Economics 316

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Solutions to problem for Tutorial 11

1. (a) Suppose that firm i uses the strategy s_i. If the other firm, j, uses s_j, then its payoff in the infinitely repeated game is $\frac{1}{2}\Pi^m$ in every period.

   If, on the other hand, firm j deviates to a price p then the closer this price is to p^m, the higher is j's profit, because the punishment does not depend on p. Thus by choosing p close enough to p^m the firm can obtain a profit as close as it wishes to Π^m in the period of its deviation. Its profit during its punishment in the following k periods is zero. Once its punishment is complete, it can either revert to p^m or deviate once again. If it can profit from deviating then its profit in the period of its deviation and the following k periods of punishment must exceed its profit during these $k+1$ periods if it adheres to its strategy. So the condition for it not to have a profitable deviation is

$$\Pi^m \leq \tfrac{1}{2}\Pi^m(1+\delta+\cdots+\delta^k) = \tfrac{1}{2}\Pi^m\frac{1-\delta^{k+1}}{1-\delta}.$$

   Thus for (s_1, s_2) to be a Nash equilibrium we need

$$\delta^{k+1} - 2\delta + 1 \leq 0.$$

   (b) Suppose that firm i uses the strategy s_i. If the other firm does so then its payoff is $\frac{1}{2}\Pi^m/(1-\delta)$. If the other firm deviates to some price p with $c < p < p^m$ in the first period, and maintains this price subsequently, then it obtains $\pi(p)$ in the first period and shares $\pi(p)$ in each subsequent period, so that its payoff is

$$\pi(p) + \tfrac{1}{2}\delta\pi(p) + \tfrac{1}{2}\delta^2\pi(p) + \cdots = \pi(p) + \frac{\frac{1}{2}\delta\pi(p)}{1-\delta}.$$

   Thus the condition for the deviation not to be profitable is

$$(2-\delta)\pi(p) \leq \Pi^m.$$

For $\delta < 1$ we have $2 - \delta > 1$, so that for p close enough to p^m the inequality is not satisfied. Thus for any $\delta < 1$ the strategy pair is not a Nash equilibrium of the infinitely repeated game.

2. Suppose that firm 2 uses the strategy.

   If firm 1 does so then its profit in every period is $\frac{1}{8}(\alpha - c)^2$.

   If it deviates in the first period, the most profit it can obtain in that period is its profit at its best response to the other firm's output of $\frac{1}{4}(\alpha - c)$. Its best response function in the stage game is $\frac{1}{2}(\alpha - c - q_2)$, so its best response is $\frac{3}{8}(\alpha - c)$, yielding a profit of $\frac{9}{64}(\alpha - c)^2$. In every subsequent period its best profit is $\frac{1}{9}(\alpha - c)^2$ (its profit in the Nash equilibrium of the stage game).

   Thus for firm 1's deviation not to be profitable we need

$$\tfrac{9}{64}(\alpha - c)^2 + \tfrac{1}{9}(\alpha - c)^2(\delta + \delta^2 + \dots) \le \tfrac{1}{8}(\alpha - c)^2(1 + \delta + \delta^2 + \dots),$$

   or

$$\delta \ge \tfrac{9}{17}.$$