

ECO316: Applied game theory

Lecture 10

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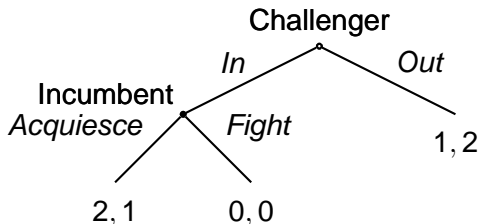
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Are more options better?

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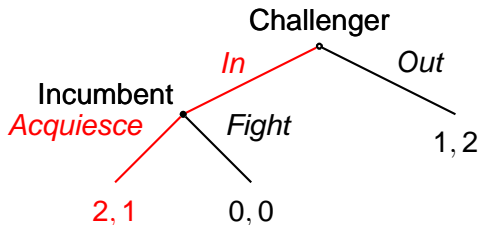
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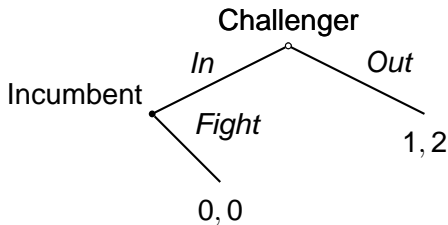
Are more options better?

- ▶ For an isolated decision-maker, more options are always better (ignoring the cost of making a decision!)
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- ▶ Unique subgame perfect equilibrium is $(In, Acquiesce)$



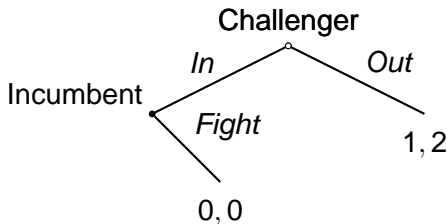
Are more options better?

- Eliminate incumbent's option to acquiesce



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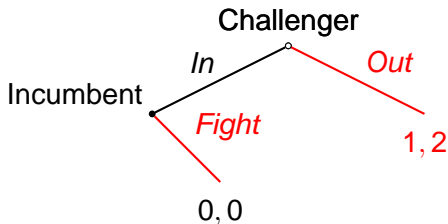
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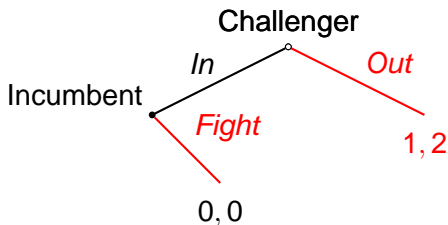
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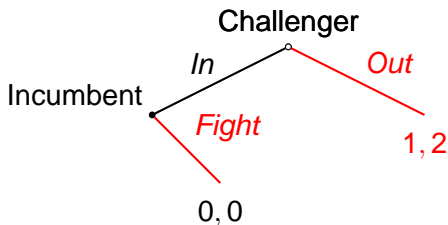
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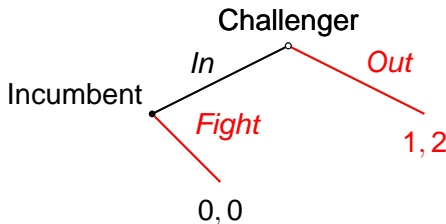
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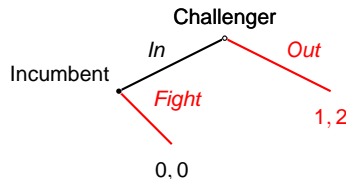
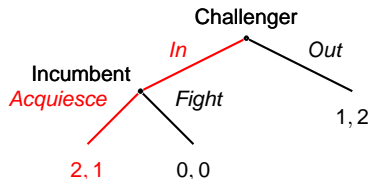
- ▶ Eliminate incumbent's option to acquiesce



- ▶ Then subgame perfect equilibrium: (*Out*, *Fight*)
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- ▶ So fewer options can be better
- ▶ Alternatively, commitment has a value
 - ▶ without option to acquiesce, threat to fight is credible

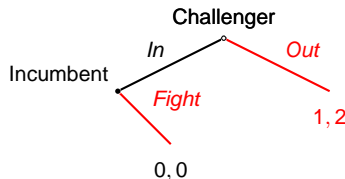
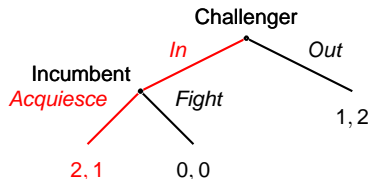
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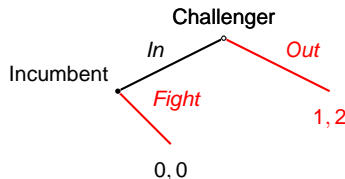
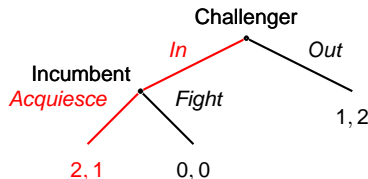
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In surrounding an enemy, leave him a way out

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- In this case, Incumbent's having more options is better for Challenger
- Corresponds to Sun Tzu's advice in *The Art of Warfare* (written between 500 BC and 300 BC): "in surrounding the enemy, leave him a way out; do not press an enemy that is cornered"

Stackelberg's duopoly model

- ▶ Variant of Cournot's duopoly model in which firms act sequentially

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- ▶ Price when total output is Q : $P_d(Q)$

Stackelberg's duopoly model

Extensive game

- Players: The two firms

Stackelberg's duopoly model

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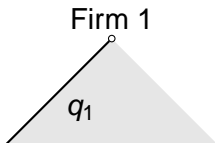
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Stackelberg's duopoly model

Strategies

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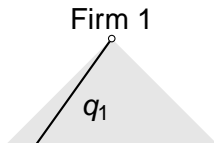
Stackelberg's duopoly model



Strategies

- Firm 1: An output, q_1

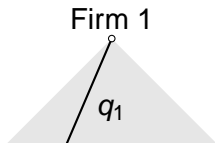
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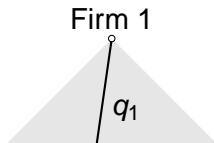
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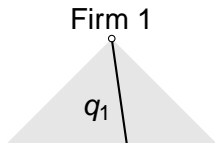
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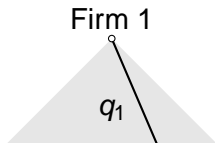
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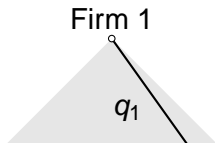
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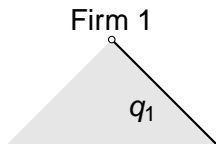
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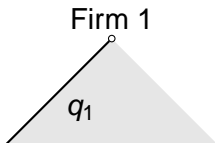
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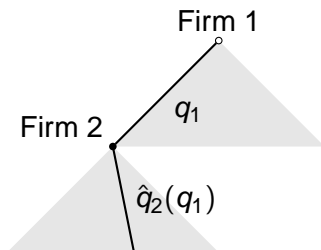
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Strategies

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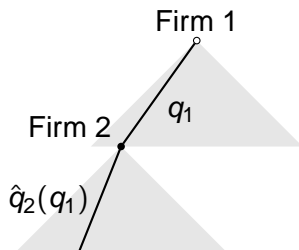
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- ▶ Firm 1: An output, q_1
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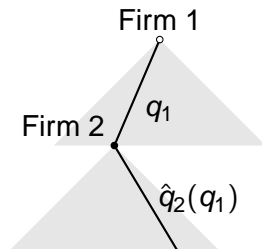
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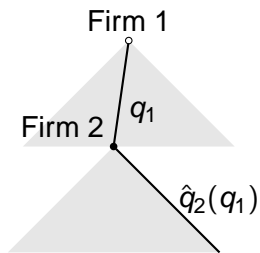
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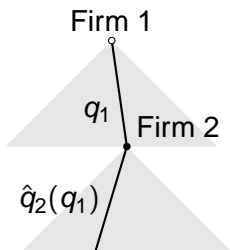
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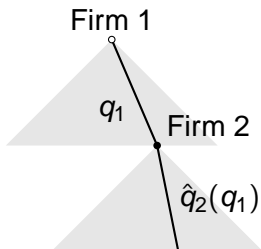
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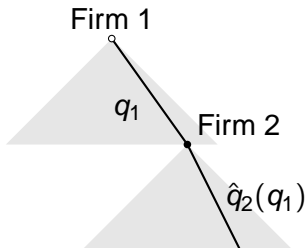
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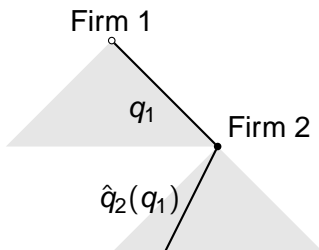
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- For every possible q_1 , find optimal value of q_2 :

