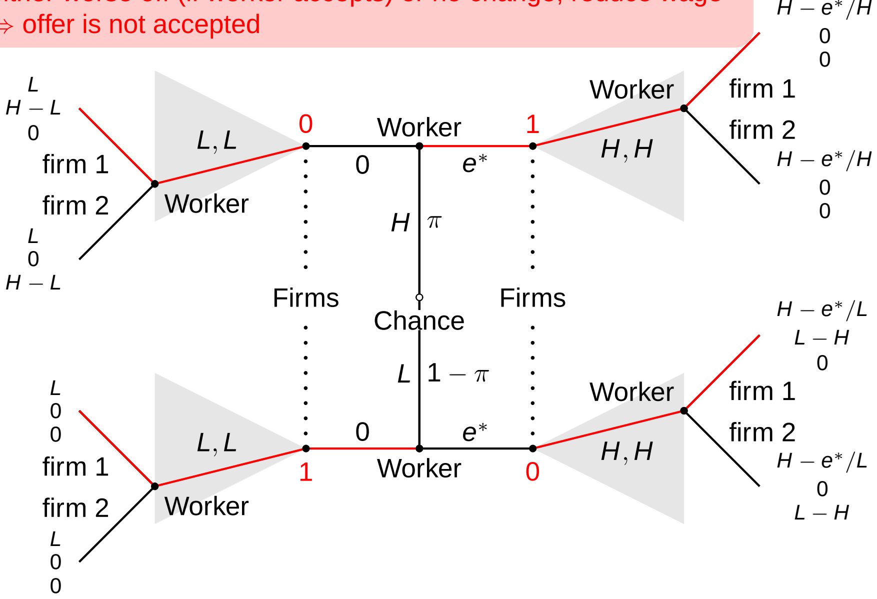
- Firm: Given belief and strategy of other firm, raise wage ⇒ either worse off (if worker accepts) or no change; reduce wage ⇒



E

Is assessment an equilibrium?

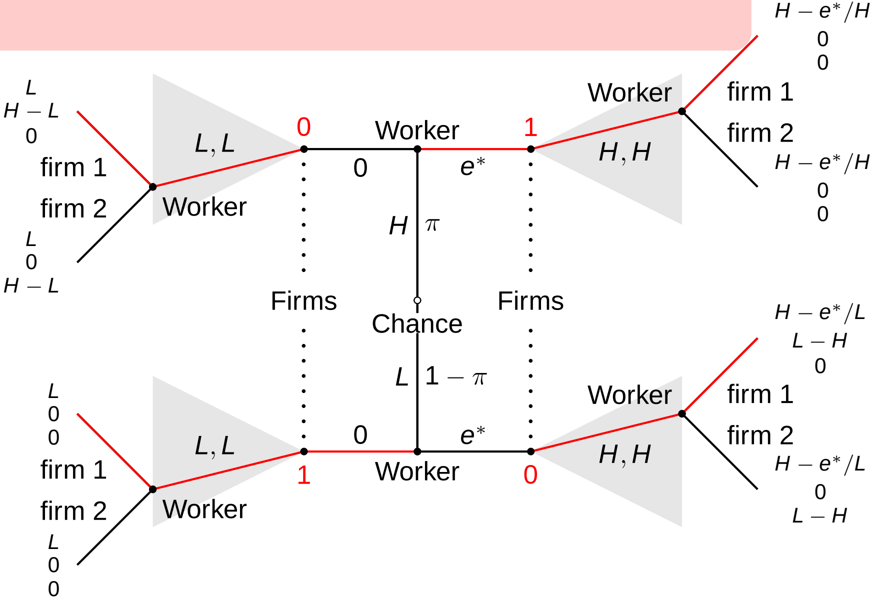
- Firm: Given belief and strategy of other firm, raise wage ⇒ either worse off (if worker accepts) or no change; reduce wage ⇒ offer is not accepted



E…

Is assessment an equilibrium?

