ECO316: Applied game theory
Lecture 10

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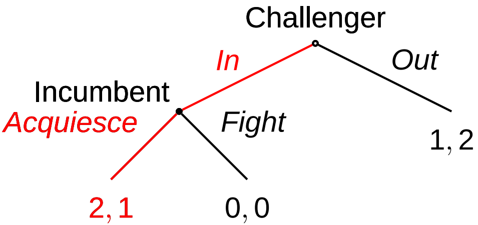
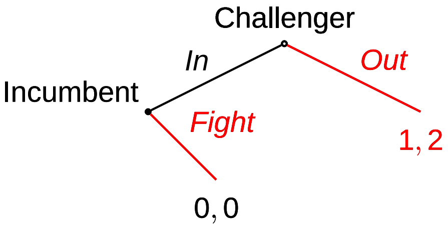
Are more options better?

- For an isolated decision-maker, more options are always better (ignoring the cost of making a decision!)
- Is the same true in a game?
- Consider entry game
- Unique subgame perfect equilibrium is $(In, Acquiesce)$



Are more options better?

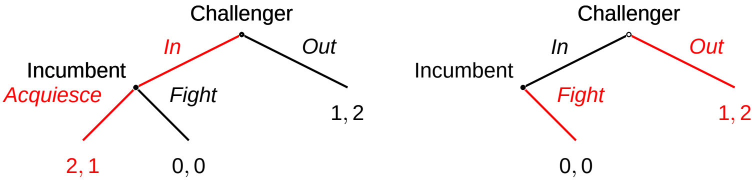
- Eliminate incumbent's option to acquiesce



- Then subgame perfect equilibrium: $(Out, Fight)$
- Incumbent is *better off* in this equilibrium than in equilibrium of original game
- So fewer options can be better
- Alternatively, commitment has a value
  - without option to acquiesce, threat to fight is credible

In surrounding an enemy, leave him a way out

- From Challenger's point of view, it is *better* for Incumbent to have option to acquiesce



- In this case, Incumbent's having more options is better for Challenger
- Corresponds to Sun Tzu's advice in *The Art of Warfare* (written between 500 BC and 300 BC): "in surrounding the enemy, leave him a way out; do not press an enemy that is cornered"

Stackelberg's duopoly model

- Variant of Cournot's duopoly model in which firms act sequentially
- Firm 1 chooses an output, then firm 2 observes this output and chooses an output
- Firm i's cost of producing q_i units of output: $C_i(q_i)$
- Price when total output is Q: $P_d(Q)$

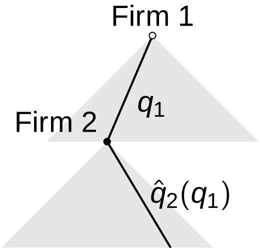
Stackelberg's duopoly model

Extensive game

- Players: The two firms
- Terminal histories: The set of all sequences (q_1, q_2) of outputs for the firms
- Player function: $P(\varnothing) = 1$ and $P(q_1) = 2$ for all q_1
- Payoffs: Payoff of each firm i to terminal history (q_1, q_2) is its profit

$$q_i P_d(q_1 + q_2) - C_i(q_i)$$

Stackelberg's duopoly model



Strategies

- Firm 1: An output, q_1
- Firm 2: A *function* $\hat{q}_2$, where $\hat{q}_2(q_1)$ is firm 2's output if firm 1's output is q_1

Stackelberg's duopoly model

Subgame perfect equilibrium

Use backward induction

- For every possible q_1, find optimal value of q_2:

$$\max_{q_2} [q_2 P_d(q_1 + q_2) - C_2(q_2)]$$

  - Call optimal value $\hat{q}_2^*(q_1)$
  - $\hat{q}_2^*$ is firm 2's best response function in Cournot's model
- Given firm 2's strategy $\hat{q}_2^*$, find optimal value of q_1:

$$\max_{q_1} [q_1 P_d(q_1 + \hat{q}_2^*(q_1)) - C_1(q_1)]$$

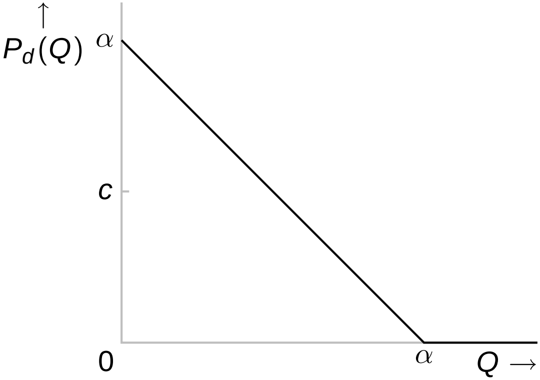
  - Call optimal value q_1^*
- Subgame perfect equilibrium is $(q_1^*, \hat{q}_2^*)$ [note: $\hat{q}_2^*$ is a function!]
- Outcome of subgame perfect equilibrium is $(q_1^*, \hat{q}_2^*(q_1^*))$