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Stackelberg's duopoly model

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Stackelberg's duopoly model

Subgame perfect equilibrium

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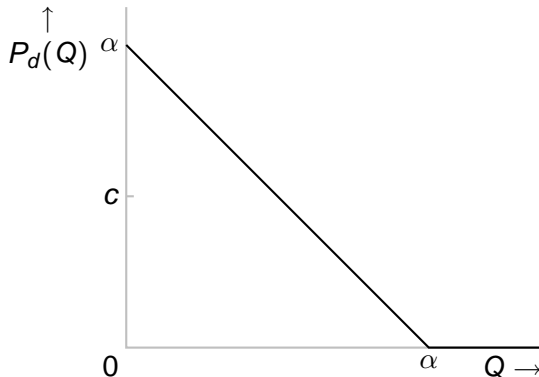
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Stackelberg's duopoly model

Example

Specification

- ▶ $C_i(q_i) = cq_i$ for $i = 1, 2$, with $c > 0$
- ▶ $P_d(Q) = \alpha - Q$ if $Q \leq \alpha$, with $\alpha > c$



Stackelberg's duopoly model

Example

Backward induction

- Firm 2: for each q_1 , $\hat{q}_2^*(q_1)$ is solution of

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$$\Rightarrow \max_{q_1} [\frac{1}{2}q_1(\alpha - c - q_1)]$$

Stackelberg's duopoly model

Example

Backward induction

- Firm 2: for each q_1 , $\hat{q}_2^*(q_1)$ is solution of

$$\max_{q_2} [q_2(\alpha - q_1 - q_2) - cq_2]$$

$$\Rightarrow \hat{q}_2^*(q_1) = \frac{1}{2}(\alpha - c - q_1)$$

(firm 2's best response function in Cournot's model)

- Firm 1: q_1^* is solution of

$$\max_{q_1} [q_1(\alpha - q_1 - \hat{q}_2^*(q_1)) - cq_1]$$

$$\Rightarrow \max_{q_1} [q_1(\alpha - q_1 - \frac{1}{2}(\alpha - c - q_1)) - cq_1]$$

$$\Rightarrow \max_{q_1} [\frac{1}{2}q_1(\alpha - c - q_1)]$$

$$\Rightarrow q_1^* = \frac{1}{2}(\alpha - c)$$

Stackelberg's duopoly model

Example

Subgame perfect equilibrium

- Unique subgame perfect equilibrium, (q_1^*, \hat{q}_2^*) :

Stackelberg's duopoly model

Example

Subgame perfect equilibrium

- ▶ Unique subgame perfect equilibrium, (q_1^*, \hat{q}_2^*) :
 - ▶ firm 1's strategy: $q_1^* = \frac{1}{2}(\alpha - c)$

Stackelberg's duopoly model

Example

Subgame perfect equilibrium

- ▶ Unique subgame perfect equilibrium, (q_1^*, \hat{q}_2^*) :
 - ▶ firm 1's strategy: $q_1^* = \frac{1}{2}(\alpha - c)$
 - ▶ firm 2's strategy: function \hat{q}_2^* given by

$$\hat{q}_2^*(q_1) = \frac{1}{2}(\alpha - c - q_1) \text{ for all } q_1$$

Stackelberg's duopoly model

Example

Subgame perfect equilibrium

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- ▶ Outcome of subgame perfect equilibrium:

Stackelberg's duopoly model

Example

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Stackelberg's duopoly model

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Subgame perfect equilibrium

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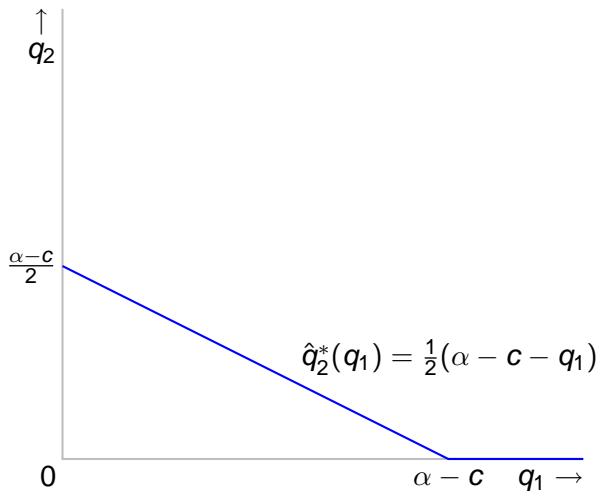
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- ▶ Outcome of subgame perfect equilibrium:
 - ▶ firm 1's output: $q_1^* = \frac{1}{2}(\alpha - c)$
 - ▶ firm 2's output:

$$\begin{aligned} q_2^* &= \hat{q}_2^*(q_1^*) \\ &= \frac{1}{2}(\alpha - c - \frac{1}{2}(\alpha - c)) \\ &= \frac{1}{4}(\alpha - c) \end{aligned}$$

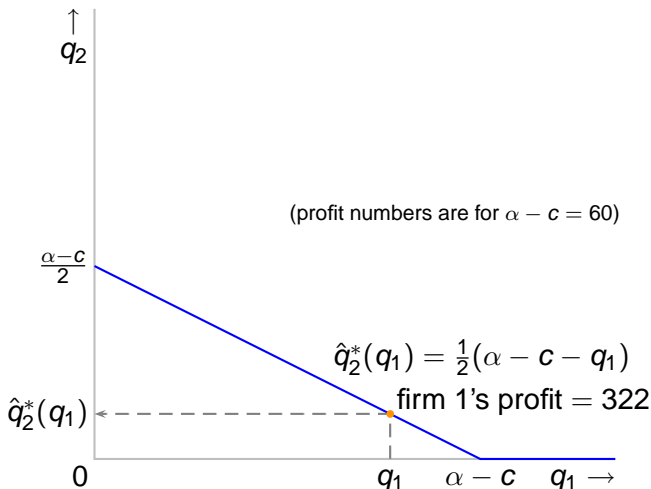
Stackelberg's duopoly model

Example



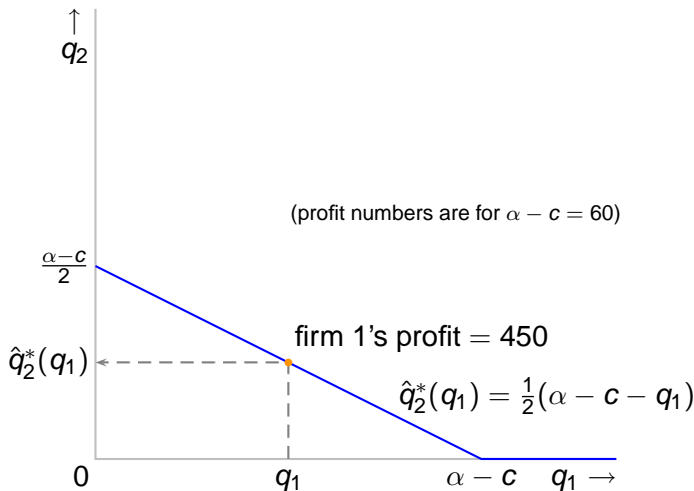
Stackelberg's duopoly model

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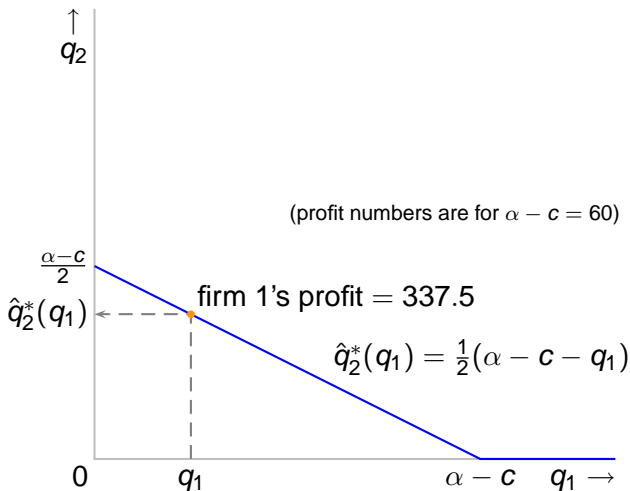
Stackelberg's duopoly model