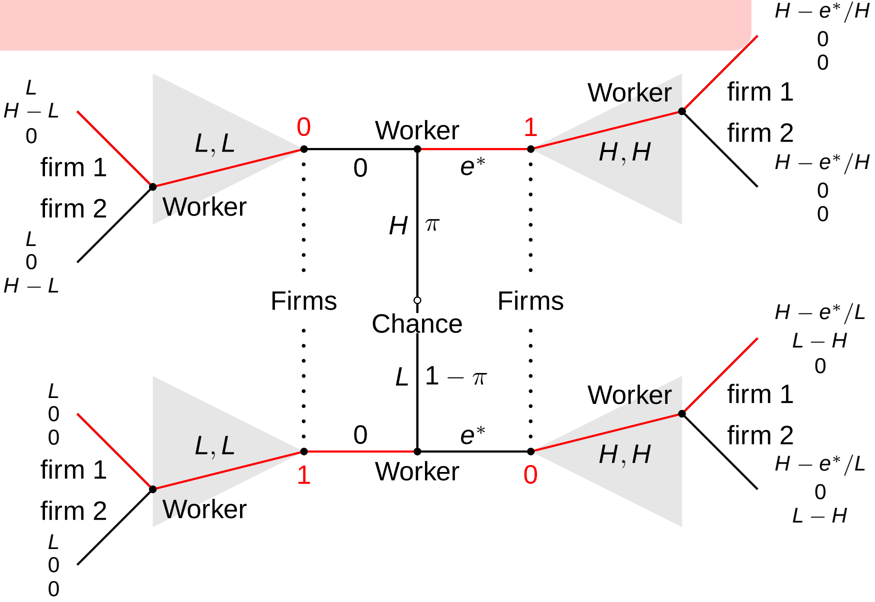
- *H* worker's choice of e:



Is assessment an equilibrium?

- *H* worker's choice of *e*: Choose $e^* \Rightarrow$

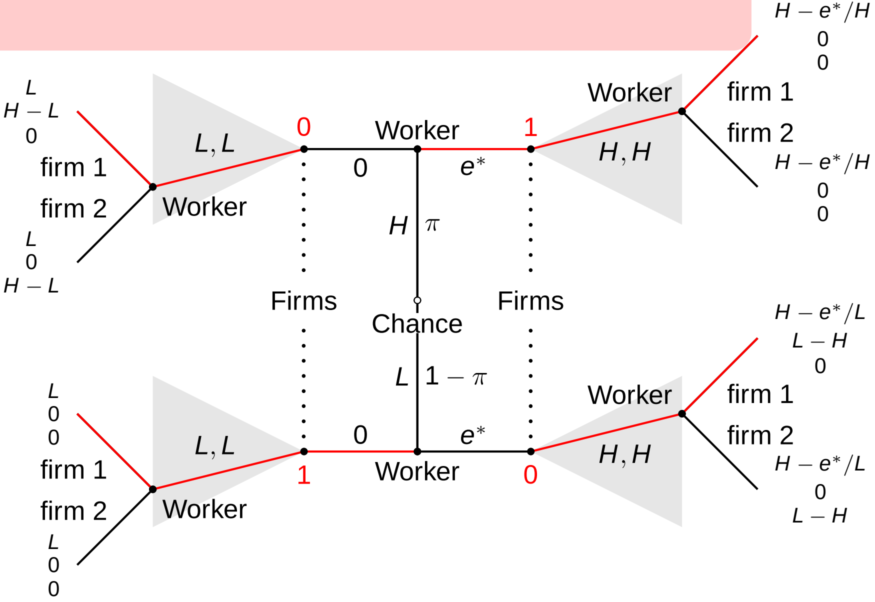
E



E…

Is assessment an equilibrium?