Stackelberg's duopoly model

Example

Specification

- $C_i(q_i) = cq_i$ for $i = 1, 2$, with $c > 0$
- $P_d(Q) = \alpha - Q$ if $Q \leq \alpha$, with $\alpha > c$



Stackelberg's duopoly model

Example

Backward induction

- Firm 2: for each q_1, $\hat{q}_2^*(q_1)$ is solution of

$$\max_{q_2}[q_2(\alpha - q_1 - q_2) - cq_2]$$

$\Rightarrow \hat{q}_2^*(q_1) = \frac{1}{2}(\alpha - c - q_1)$
(firm 2's best response function in Cournot's model)

- Firm 1: q_1^* is solution of

$$\max_{q_1}[q_1(\alpha - q_1 - \hat{q}_2^*(q_1)) - cq_1]$$
$$\Rightarrow \max_{q_1}[q_1(\alpha - q_1 - \tfrac{1}{2}(\alpha - c - q_1)) - cq_1]$$
$$\Rightarrow \max_{q_1}[\tfrac{1}{2}q_1(\alpha - c - q_1)]$$

$\Rightarrow q_1^* = \frac{1}{2}(\alpha - c)$

Stackelberg's duopoly model

Example

Subgame perfect equilibrium

- Unique subgame perfect equilibrium, $(q_1^*, \hat{q}_2^*)$:
  - firm 1's strategy: $q_1^* = \frac{1}{2}(\alpha - c)$
  - firm 2's strategy: function $\hat{q}_2^*$ given by

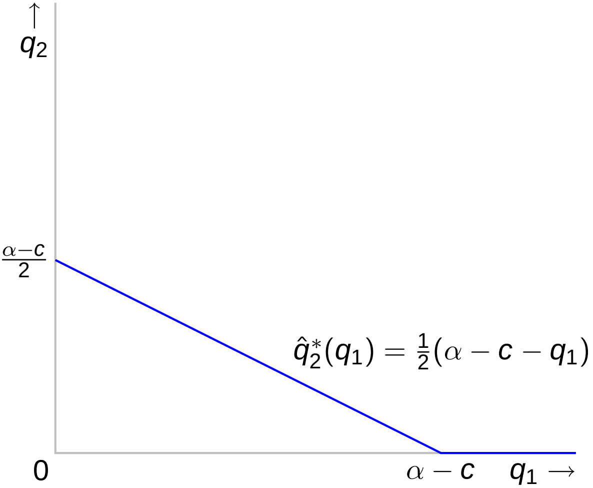
$$\hat{q}_2^*(q_1) = \tfrac{1}{2}(\alpha - c - q_1) \text{ for all } q_1$$

- Outcome of subgame perfect equilibrium:
  - firm 1's output: $q_1^* = \frac{1}{2}(\alpha - c)$
  - firm 2's output:

$$\begin{aligned} q_2^* &= \hat{q}_2^*(q_1^*) \\ &= \tfrac{1}{2}(\alpha - c - \tfrac{1}{2}(\alpha - c)) \\ &= \tfrac{1}{4}(\alpha - c) \end{aligned}$$

