Economics 316

Fall 2017 Martin J. Osborne

Problems for Tutorial 10

1. Consider an example of Stackelberg's duopoly model in which firm 2's cost function has a fixed component. Specifically,

$$C_2(q_2) = \begin{cases} 0 & \text{if } q_2 = 0 \\ f + cq_2 & \text{if } q_2 > 0. \end{cases}$$

   Suppose that firm 1's cost function is $C_1(q_1) = cq_1$ and that the inverse demand function is $P_d(Q) = \alpha - Q$ if $Q \leq \alpha$ ($P_d(Q) = 0$ if $Q > \alpha$).

   (a) Find firm 2's strategy in a subgame perfect equilibrium.

   (b) Find the range of values of f for which firm 2's output is zero in the subgame perfect equilibrium.

2. A child's action a (a number) affects both her own private income $c(a)$ and her parent's income $p(a)$; for all values of a we have $c(a) < p(a)$. The child is selfish: she cares only about the amount of money she has. Her loving parent cares both about how much money she has and how much her child has. Specifically, her preferences are represented by a payoff equal to the smaller of the amount of money she has and the amount of money her child has. The parent may transfer money to the child. First the child takes an action, then the parent decides how much money to transfer.

   (a) Model this situation as an extensive game.

   (b) Show that in a subgame perfect equilibrium of the game the child takes an action that maximizes the sum of her private income and her parent's income. (In particular, the child's action does not maximize her own private income. This result, known as the "rotten kid theorem", is not limited to the specific form of the parent's preferences, but holds for any preferences with the property that a parent who is allocating a fixed amount x of money between herself and her child wishes to give more to the child when x is larger.)