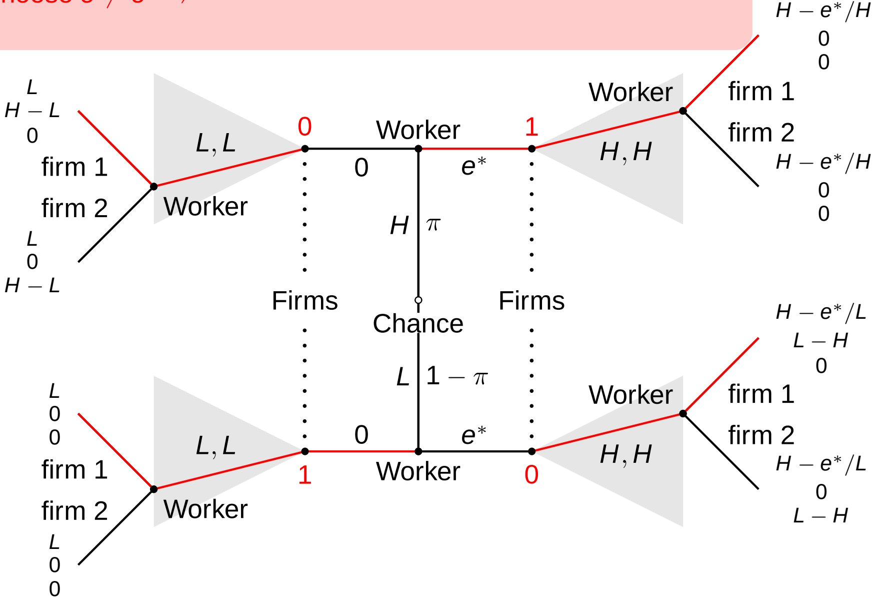
- H worker's choice of e: Choose e^* $\Rightarrow$ payoff $H - e^*/H$



E

Is assessment an equilibrium?

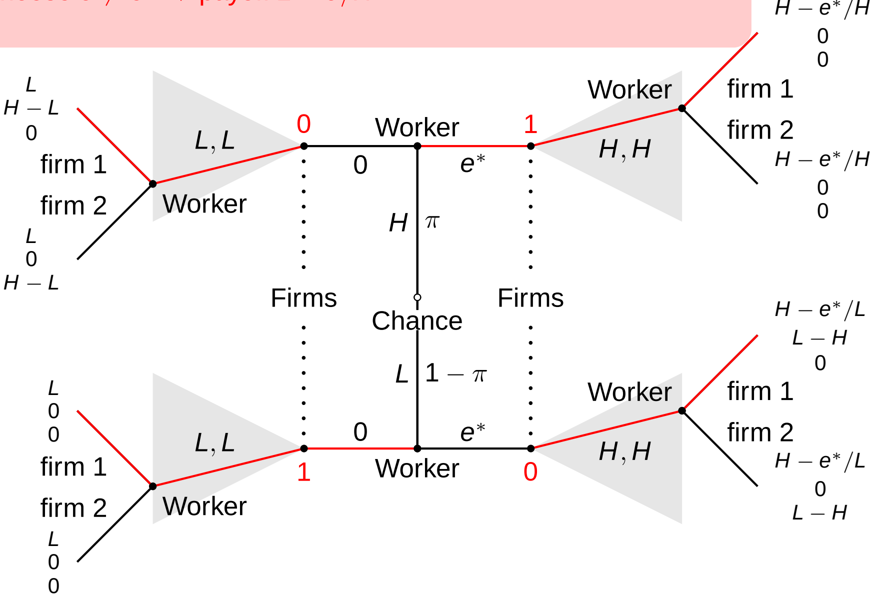
- H worker's choice of e: Choose $e^* \Rightarrow$ payoff $H - e^*/H$; choose $e \neq e^* \Rightarrow$



E

Is assessment an equilibrium?