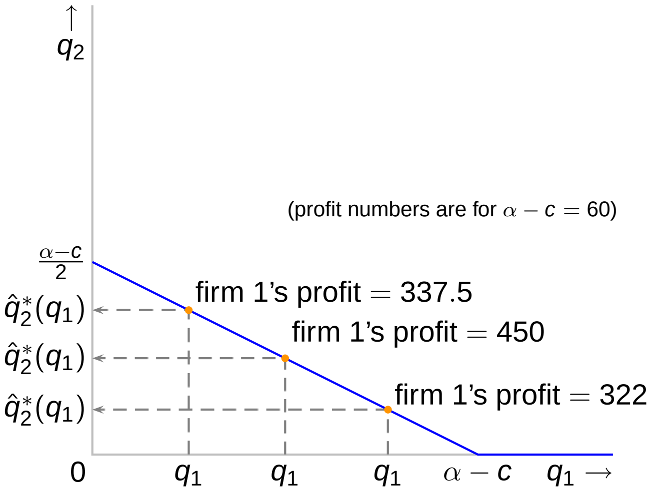
Stackelberg's duopoly model

Example



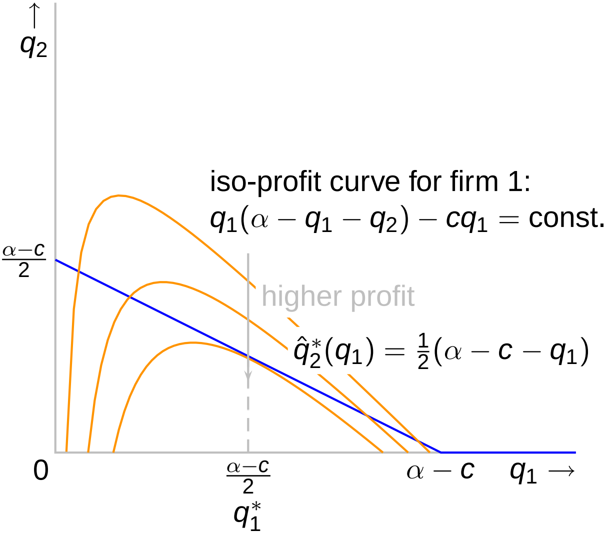
Stackelberg's duopoly model

Example



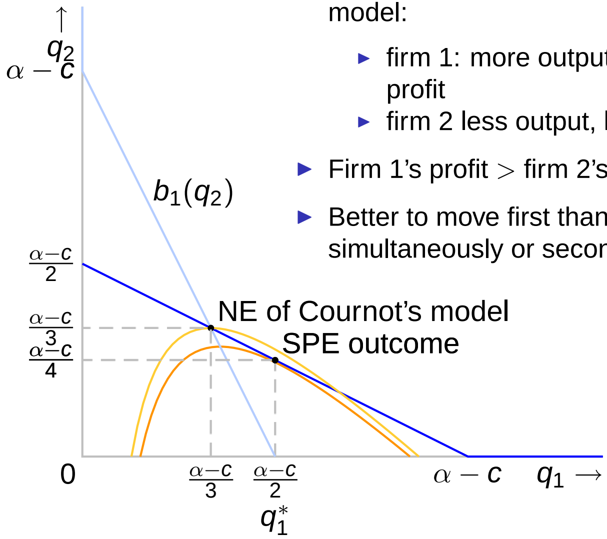
Stackelberg's duopoly model

Example



Stackelberg's duopoly model

Example



- Compared with NE of Cournot's model:
  - firm 1: more output, more profit
  - firm 2 less output, less profit
- Firm 1's profit $>$ firm 2's profit
- Better to move first than simultaneously or second

General result: first-mover advantage

- Consider two-player strategic game (simultaneous moves)
  - Let (a_1^*, a_2^*) be Nash equilibrium
- Consider extensive game in which player 1 moves first
  - Suppose player 1 chooses a_1^*
  - What action does player 2 choose?
  - (a_1^*, a_2^*) Nash equilibrium of simultaneous move game

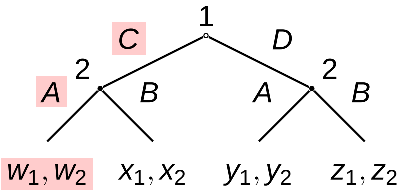
    $\Rightarrow$ a_2^* is best response to a_1^*

    $\Rightarrow$ a_2^* is an optimal choice of player 2 following a_1^* in extensive game

Example

	A	B
C	w_1, w_2	x_1, x_2
D	y_1, y_2	z_1, z_2

$$w_1 \geq y_1,\ w_2 \geq x_2$$



General result: first-mover advantage

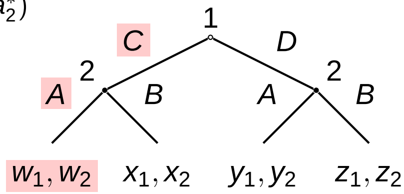
- Consider two-player strategic game (simultaneous moves)
  - Let (a_1^*, a_2^*) be Nash equilibrium
- Consider extensive game in which player 1 moves first
  - *If* a_2^* is *only* optimal choice of player 2, player 1's choice of a_1^* *guarantees* her the payoff $u_1(a_1^*, a_2^*)$