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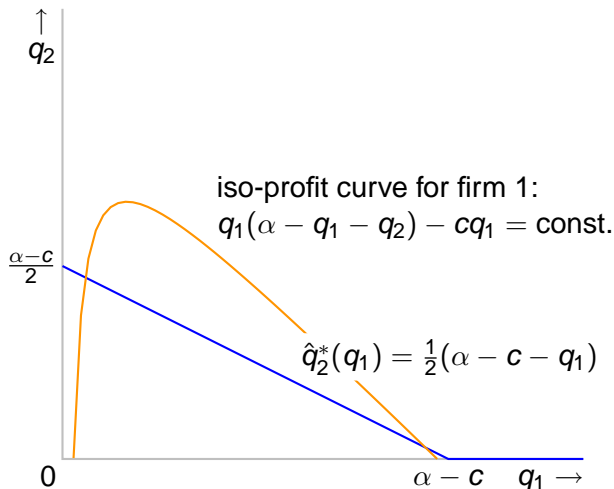
Stackelberg's duopoly model

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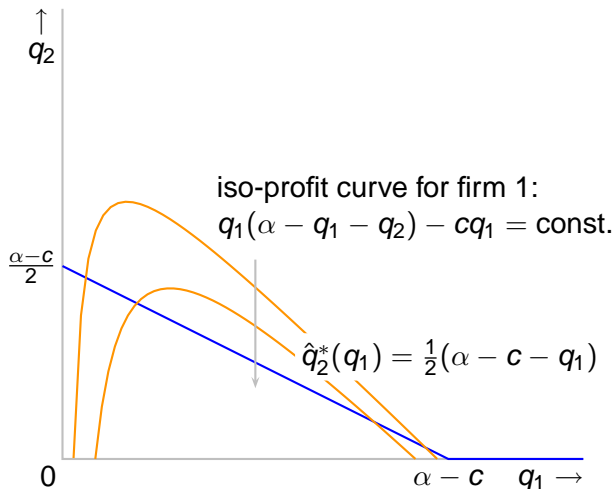
Stackelberg's duopoly model

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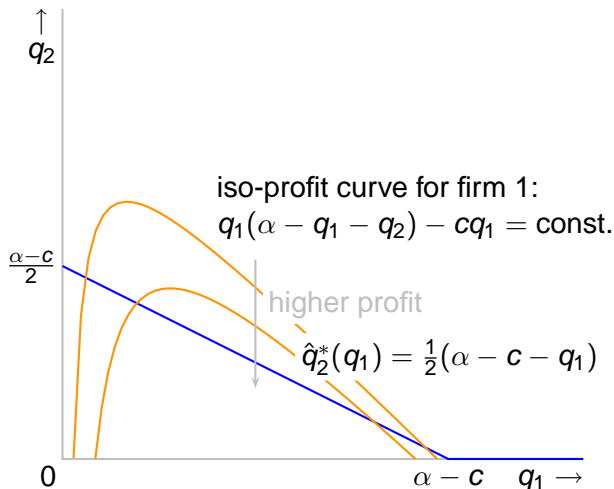
Stackelberg's duopoly model

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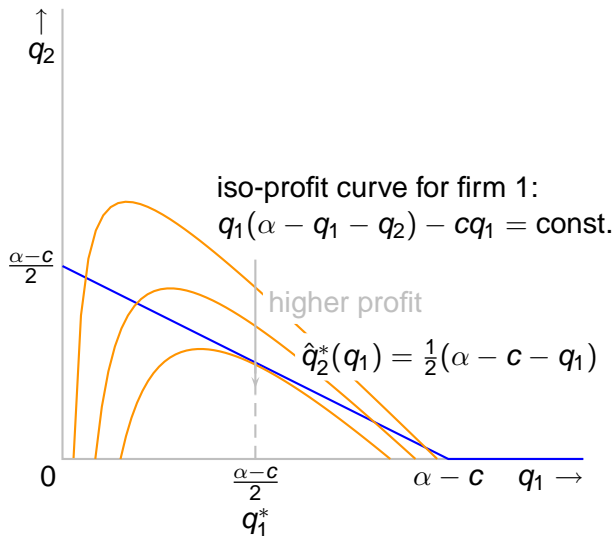
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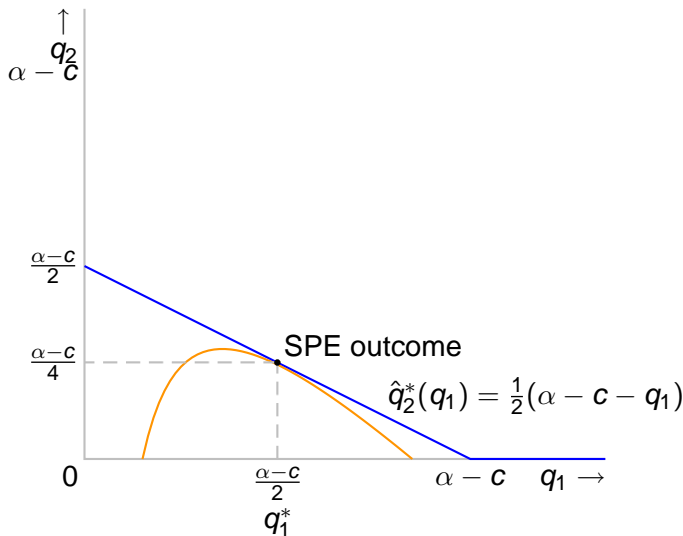
Stackelberg's duopoly model

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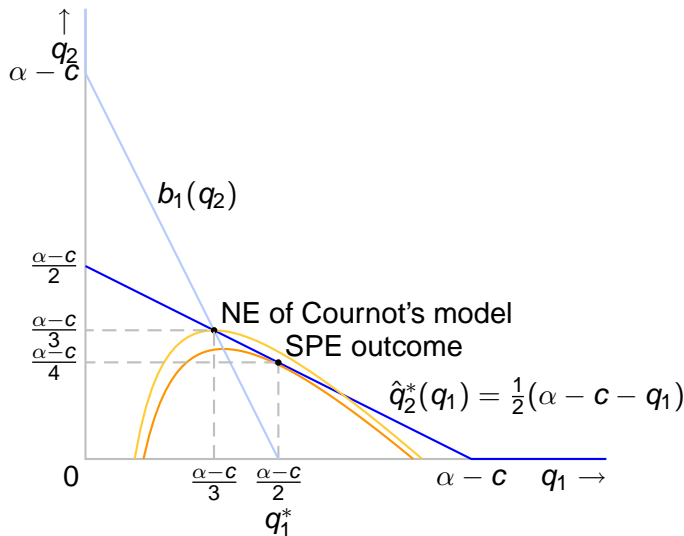
Stackelberg's duopoly model

Example



Stackelberg's duopoly model

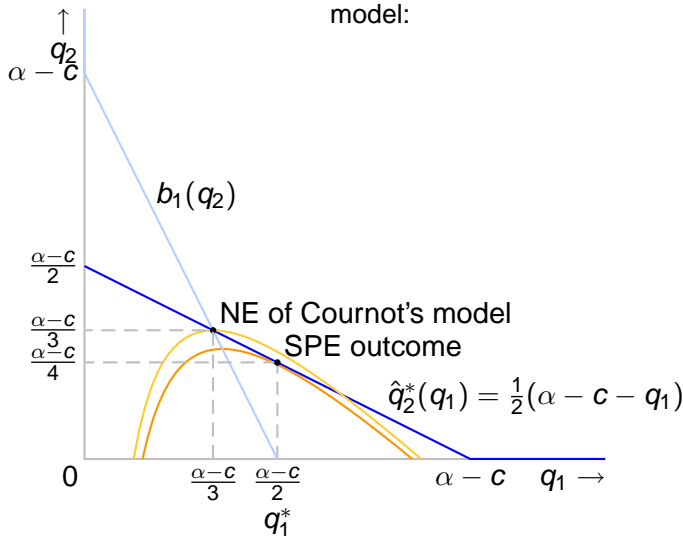
Example



Stackelberg's duopoly model

Example

- Compared with NE of Cournot's model:

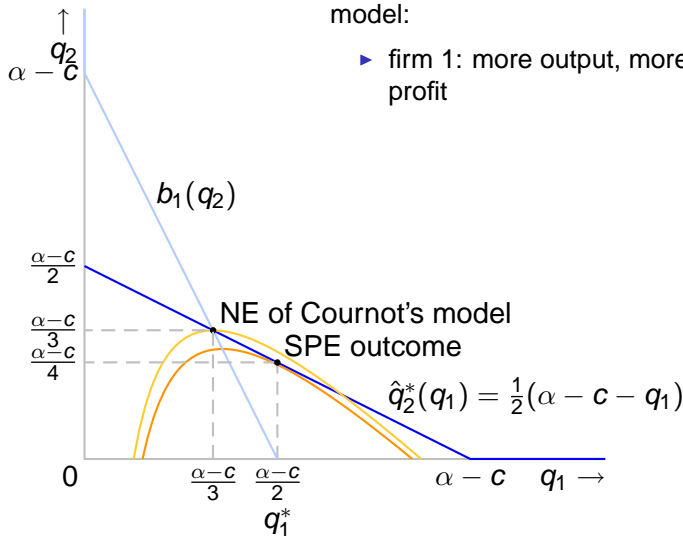


Stackelberg's duopoly model

Example

- ▶ Compared with NE of Cournot's model:

- ▶ firm 1: more output, more profit

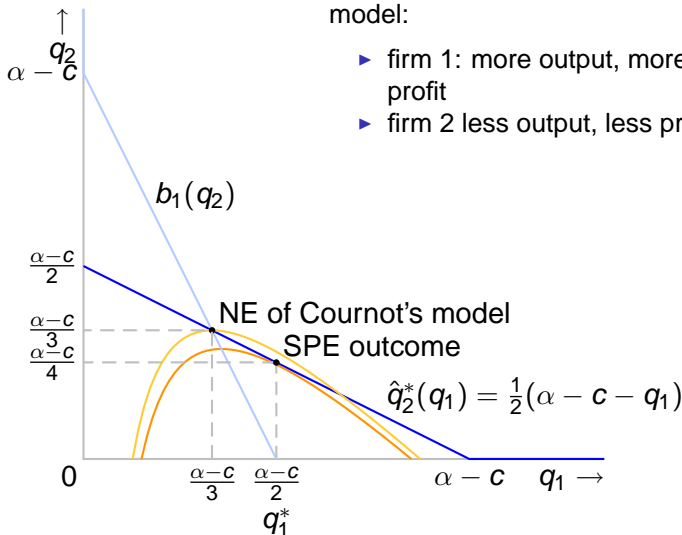


Stackelberg's duopoly model

Example

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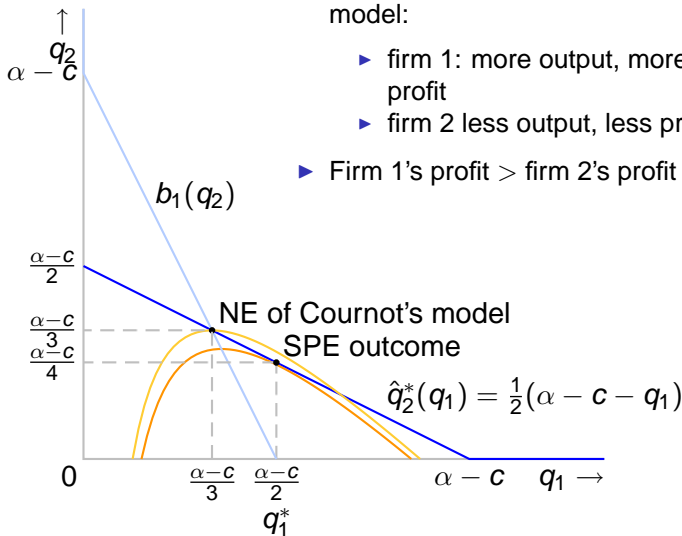
- ▶ firm 1: more output, more profit
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Stackelberg's duopoly model

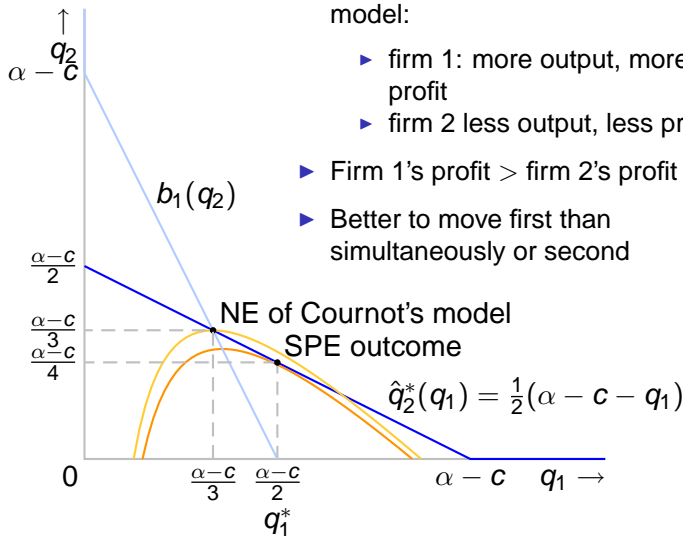
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Stackelberg's duopoly model

Example



- ▶ Compared with NE of Cournot's model:
 - ▶ firm 1: more output, more profit
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- ▶ Firm 1's profit > firm 2's profit
- ▶ Better to move first than simultaneously or second

General result: first-mover advantage

- ▶ Consider two-player strategic game (simultaneous moves)

General result: first-mover advantage

- Consider two-player strategic game (simultaneous moves)

Example

	A	B
C	w_1, w_2	x_1, x_2
D	y_1, y_2	z_1, z_2

General result: first-mover advantage

- ▶ Consider two-player strategic game (simultaneous moves)
 - ▶ Let (a_1^*, a_2^*) be Nash equilibrium

Example

	A	B
C	w_1, w_2	x_1, x_2
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$$w_1 \geq y_1, w_2 \geq x_2$$

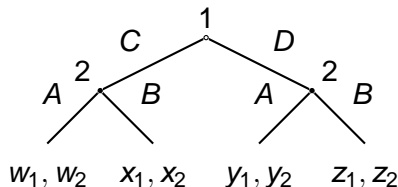
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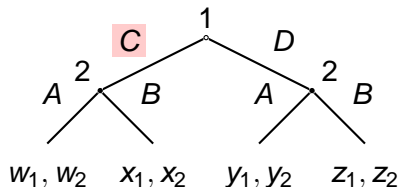
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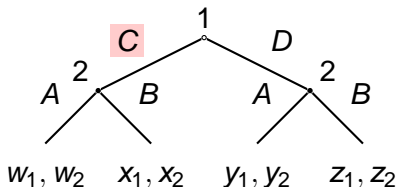
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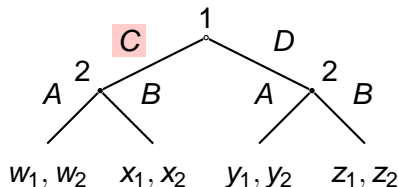
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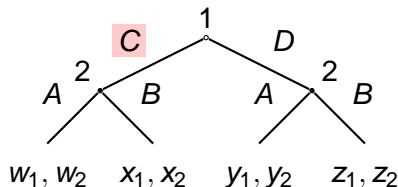
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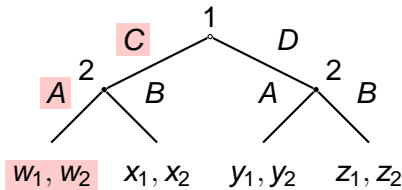
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 - $\Rightarrow a_2^*$ is an optimal choice of player 2 following a_1^* in extensive game