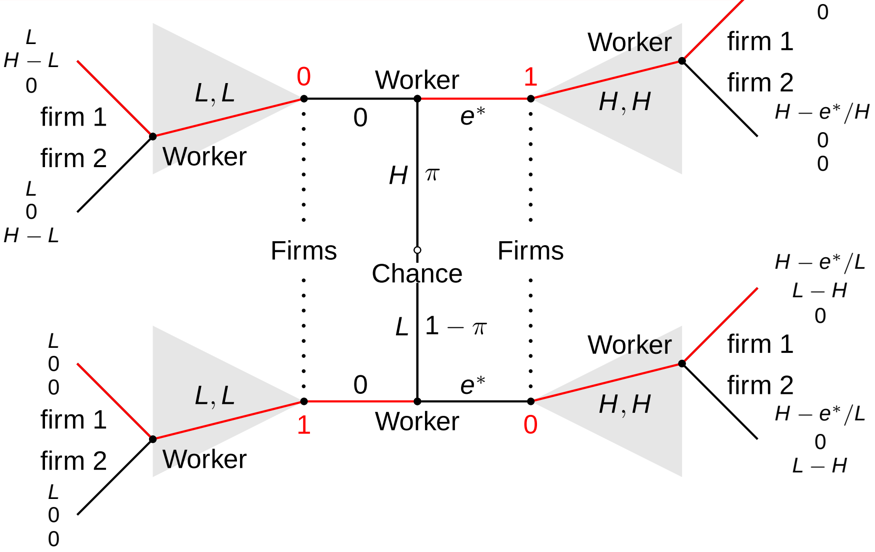
• H worker's choice of e: Choose e^* ⇒ payoff $H - e^*/H$; choose $e \neq e^*$ ⇒ payoff $L - e/H$.



Is assessment an equilibrium?

- H worker's choice of e: Choose e^* $\Rightarrow$ payoff $H - e^*/H$; choose $e \neq e^*$ $\Rightarrow$ payoff $L - e/H$.

So need $H - e^*/H \geq L - e/H$ for all $e \neq e^*$

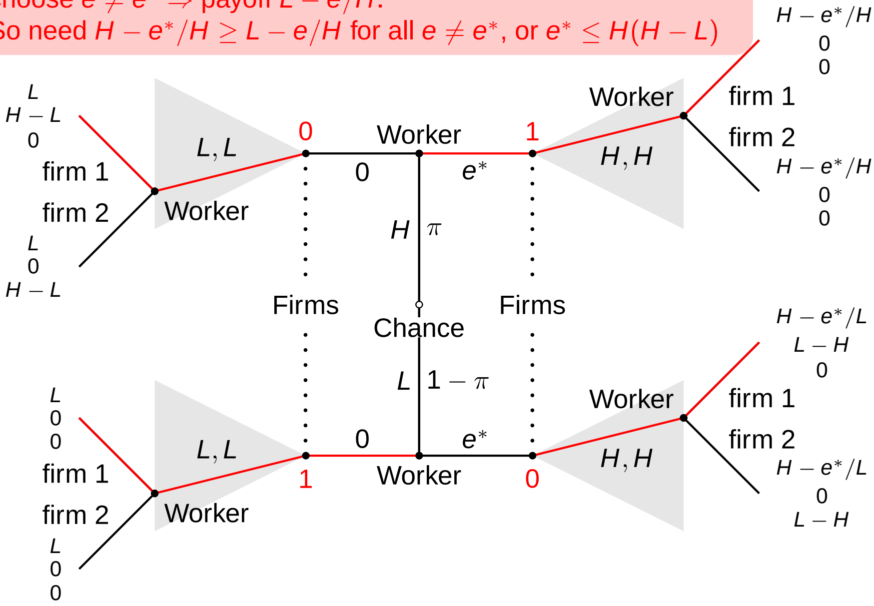


E…

Is assessment an equilibrium?

• H worker's choice of e: Choose $e^* \Rightarrow$ payoff $H - e^*/H$; choose $e \neq e^* \Rightarrow$ payoff $L - e/H$.