    $\Rightarrow$ in any subgame perfect equilibrium, player 1's payoff $\geq u_1(a_1^*, a_2^*)$

Example

	A	B
C	w_1, w_2	x_1, x_2
D	y_1, y_2	z_1, z_2

$w_1 \geq y_1$, $w_2 > x_2$



General result: first-mover advantage

- Suppose two-player strategic game (simultaneous moves) has pure strategy Nash equilibrium a^* in which a_2^* is player 2's *only* best response to a_1^*
- Then in *every* subgame perfect equilibrium of extensive game in which player 1 moves first, player 1's payoff is *at least* $u_1(a_1^*, a_2^*)$
- That is: first-mover's payoff in every subgame perfect equilibrium of extensive game is at least as high as her payoff in every Nash equilibrium of simultaneous move game

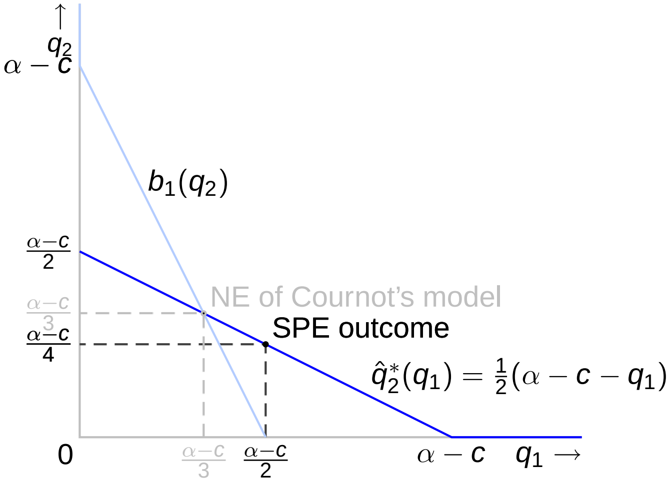
Notes

- What about *Matching Pennies*?

Stackelberg's duopoly model

Value of commitment

- In subgame perfect equilibrium, is firm 1's output a best response to firm 2's output?



Stackelberg's duopoly model

Value of commitment

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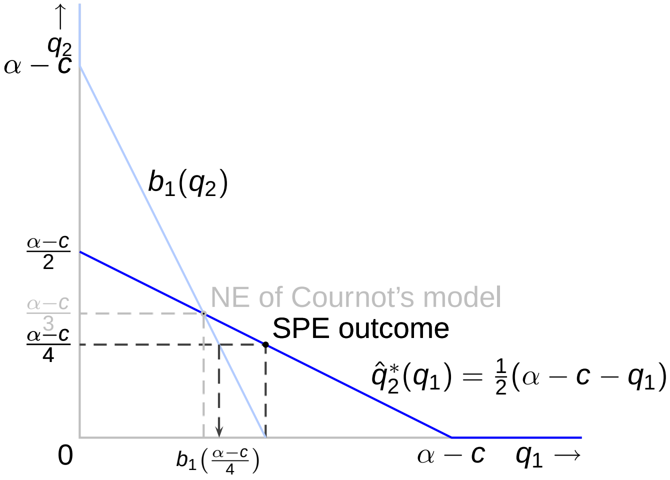


Stackelberg's duopoly model

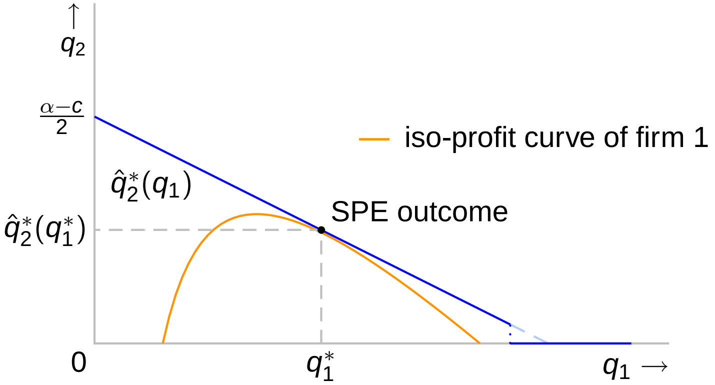
Value of commitment

- In subgame perfect equilibrium, firm 1's output is *not* a best response to firm 2's output
- So after firm 2 moves, firm 1 would like to change its output
- Would firm 1 be better off if it had the opportunity to change its output after firm 2 has acted?
- No! If firm 1 has such an opportunity,
  - first stage of game is irrelevant, and *firm 2* is effectively first-mover
  - $\Rightarrow$ in subgame perfect equilibrium firm 1 is worse off than it is in Nash equilibrium of simultaneous-move game
- Firm 1 prefers to be *committed* not to change its mind

