Economics 316

Fall 2017 Martin J. Osborne

Problems for Tutorial 11

1. Consider the version of Bertrand's duopoly game considered in class, in which each firm's unit cost is constant, equal to c and the total demand for the good at the price p is $D(p)$, where D is a continuous decreasing function. Denote the profit of a monopolist charging p by $\pi(p) = (p-c)D(p)$ and assume that D is such that the function π is continuous and has a single maximizer, p^m (the "monopoly price"); denote $\pi(p^m)$ by Π^m.

   (a) Let s_i be the strategy of firm i in the infinitely repeated game of this strategic game that charges p^m in the first period and subsequently as long as the other firm continues to charge p^m, and punishes any deviation from p^m by the other firm by choosing the price c for k periods. Once a punishment is over, firm i again charges p^m until the other firm deviates, when it again initiates k periods of punishment. Given any value of δ, for what values of k is the strategy pair (s_1, s_2) a Nash equilibrium of the infinitely repeated game?

   (b) Let s_i be the following strategy for firm i in the infinitely repeated game:

   - in the first period charge the price p^m
   - in every subsequent period charge the lowest of all the prices charged by the other firm in all previous periods.

   (That is, firm i matches the other firm's lowest price.) Is the strategy pair (s_1, s_2) a Nash equilibrium of the infinitely repeated game for any discount factor less than 1?

2. Suppose that two quantity-setting firms repeatedly interact. The cost function of each firm i is cq_i and the inverse demand function is given by $P(Q) = \alpha - Q$, where $\alpha > c$. Suppose that each firm produces half of the monopoly output in period 1 and after any history in which the other firm produced half of the monopoly output in every previous period, and produces its Nash equilibrium output in the one-period

game after any other history. (That is, each firm punishes the other for deviations from half of the monopoly output by producing it one-period Nash equilibrium output in every future period.)

Find the condition on the discount factor δ under which this strategy pair is a Nash equilibrium of the infinitely repeated game.