Example

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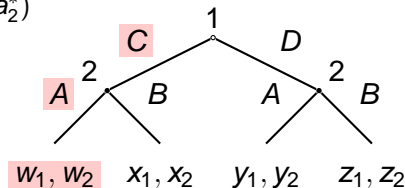
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Example

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$$w_1 \geq y_1, w_2 > x_2$$



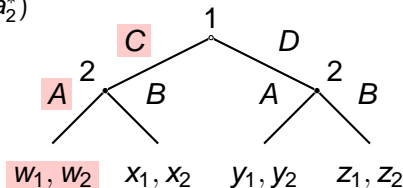
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 \Rightarrow in any subgame perfect equilibrium, player 1's payoff $\geq u_1(a_1^*, a_2^*)$

Example

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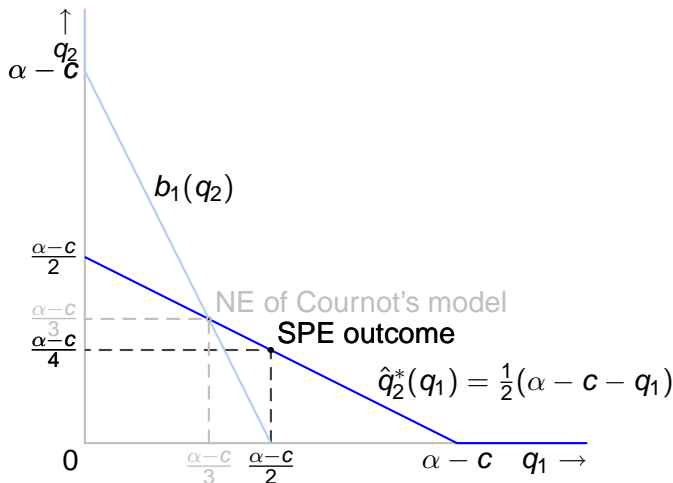
Notes

- ▶ What about *Matching Pennies*?

Stackelberg's duopoly model

Value of commitment

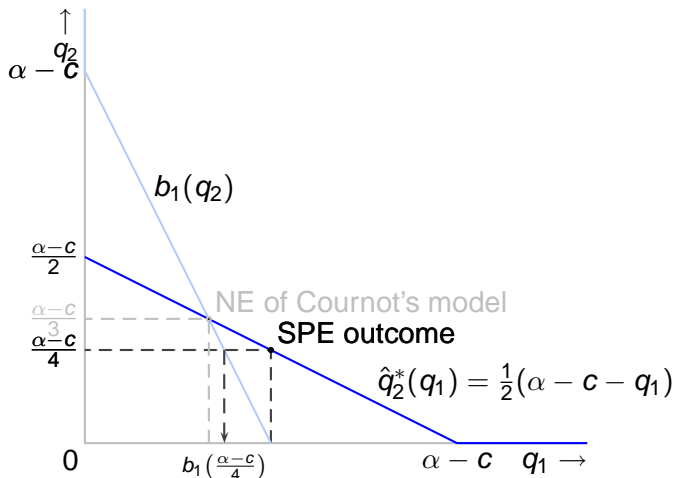
- In subgame perfect equilibrium, is firm 1's output a best response to firm 2's output?



Stackelberg's duopoly model

Value of commitment

- In subgame perfect equilibrium, is firm 1's output a best response to firm 2's output?



Stackelberg's duopoly model

Value of commitment

- In subgame perfect equilibrium, firm 1's output is *not* a best response to firm 2's output

Stackelberg's duopoly model

Value of commitment

- ▶ In subgame perfect equilibrium, firm 1's output is *not* a best response to firm 2's output
- ▶ So after firm 2 moves, firm 1 would like to change its output

Stackelberg's duopoly model

Value of commitment

- ▶ In subgame perfect equilibrium, firm 1's output is *not* a best response to firm 2's output
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Stackelberg's duopoly model

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Stackelberg's duopoly model

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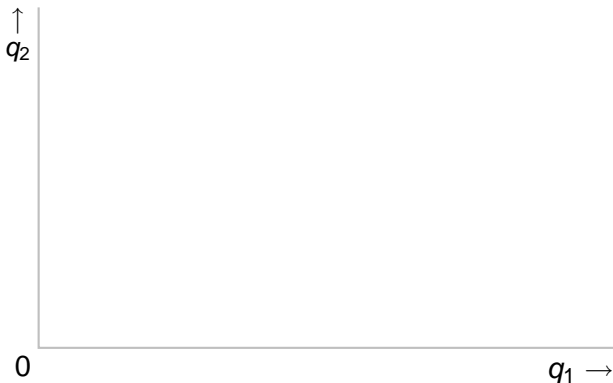
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- ▶ No! If firm 1 has such an opportunity,
 - ▶ first stage of game is irrelevant, and *firm 2* is effectively first-mover
 - ⇒ in subgame perfect equilibrium firm 1 is worse off than it is in Nash equilibrium of simultaneous-move game
- ▶ Firm 1 prefers to be *committed* not to change its mind

Stackelberg's duopoly model

Example with fixed cost

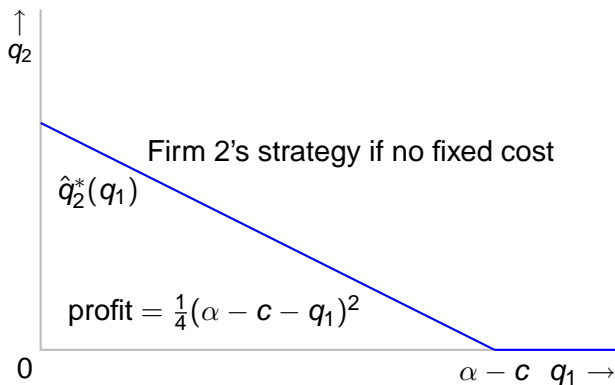
- Suppose firm 2 incurs a fixed cost (independent of its output) if it produces a positive output



Stackelberg's duopoly model

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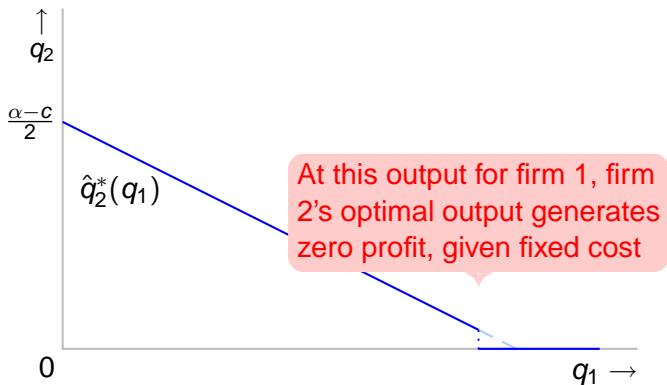
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Stackelberg's duopoly model

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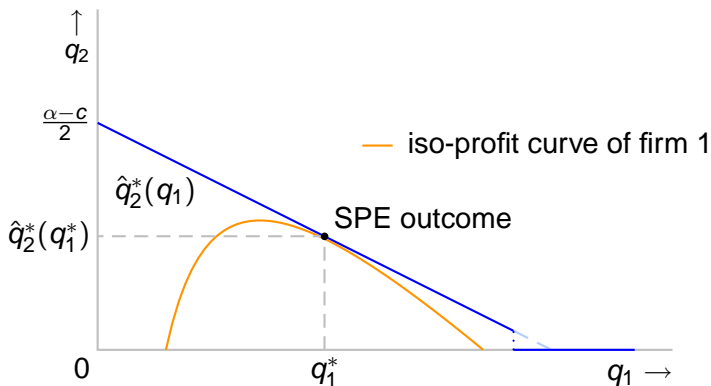
- ▶ Suppose firm 2 incurs a fixed cost (independent of its output) if it produces a positive output
- ▶ When fixed cost is small