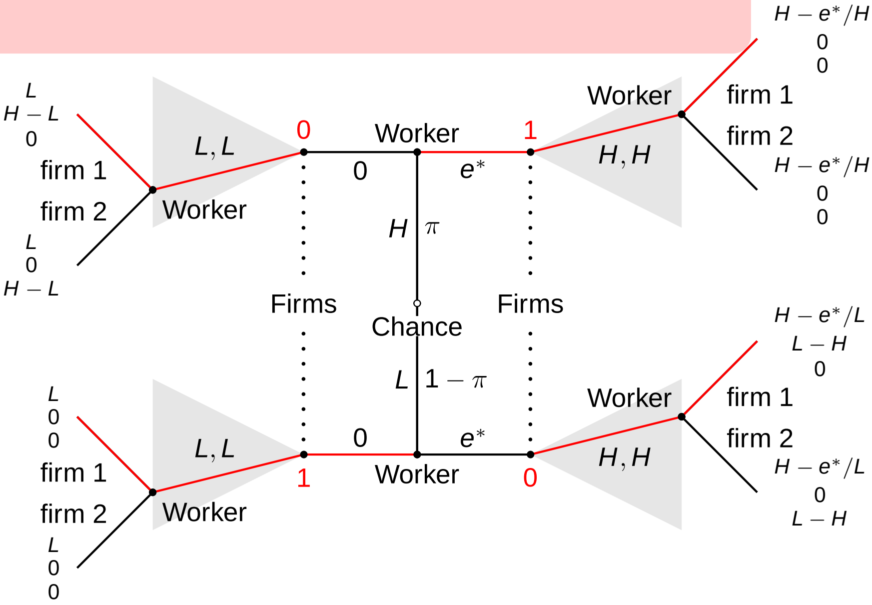
So need $H - e^*/H \geq L - e/H$ for all $e \neq e^*$, or $e^* \leq H(H - L)$



E…

Is assessment an equilibrium?

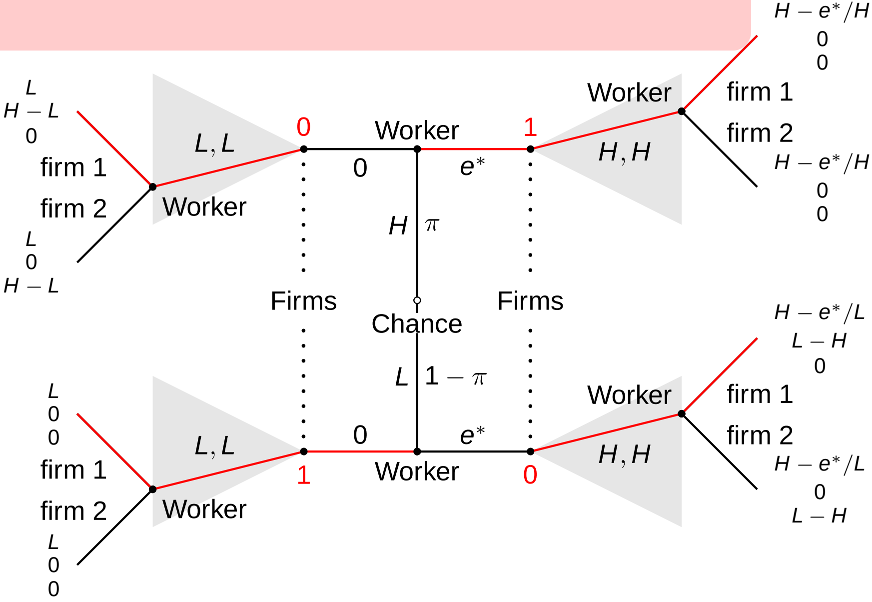
- L worker's choice of e:



E

Is assessment an equilibrium?