Stackelberg's duopoly model

Example with fixed cost

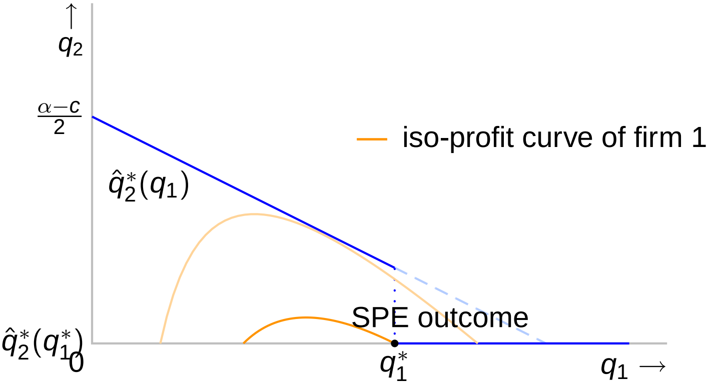
- Suppose firm 2 incurs a fixed cost (independent of its output) if it produces a positive output
- When fixed cost is small, it does not affect SPE outcome



Stackelberg's duopoly model

Example with fixed cost

- Suppose firm 2 incurs a fixed cost (independent of its output) if it produces a positive output
- When fixed cost is larger, it *does* affect SPE outcome



Stackelberg's duopoly model

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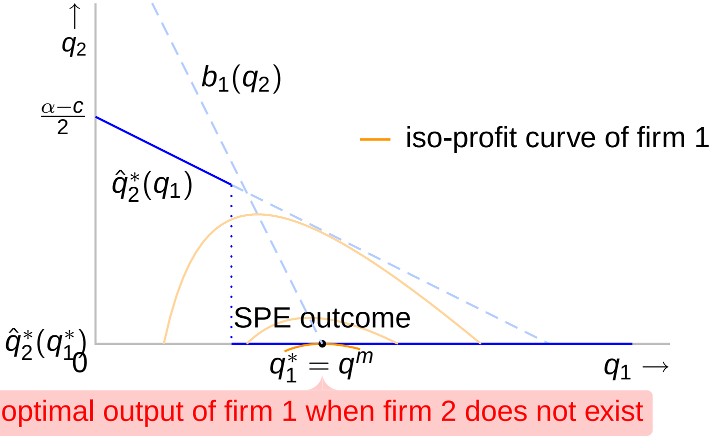


Stackelberg's duopoly model

Example with fixed cost

Summary

- For medium-size fixed cost, subgame perfect equilibrium in which firm 1 produces more than monopoly output and firm 2 produces no output
- If firm 2 were entirely absent from the market, firm 1 would produce q^m, less than q_1^*
- Thus firm 2's presence affects the outcome, even though it produces no output

Stackelberg's duopoly model

Lessons

- First-mover advantage
- Value of commitment
- Existence of potential entrant may affect outcome even if in equilibrium firm does not in fact enter

Ultimatum game

- Two players: proposer and responder
- Pie of size c
- Proposer offers an amount of pie (from 0 to c) to responder
- Responder either accepts or rejects offer
  - If responder accepts an offer of x, proposer gets $c - x$ and responder gets x
  - If responder rejects an offer, both proposer and responder get 0

Ultimatum game

Experiment

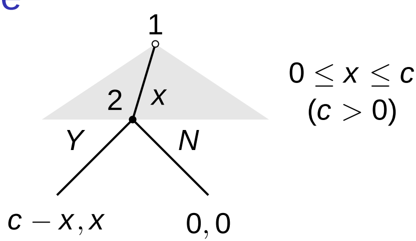
- Pie of size \$20
- Every participant will first act as a proposer
- Every participant will choose an amount from \$0 to \$20 to offer a responder
- After all participants have chosen offers, every participant's offer will be presented to another randomly chosen participant—a responder—who will either accept or reject it
  - If responder accepts an offer of x, proposer will get $\$(20 - x)$ and responder will get $\$x$
  - If responder rejects an offer, both proposer and responder get \$0
  - Your total payoff will be the sum of the payoffs you get as a proposer and as a responder