Stackelberg's duopoly model

Example with fixed cost

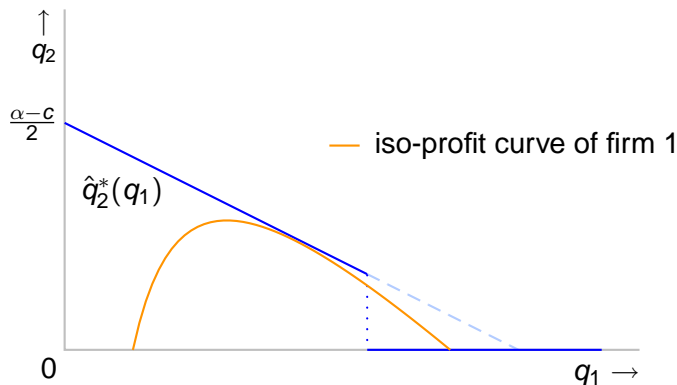
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Stackelberg's duopoly model

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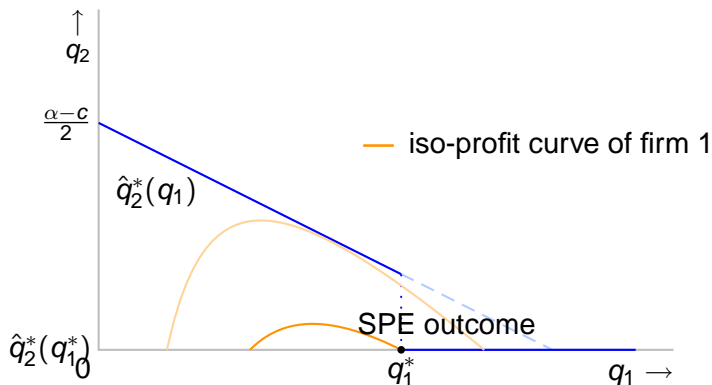
- ▶ Suppose firm 2 incurs a fixed cost (independent of its output) if it produces a positive output
- ▶ When fixed cost is larger



Stackelberg's duopoly model

Example with fixed cost

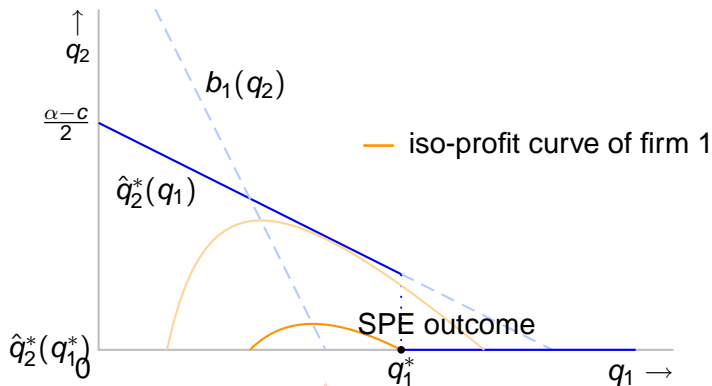
- Suppose firm 2 incurs a fixed cost (independent of its output) if it produces a positive output
- When fixed cost is larger, it *does* affect SPE outcome



Stackelberg's duopoly model

Example with fixed cost

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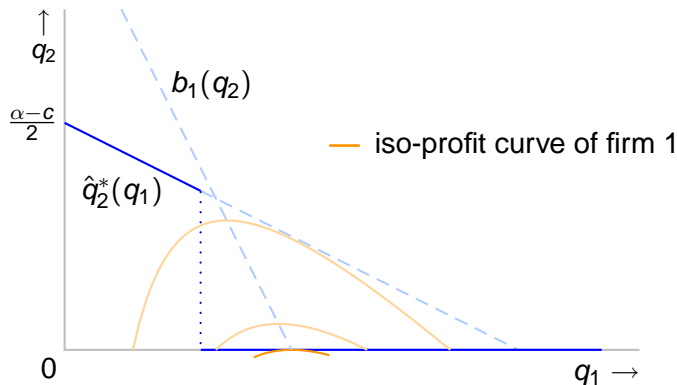


optimal output of firm 1 when firm 2 does not exist

Stackelberg's duopoly model

Example with fixed cost

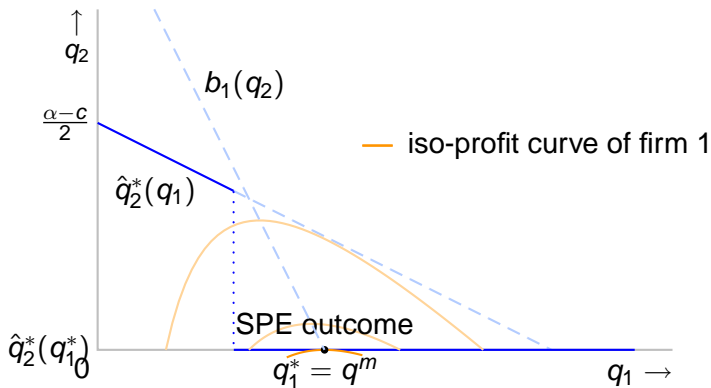
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Stackelberg's duopoly model

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Summary

- ▶ For medium-size fixed cost, subgame perfect equilibrium in which firm 1 produces more than monopoly output and firm 2 produces no output

Stackelberg's duopoly model

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- ▶ If firm 2 were entirely absent from the market, firm 1 would produce q^m , less than q_1^*

Stackelberg's duopoly model

Example with fixed cost

Summary

- ▶ For medium-size fixed cost, subgame perfect equilibrium in which firm 1 produces more than monopoly output and firm 2 produces no output
- ▶ If firm 2 were entirely absent from the market, firm 1 would produce q^m , less than q_1^*
- ▶ Thus firm 2's presence affects the outcome, even though it produces no output

Stackelberg's duopoly model

Lessons

- ▶ First-mover advantage
- ▶ Value of commitment
- ▶ Existence of potential entrant may affect outcome even if in equilibrium firm does not in fact enter

Ultimatum game

- ▶ Two players: proposer and responder

Ultimatum game

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- ▶ Pie of size c

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 - ▶ If responder accepts an offer of x , proposer gets $c - x$ and responder gets x

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- ▶ Pie of size c
- ▶ Proposer offers an amount of pie (from 0 to c) to responder
- ▶ Responder either accepts or rejects offer
 - ▶ If responder accepts an offer of x , proposer gets $c - x$ and responder gets x
 - ▶ If responder rejects an offer, both proposer and responder get 0

Ultimatum game

Experiment

- ▶ Pie of size \$20

Ultimatum game

Experiment

- ▶ Pie of size \$20
- ▶ Every participant will first act as a proposer

Ultimatum game

Experiment

- ▶ Pie of size \$20
- ▶ Every participant will first act as a proposer
- ▶ Every participant will choose an amount from \$0 to \$20 to offer a responder

Ultimatum game

Experiment

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- ▶ Every participant will choose an amount from \$0 to \$20 to offer a responder
- ▶ After all participants have chosen offers, every participant's offer will be presented to another randomly chosen participant—a responder—who will either accept or reject it

Ultimatum game

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 - ▶ Your total payoff will be the sum of the payoffs you get as a proposer and as a responder

Ultimatum game

Experiment

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Ultimatum game

Experiment

- ▶ All interaction will be anonymous
- ▶ No participant will know identity of participant with whom s/he is matched

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Ultimatum game

Experiment

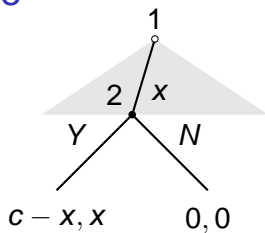
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Ultimatum game

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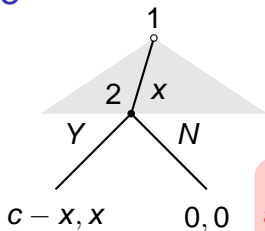
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- ▶ If participant A's offer is presented to participant B for a response, then participant B's offer will *not* (except by chance) be presented to participant A for response
- ▶ Names of participants with top 3 payoffs will be revealed, but *not* their payoffs

Ultimatum game



$$0 \leq x \leq c \\ (c > 0)$$

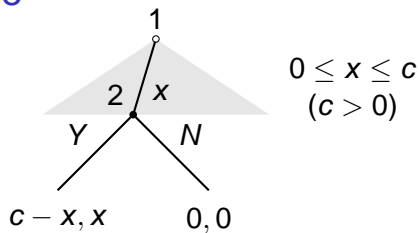
Ultimatum game



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Note: Only one representative action of player 1 and one subgame of length one are indicated