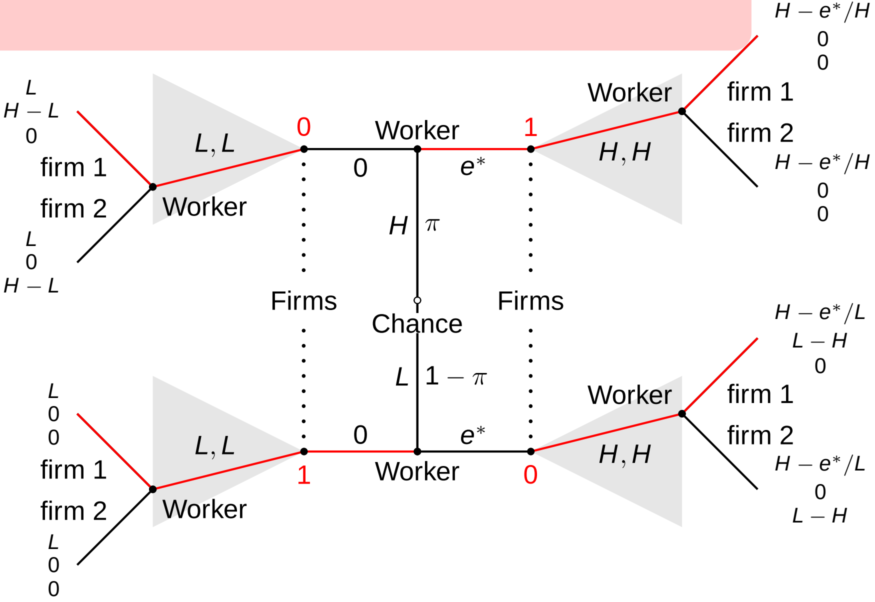
- L worker's choice of e: Choose 0 $\Rightarrow$



E…

Is assessment an equilibrium?

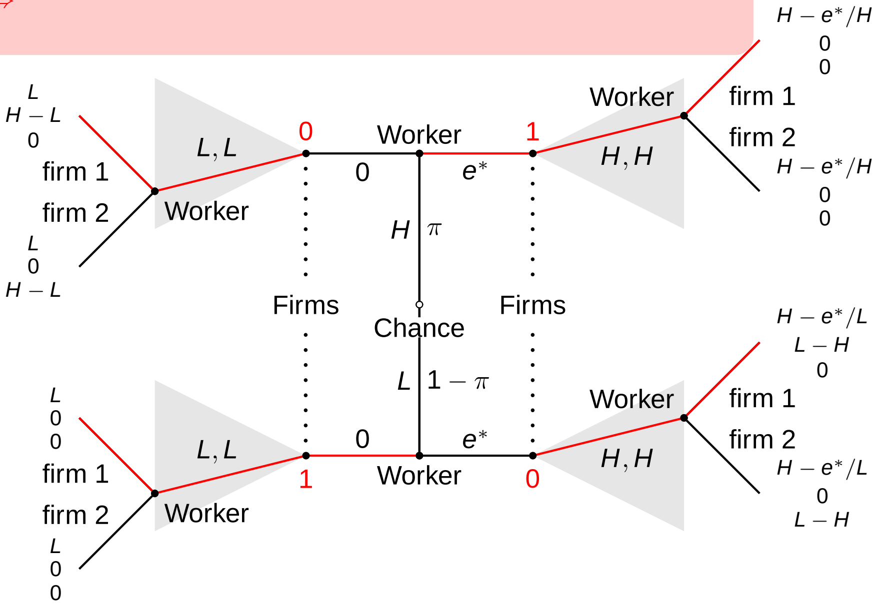
- L worker's choice of e: Choose 0 $\Rightarrow$ payoff L



Is assessment an equilibrium?

- L worker's choice of e: Choose 0 $\Rightarrow$ payoff L; choose $e \neq e^*$ $\Rightarrow$