Ultimatum game

Experiment

- All interaction will be anonymous
- No participant will know identity of participant with whom s/he is matched
- Matching will be random
- If participant A's offer is presented to participant B for a response, then participant B's offer will *not* (except by chance) be presented to participant A for response
- Names of participants with top 3 payoffs will be revealed, but *not* their payoffs

Ultimatum game



Extensive game

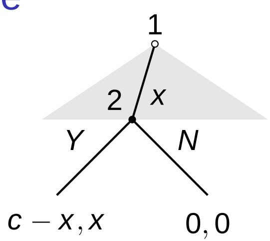
Players 1 and 2

Terminal histories Set of sequences (x, d) where $0 \leq x \leq c$ and d is Y or N

Player function $P(\varnothing) = 1$, $P(x) = 2$ for all x

Payoffs $u_1(x, Y) = c - x$, $u_2(x, Y) = x$ for all x, and $u_1(x, N) = u_2(x, N) = 0$

Ultimatum game



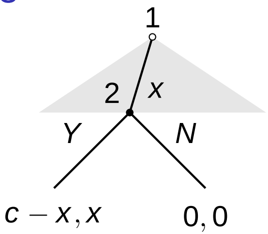
$0 \leq x \leq c$
$(c > 0)$

Strategies

Player 1 Set of x with $0 \leq x \leq c$

Player 2 Functions s_2 for which, for each value of x, $s_2(x)$ is either Y or N

Ultimatum game



$0 \leq x \leq c$
$(c > 0)$

Backward induction

In the subgame following x, Y is optimal if $x > 0$, and *both* Y and N are optimal if $x = 0$

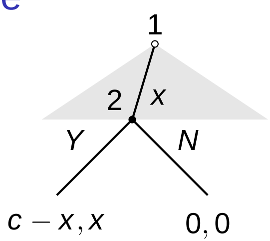
So *two* optimal strategies in subgame:

$$s_2^1(x) = Y \text{ for all } x \qquad\qquad s_2^2(x) = \begin{cases} Y & \text{if } x > 0 \\ N & \text{if } x = 0 \end{cases}$$

$$\Downarrow \qquad\qquad\qquad\qquad \Downarrow$$

Optimal action of P1 is 0   *No* optimal action of P1

Ultimatum game



$0 \leq x \leq c$
$(c > 0)$

Subgame perfect equilibria

Hence *unique* subgame perfect equilibrium: $s_1 = 0$ and $s_2(x) = Y$ for all x.

Ultimatum game: Experimental evidence

- Experiment at University of Cologne (West Germany) in late 1970s among graduate students of economics (authors say “It is almost sure that none of the students was familiar with game theory”):
  - Size of pie: DM 4–10 (about \$2–5 then, worth \$6–14 now)
  - Average offer of player 1 around $0.3c$ to $0.35c$ (versus subgame perfect equilibrium offer of 0)
  - About 20% of offers rejected

  Source: Güth et al., *Journal of Economic Behavior and Organization* 3 (1982), 367-388

- Many other experiments yield similar results
- If stakes are high, some evidence that proposers offer lower fraction of pie and fewer offers are rejected

Ultimatum game: Experimental evidence

Pie size: one or two days' wages

Group	Country	Avg. offer	Rejection rate
Machiguenga	Perú	26%	5%
Torguud	Mongolia	35%	5%
Tsimané	Bolivia	37%	0%
Sangu	Tanzania	41%	10%
Lamalera	Indonesia	58%	0%

Source: Henrich et al., *American Economic Review, Papers and Proceedings* 91 (2001), 73–78

Ultimatum game: Experimental evidence

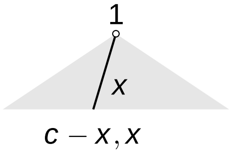
Subjects: 6 chimpanzees in Atlanta
Two possible divisions of pie: one equitable, one not equitable

	% choosing equitable division
Pair 1	58%
Pair 2	71%
Pair 3	67%
Pair 4	92%

Source: Proctor et al., *Proceedings of the National Academy of Sciences of the United States of America*, January 14, 2013