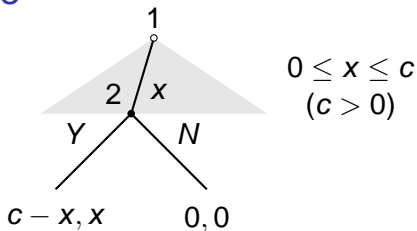
Ultimatum game



Extensive game

Players 1 and 2

Ultimatum game

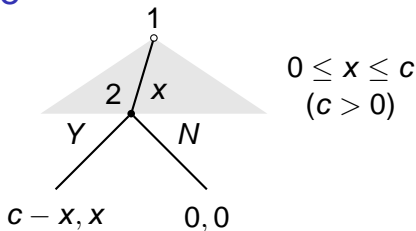


Extensive game

Players 1 and 2

Terminal histories Set of sequences (x, d) where $0 \leq x \leq c$ and d is Y or N

Ultimatum game



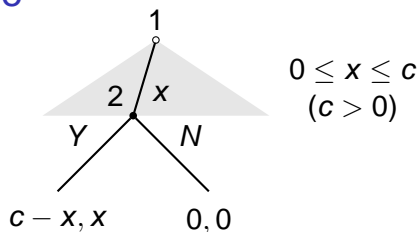
Extensive game

Players 1 and 2

Terminal histories Set of sequences (x, d) where $0 \leq x \leq c$ and d is Y or N

Player function $P(\emptyset) = 1$, $P(x) = 2$ for all x

Ultimatum game



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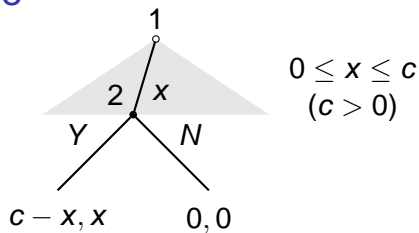
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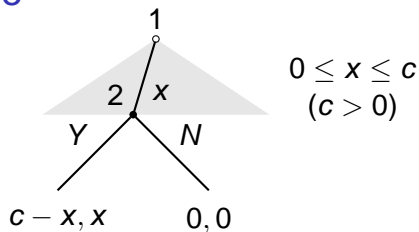
Payoffs $u_1(x, Y) = c - x$, $u_2(x, Y) = x$ for all x , and
 $u_1(x, N) = u_2(x, N) = 0$

Ultimatum game



Strategies

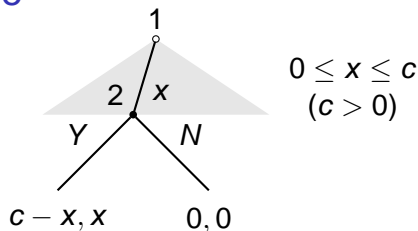
Ultimatum game



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Ultimatum game

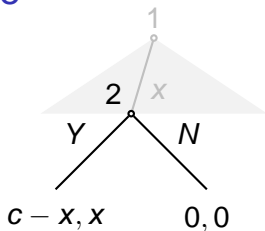


Strategies

Player 1 Set of x with $0 \leq x \leq c$

Player 2 Functions s_2 for which, for each value of x , $s_2(x)$ is either Y or N

Ultimatum game



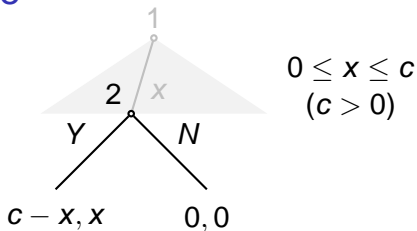
$$0 \leq x \leq c$$

$$(c > 0)$$

Backward induction

In the subgame following x ,

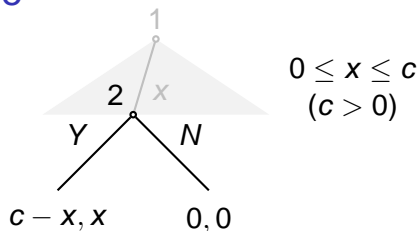
Ultimatum game



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In the subgame following x , Y is optimal if $x > 0$, and *both* Y and N are optimal if $x = 0$

Ultimatum game



Backward induction

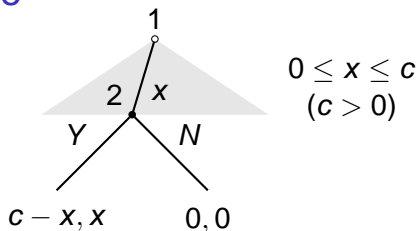
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So *two* optimal strategies in subgame:

$$s_2^1(x) = Y \text{ for all } x$$

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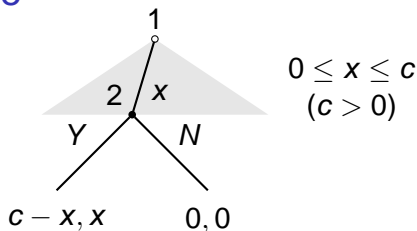
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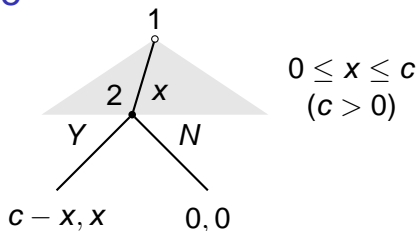
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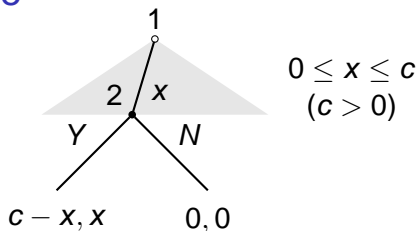


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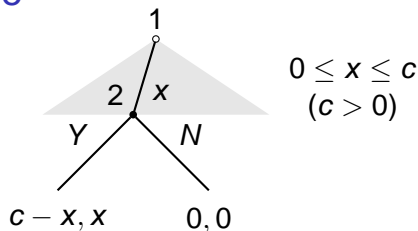
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No optimal action of P1

Ultimatum game



Subgame perfect equilibria

Hence *unique* subgame perfect equilibrium: $s_1 = 0$ and $s_2(x) = Y$ for all x .

Ultimatum game: Experimental evidence

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- ▶ Many other experiments yield similar results
- ▶ If stakes are high, some evidence that proposers offer lower fraction of pie and fewer offers are rejected

Ultimatum game: Experimental evidence

Pie size: one or two days' wages

Group	Country	Avg. offer	Rejection rate
Machiguenga	Perú	26%	5%
Torguud	Mongolia	35%	5%
Tsimané	Bolivia	37%	0%
Sangu	Tanzania	41%	10%
Lamalera	Indonesia	58%	0%

Source: Henrich et al., *American Economic Review, Papers and Proceedings* 91 (2001), 73–78

Ultimatum game: Experimental evidence

Subjects: 6 chimpanzees in Atlanta

Two possible divisions of pie: one equitable, one not equitable

Ultimatum game: Experimental evidence

Subjects: 6 chimpanzees in Atlanta

Two possible divisions of pie: one equitable, one not equitable

	% choosing equitable division
Pair 1	58%
Pair 2	71%
Pair 3	67%
Pair 4	92%

Source: Proctor et al., *Proceedings of the National Academy of Sciences of the United States of America*, January 14, 2013

Ultimatum game: Experimental evidence

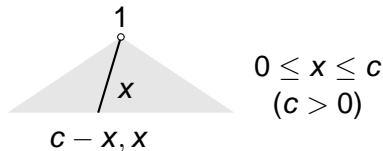
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Ultimatum game: Experimental evidence

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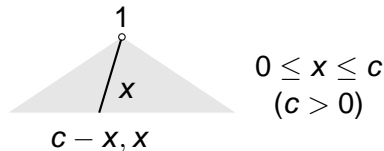
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Ultimatum game: Experimental evidence

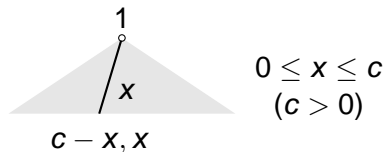
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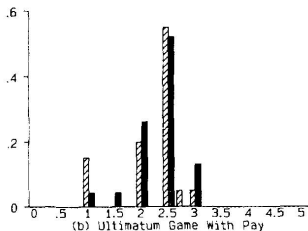
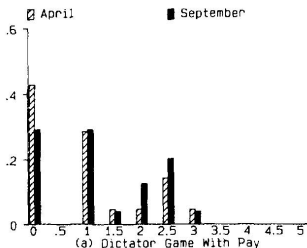
- ▶ Unique subgame perfect equilibrium: player 1 offers 0
- ▶ If non-zero offers in ultimatum game are result of subjects' concern for fairness, should get similar outcomes in dictator game

Ultimatum game: Experimental evidence

Dictator game

Subjects: students at University of Iowa

Pie size: \$5



- ▶ Dictators offer less than proposers in ultimatum game, but still offer significant positive amounts

Source: Forsythe et al., *Games and Economic Behavior* 6 (1994), 347–369. See also Bolton et al., *International Journal of Game Theory* 27 (1998), 269–299 and Eckel et al., *Journal of Economic Behavior and Organization* 80 (2011), 603–612.