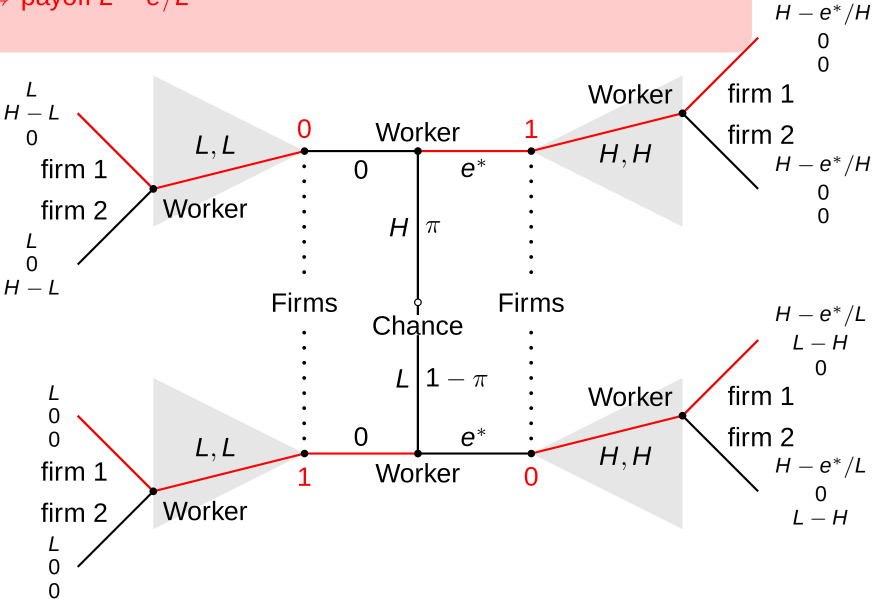
E



E

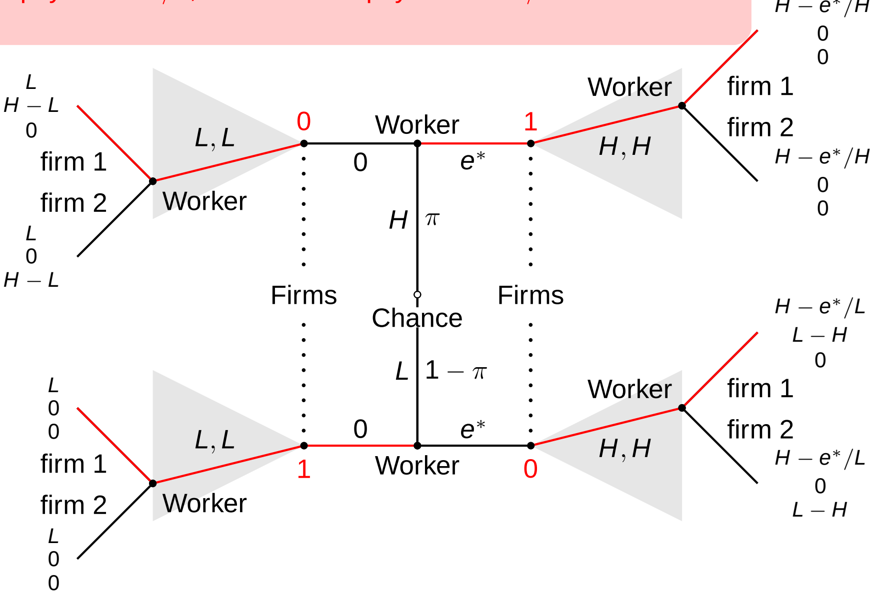
Is assessment an equilibrium?

- L worker's choice of e: Choose 0 $\Rightarrow$ payoff L; choose $e \neq e^*$ $\Rightarrow$ payoff $L - e/L$



Is assessment an equilibrium?

- L worker's choice of e: Choose 0 $\Rightarrow$ payoff L; choose $e \neq e^*$ $\Rightarrow$ payoff $L - e/L$; choose e^* $\Rightarrow$ payoff $H - e^*/L$

