Economics 316

Fall 2017 Martin J. Osborne

Problem Set 11

1. Find the range of values of the discount factor δ (in terms of a and b) for which the strategy pair in which both players use the strategy *Unrelenting punishment* is a Nash equilibrium of the following *Prisoner's Dilemma*, where $a > b > 1$.

	C	D
C	b, b	$0, a$
D	$a, 0$	$1, 1$

   How does the range change as a increases?

2. Suppose that each player uses the strategy *tit-for-tat* in the infinitely repeated game in which the stage game is the *Prisoner's Dilemma* with payoffs as in the previous question.

   Consider the condition on δ under which this strategy pair is a Nash equilibrium of the repeated game.

   (a) Suppose that player 1 deviates to D in period 1 and then returns to choosing C in every subsequent period. Find the condition on δ under which this deviation is not profitable.

   (b) Suppose that player 1 deviates to D in every period. Find the condition on δ under which this deviation is not profitable.

   (c) If player 1 has a profitable deviation, then either the deviation in (a) is profitable or the deviation in (b) is profitable (or both). Thus find the condition on δ for the strategy pair to be a Nash equilibrium. Is it the case that for any values of a and b the strategy pair is a Nash equilibrium when the players are sufficiently patient?

3. For the *Prisoner's Dilemma* in question 1, suppose that each player uses the following strategy: in every odd period following a history in which the other player chose C in every odd period and D in every even period, choose C; in every odd period following some other history, and in every even period, choose D.

   Find the range of values of the discount factor δ under which this strategy pair is a Nash equilibrium of the infinitely repeated game.

4. In the standard model, each player observes the other player's action in period t in the following period, $t+1$. Suppose instead that each player does not learn the other player's action in period t until period $t+k$, where $k \geq 2$. For the *Prisoner's Dilemma* in question 1, find the values of δ for which the strategy pair in which each player use the following strategy is a Nash equilibrium: chooses C until she detects that the other player has chosen D, when she switches permanently to D.

5. Consider a generalization of Bertrand's duopoly game in which there are $n \geq 2$ firms, rather than two. Assume that if all firms charge the monopoly price p^m, the demand is split equally, so that each firm's profit is Π^m/n. Find the conditions on the discount factor δ such that the strategy pair in which each firm uses the following strategy is a Nash equilibrium: charge the price p^m after any history in which every other firm charged p^m in every previous period, and charge c after any other history. How does the condition depend on n?

6. If the *Prisoner's Dilemma* is played a fixed finite number of times, then every Nash equilibrium yields the outcome (D, D) every period. (An argument is on pp. 424–425 of the book.) But for some other games a similar result does not hold. Consider the following game.

	A	B	C
A	$4,4$	$0,0$	$0,5$
B	$0,0$	$1,1$	$0,0$
C	$5,0$	$0,0$	$3,3$

   (a) Find the Nash equilibria of this game.

   (b) Suppose that the game is played twice, and the players' payoffs are the sums of their payoffs in the two periods. (There is no discounting.) Find a Nash equilibrium of the two-period game in which the outcome is (A, A) in the first period and (C, C) in the second period.