Ultimatum game: Experimental evidence

Another hypothesis

- ▶ Significant offers of proposer consistent with proposer's fear that responder will reject offer

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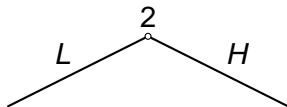
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- ▶ In a long-term relationship, "punishing" a proposer who makes a low offer by rejecting it may have benefit of discouraging low offers in the future

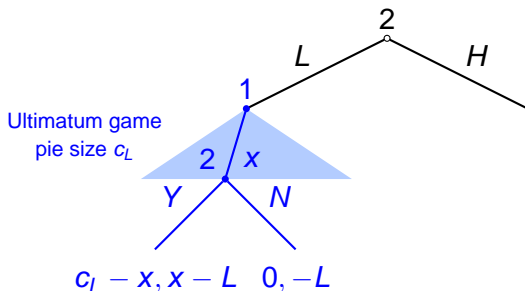
Holdup game

- ▶ Before playing ultimatum game, responder decides whether to expend low effort (L) or high effort (H)



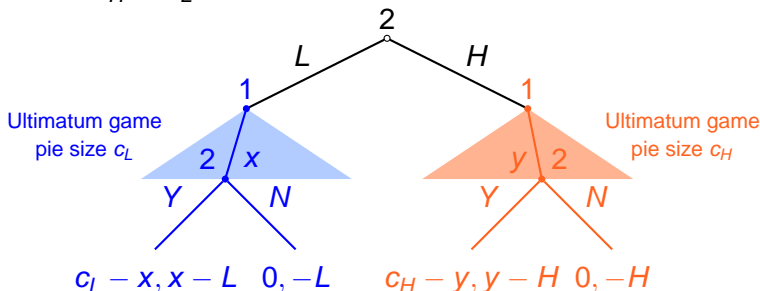
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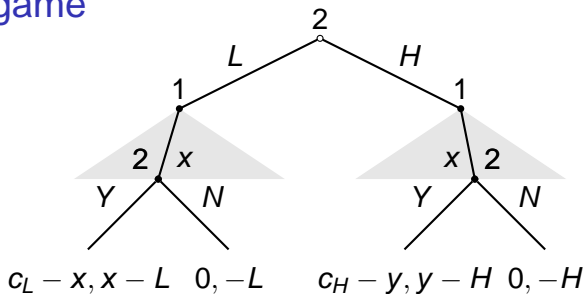


Holdup game

- ▶ Before playing ultimatum game, responder decides whether to expend low effort (L) or high effort (H)
- ▶ More effort is more costly, but produces bigger pie: $H > L$ and $c_H > c_L$



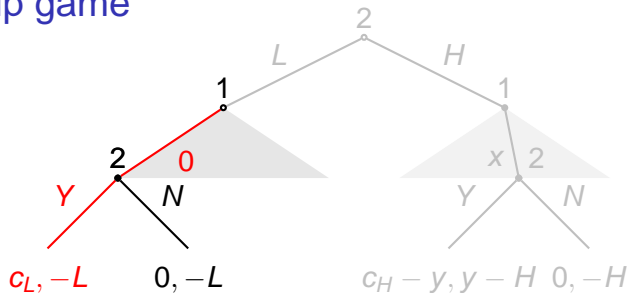
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Analysis

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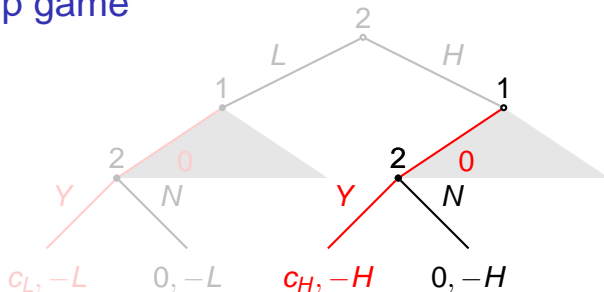
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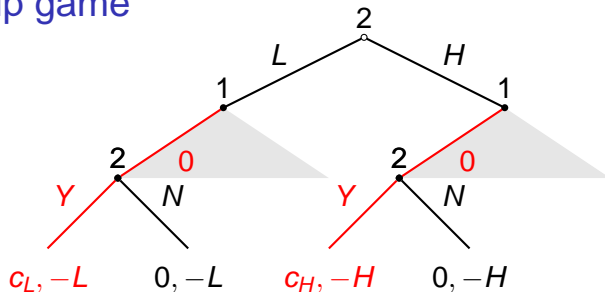
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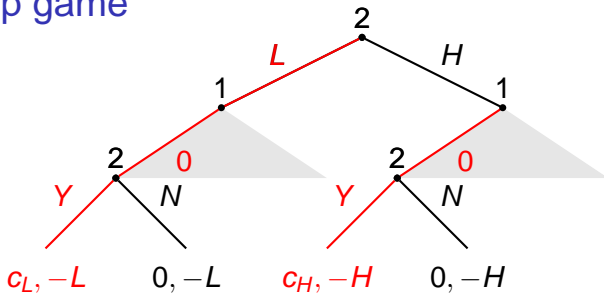
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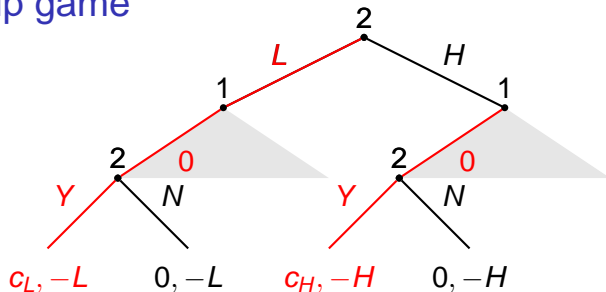
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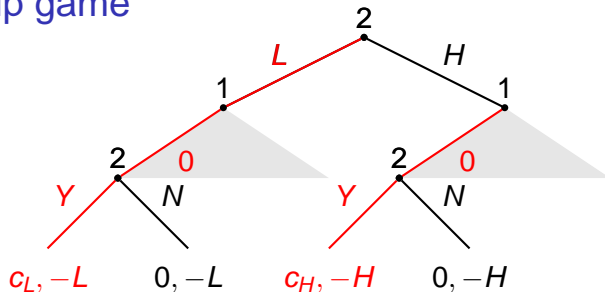
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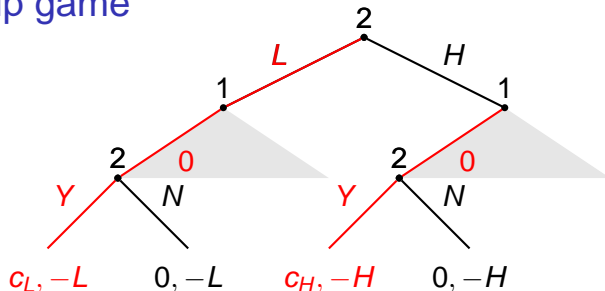
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- ▶ SPE of whole game: P2 chooses L
- ⇒ inefficient outcome if $c_H - H > c_L - L$
- ▶ P2 is “held up” for all the surplus her extra effort produces
- ▶ Even with less extreme outcome of bargaining, SPE outcome may still be inefficient

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- ▶ After 10 rounds of play, with probability 0.07 play stops after each round \Rightarrow random number of rounds
- ▶ At all points in the game, each player observes the history of play (his/her actions and actions of opponent)