Economics 316

Fall 2017 Martin J. Osborne

Problem Set 10

1. (a) Find the subgame perfect equilibrium of Stackelberg's duopoly game when the inverse demand function is the same as the one in the example considered class and the cost function of each firm i is $C_i(q_i) = q_i^2$. Compare the equilibrium outcome with the Nash equilibrium of Cournot's game under the same assumptions (see the problems for Tutorial 2).

   (b) Consider the strategy pair in the extensive game in which firm 1 chooses its Nash equilibrium output in Cournot's game and firm 2's strategy is to choose its Nash equilibrium output in Cournot's game *regardless* of firm 1's output. Why is this strategy pair not a subgame perfect equilibrium of the extensive game? Is it a Nash equilibrium of the extensive game?

   (c) Consider the strategy pair in the extensive game in which firm 1 chooses the output 0 and firm 2's strategy is to choose the monopoly output if firm 1 chooses the output 0, and the output α if firm 1 chooses any other output. Why is this strategy pair not a subgame perfect equilibrium of the extensive game? Is it a Nash equilibrium of the extensive game?

2. In class I argued that if a two-player strategic game has a pure strategy Nash equilibrium a^* in which a_2^* is player 2's *only* best response to a_1^* then in any subgame perfect equilibrium of the extensive game in which the players choose their actions sequentially, with player 1 moving first, player 1's payoff is at least $u_1(a^*)$.

   (a) Give an example in which a^* is the unique pure strategy Nash equilibrium of the strategic game and player 1's payoff in the unique subgame perfect equilibrium of the extensive game is *greater* than $u_1(a^*)$. (You can give an example in which each player has two actions.)

   (b) Give an example in which a^* is the unique pure strategy Nash equilibrium of the strategic game, a_2^* is not the only best response to a_1^*, and the extensive game has a subgame perfect equilibrium in which player 1's payoff is less than $u_1(a^*)$.

3. Study whether the result in the previous question generalizes to three players by considering the following three-player strategic game.

	L	R
T	$1,1,1$	$0,0,0$
B	$0,1,1$	$0,3,0$

L

	L	R
T	$0,3,0$	$0,2,1$
B	$0,3,0$	$0,2,1$

R

As usual, player 1 chooses a row and player 2 chooses a column. Player 3 chooses a table. Thus, for example, the payoff profile for the action profile (T, R, L) is $(0, 0, 0)$ and the payoff profile for the action profile (B, L, R) is $(0, 3, 0)$.

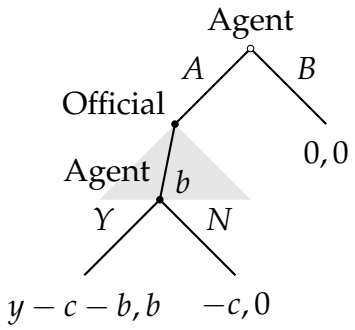
   (a) Find the pure strategy Nash equilibria of this game.

   (b) Now suppose that the players move sequentially. First player 1 chooses T or B, then player 2 chooses L or R, and finally player 3 chooses L or R. Suppose that the players' payoffs to each terminal history (x, y, z) are their payoffs to the action profile (x, y, z) in the strategic game. Find the subgame perfect equilibria of this extensive game.

4. Find the subgame perfect equilibria of the variant of the ultimatum game in which all offers must be integral multiples of a monetary unit that is small relative to the size of the pie. (For example, the pie is of size \$20 and all offers must be a multiple of \$1.)

5. An agent can pursue activity A or activity B. Activity B yields the return 0. Activity A yields the return y but requires both an unrecoverable investment of $c > 0$ and a permit from an official. The permit is free, but the official may demand a bribe of any magnitude; the permit is granted only after the investment has been made and the bribe paid. If the agent pays the bribe b, her payoff is $y - c - b$ and the official's payoff is b.