Ultimatum game: Experimental evidence

- Could a preference for fairness explain the results?
- Consider variant of ultimatum game in which player 2 has no option to reject offer
- Called *dictator game*



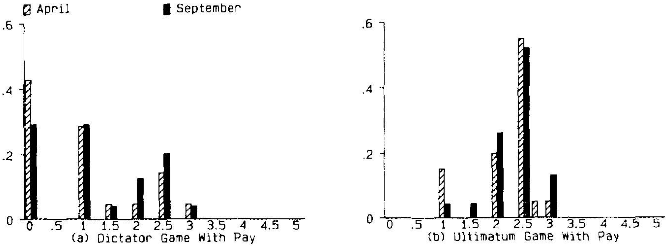
$$0 \le x \le c$$
$$(c > 0)$$

- Unique subgame perfect equilibrium: player 1 offers 0
- If non-zero offers in ultimatum game are result of subjects' concern for fairness, should get similar outcomes in dictator game

Ultimatum game: Experimental evidence

Dictator game

Subjects: students at University of Iowa
Pie size: $5



- Dictators offer less than proposers in ultimatum game, but still offer significant positive amounts

Source: Forsythe et al., *Games and Economic Behavior* 6 (1994), 347–369. See also Bolton et al., *International Journal of Game Theory* 27 (1998), 269–299 and Eckel et al., *Journal of Economic Behavior and Organization* 80 (2011), 603–612.

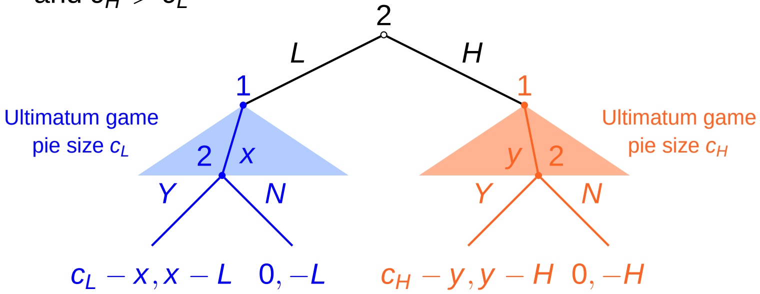
Ultimatum game: Experimental evidence

Another hypothesis

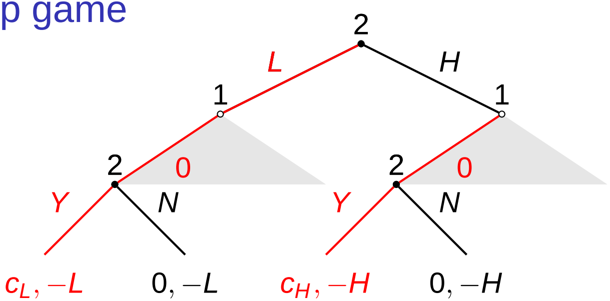
- Significant offers of proposer consistent with proposer's fear that responder will reject offer
- And in fact responders do reject offers
- Why do responders reject offers?
- They may fail to comprehend fully the isolated nature of the interaction, and instead follow their instinct, which is shaped by the long-term relationships to which they are accustomed
- In a long-term relationship, "punishing" a proposer who makes a low offer by rejecting it may have benefit of discouraging low offers in the future

Holdup game

- Before playing ultimatum game, responder decides whether to expend low effort (L) or high effort (H)
- More effort is more costly, but produces bigger pie: $H > L$ and $c_H > c_L$



Holdup game



Analysis

- SPE: in each ultimatum game, P1 offers 0 and P2 accepts all offers
- SPE of whole game: P2 chooses L

$\Rightarrow$ inefficient outcome if $c_H - H > c_L - L$

- P2 is "held up" for all the surplus her extra effort produces
- Even with less extreme outcome of bargaining, SPE outcome may still be inefficient

Repeated games: Experiment

- Each participant will be matched with another participant to play the Prisoner's Dilemma repeatedly
- Game interpreted as duopoly, with choices high price and low price

	high price	low price
high price	$7, 7$	$0, 10$
low price	$10, 0$	$3, 3$

- Each participant plays against the *same* opponent repeatedly
- After 10 rounds of play, with probability 0.07 play stops after each round $\Rightarrow$ random number of rounds
- At all points in the game, each player observes the history of play (his/her actions and actions of opponent)