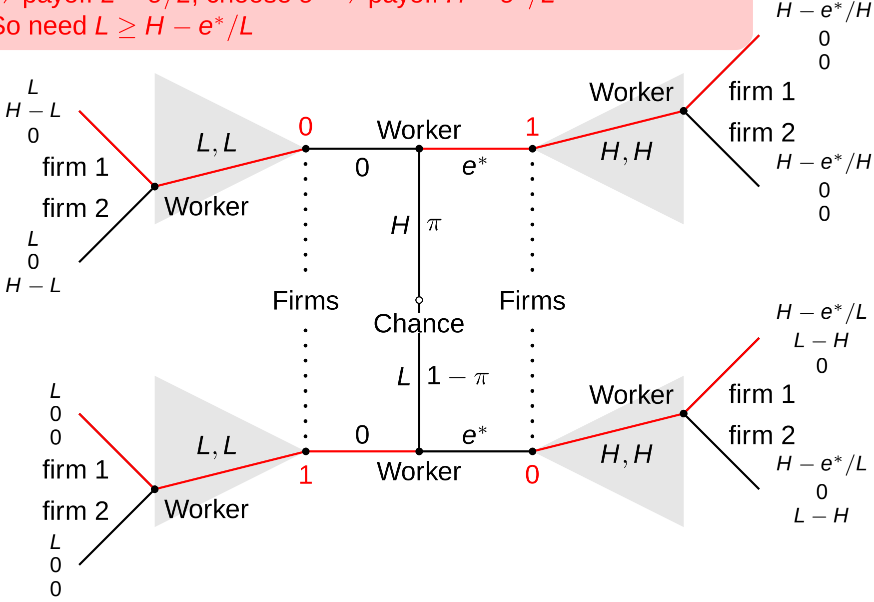


E…

Is assessment an equilibrium?

- L worker's choice of e: Choose 0 $\Rightarrow$ payoff L; choose $e \neq e^*$ $\Rightarrow$ payoff $L - e/L$; choose e^* $\Rightarrow$ payoff $H - e^*/L$

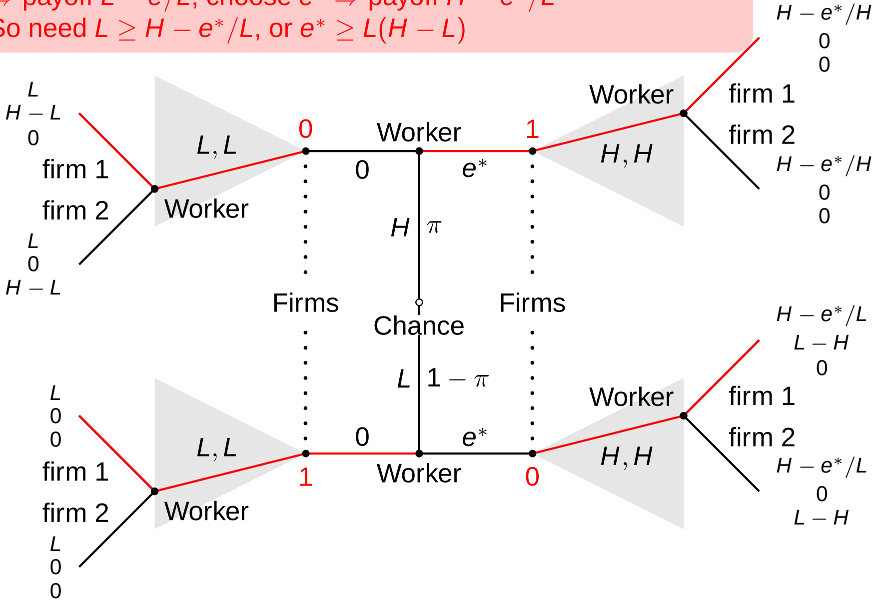
So need $L \geq H - e^*/L$



Is assessment an equilibrium?

- L worker's choice of e: Choose 0 $\Rightarrow$ payoff L; choose $e \neq e^*$ $\Rightarrow$ payoff $L - e/L$; choose e^* $\Rightarrow$ payoff $H - e^*/L$

So need $L \geq H - e^*/L$, or $e^* \geq L(H - L)$



Education as a signal of ability

Summary

Assessment is a (separating) equilibrium if and only if

$$L(H-L) \leq e^* \leq H(H-L)$$

That is, if e^* satisfies this condition, the game, in which education has no impact on productivity, has equilibrium in which

- high ability worker gets education e^*
- low ability worker gets no education
- firms deduce that education $e^* \Rightarrow$ high ability

Game *also* has (pooling) equilibrium in which all workers get the same amount of education (exercise)