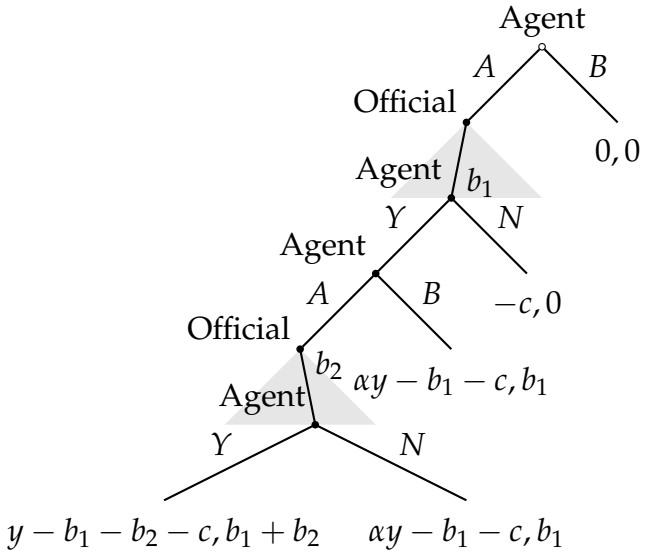
   Model this situation as the following extensive game with perfect information.



(a) Show that this game has a unique subgame perfect equilibrium, in which the agent pursues activity B and the payoffs of both the agent and the official are 0.

Now suppose that activity A takes time and that the official requires a sequence of two permits. One permit is required at the start, after the agent has invested, and another is required after the fraction of time α has passed, during which the activity yields the return αy. In the remainder of the time, the activity yields the return $(1 - \alpha)y$. The official may demand a bribe for each permit. If the agent pays the bribe b_1 demanded initially but does not pay the second bribe, her payoff is $\alpha y - b_1 - c$ and the official's is b_1. If she pays both bribes then her payoff is $y - b_1 - b_2 - c$ and the official's is $b_1 + b_2$, where b_2 is the second bribe.

Model this situation as the following extensive game with perfect information.



(b) Study the subgame perfect equilibria of this game as follows.

i. For any value of b_1, find the subgame perfect equilibrium of the subgame following the history (A, b_1, Y, A).

ii. You should have found that in the subgame following any history (A, b_1, Y), the agent is indifferent between A and B, in both cases obtaining the payoff $\alpha y - b_1 - c$. Suppose that for some number b_1^* with $(2\alpha - 1)y \leq b_1^* \leq \alpha y - c$ the agent chooses A after any history (A, b_1, Y) with $b_1 \leq b_1^*$ and B after any history (A, b_1, Y) with $b_1 > b_1^*$. (Such a number b_1^* exists because $\alpha \leq 1 - c/y$.)
For each possible value of b_1, find the optimal action of the agent following the history (A, b_1).

iii. Show that if $\alpha \leq 1 - c/y$ then the game has a subgame perfect equilibrium in which the official chooses the bribe b_1^* after the history A, and the agent chooses A at the start of the game.

iv. Does the game also have a subgame perfect equilibrium in which the agent chooses B initially?

6. Watch http://www.youtube.com/watch?v=S0qjK3TWZE8.

(a) Formulate the choice of *Split* or *Steal* for the two players as a strategic game and find its Nash equilibria.

(b) The person on the right wants to commit to choose *Steal* and transfer half of his winnings to the person on the left after the show. Of course he cannot write a contract, but promising in front of millions of viewers is pretty close to a commitment.

Suppose that before playing the strategic game you formulated in the first part, player 1 can commit to choosing an action and transferring payoff to player 2. Player 1 has a positive payoff to transfer only if player 2 chooses *Split*, so you can model the situation as an extensive game in which player 1 first chooses either *Split* and an amount to transfer to player 2 in the event player 2 also chooses *Split*, or *Steal* and an amount to transfer to player 2 in the event player 2 chooses *Split*, and then player 2 chooses either *Split* or *Steal*. Assume that no transfer can be more than the amount the player wins in the game. (For example, if the outcome is $(Split, Split)$, player 1 can transfer no more than 50 to player 2.) Find the subgame perfect equilibria of this game.

7. Prepare your strategy for the repeated *Prisoner's Dilemma* experiment in next week's class.