Economics 316

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Solutions to Problem Set 11

1. The condition for a player not to want to deviate is
$$\frac{b}{1-\delta} \geq a + \frac{\delta}{1-\delta}$$
or
$$\delta \geq \frac{a-b}{a-1}.$$
As a increases, this lower bound increases. That is, if the payoff to D increases then the players need to be more patient to sustain cooperation as a Nash equilibrium.

2. (a) The outcome path will be $((D,C),(C,D),(C,C),(C,C),\ldots)$, which differs from the outcome path if player 1 adheres to *tit-for-tat* only in the first two periods. Thus the condition for the deviation not to be profitable is
$$b + \delta b \geq a$$
or
$$\delta \geq \frac{a-b}{b}.$$
(b) The outcome path will be $((D,C),(D,D),(D,D),\ldots)$, so player 1's payoff is
$$a + \delta + \delta^2 + \cdots = a + \frac{\delta}{1-\delta}.$$
Her payoff if she adheres to *tit-for-tat* is $b/(1-\delta)$, so the condition for the deviation not to be profitable is
$$b \geq a(1-\delta) + \delta$$
or
$$\delta \geq \frac{a-b}{a-1}.$$

(c) For neither deviation to be profitable we need both conditions to be satisfied, or

$$\delta \geq \max\left\{\frac{a-b}{b}, \frac{a-b}{a-1}\right\}.$$

If $a > 2b$ then $(a-b)/b > 1$, so there is no value of δ such that the strategy pair is a Nash equilibrium.

3. Suppose player 2 uses the strategy. If player 1 adheres to the strategy her payoff is

$$b + \delta + \delta^2 b + \delta^3 + \cdots = \frac{b+\delta}{1-\delta^2}.$$

If she deviates in the first period her payoff is

$$a + \delta + \delta^2 + \cdots = a + \frac{\delta}{1-\delta}.$$

If she deviates in the second period she is worse off in that period and not better off in any subsequent period, so such a deviation is not profitable.

Thus for the strategy pair to be a Nash equilibrium we need

$$a + \frac{\delta}{1-\delta} \leq \frac{b+\delta}{1-\delta^2},$$

or

$$\delta \geq \sqrt{(a-b)/(a-1)}.$$

4. Suppose player 2 uses the strategy. If player 1 adheres to the strategy her payoff is $b/(1-\delta)$. If she deviates to D in period 1 and continues to choose D subsequently her payoff is

$$a + \delta a + \delta^2 a + \cdots + \delta^{k-1} a + \delta^k + \delta^{k+1} + \cdots = \frac{a(1-\delta^k)}{1-\delta} + \frac{\delta^k}{1-\delta}.$$

Thus for the strategy pair to be a Nash equilibrium we need

$$\frac{b}{1-\delta} \geq \frac{a(1-\delta^k)}{1-\delta} + \frac{\delta^k}{1-\delta},$$

or

$$\delta^k \geq \frac{a-b}{a-1}$$

or

$$\delta \geq \left(\frac{a-b}{a-1}\right)^{1/k}.$$

5. Suppose that firm 2 uses the strategy. If firm 1 does so then its payoff is $\Pi^m/(n(1-\delta))$. If it deviates to a slightly lower price in the first period, then its profit is close to Π^m in the first period, and zero subsequently. Thus the strategy pair is a Nash equilibrium if and only if

$$\delta \geq 1 - 1/n.$$

As n increases, this cutoff increases. That is, collusion is sustained when n is large only if the firms are very patient.

6. (a) The Nash equilibria are (B, B) and (C, C).

   (b) The strategy pair in which each player's strategy chooses A in the first period and, in the second period, chooses C if the other player chose A in the first period and chooses B if the other player did not choose A in the first period.

   Suppose player 2 uses this strategy. Then if player 1 does so the outcome if (A, A) in the first period and (C, C) in the second period, so that player 1's payoff is 7.

   If player 1 deviates from A in the first period, the best deviation is to C, which induces player 2 to choose B in the second period, so that the best action for player 1 in the second period is also B. This deviation yields player 1 the payoff of 6. Thus it is not profitable.

   Hence the strategy pair is a Nash equilibrium.