UNIVERSITY OF TORONTO
Faculty of Arts and Science

ECO 316 (Applied Game Theory)
Instructor: Martin J. Osborne

Midterm Exam
20 October 2017

Duration: 1 hour 50 minutes

No aids allowed

This examination paper consists of **12** pages and **5** questions. Please alert an invigilator to any discrepancy. The number in brackets at the start of each question is the number of points the question is worth.

Answer all questions.

TO OBTAIN CREDIT, YOU MUST GIVE ARGUMENTS TO SUPPORT YOUR ANSWERS.

Question	Points
1	30
2	15

Question	Points
3	20
4	15
5	20

Total	100

1. (a) [8] Find all mixed strategy Nash equilibria of the following strategic game. (Remember that in a mixed strategy Nash equilibrium a player may assign probability 1 to a single action.)

	L	R
T	$2, 0$	$2, 4$
B	$3, 1$	$0, 0$

Question continues on next page

(b) [12] Does the following strategic game have a mixed strategy Nash equilibrium in which each player assigns positive probability *only* to her actions A and C?

	A	B	C
A	$2, 2$	$2, 4$	$6, 3$
B	$4, 2$	$6, 6$	$0, 3$
C	$3, 6$	$3, 1$	$3, 3$

Question continues on next page

(c) [5] In the following strategic game, is any action of either player strictly dominated by a mixed strategy?

	A	B	C
T	$1, 5$	$3, 6$	$2, 5$
B	$5, 3$	$2, 1$	$0, 2$

Question continues on next page

(d) [5] Consider the following strategic game. Which actions for each player are consistent with her being rational (in the sense defined in class), with her knowing that the other player is rational, with her knowing that the other player knows she is rational, and so on?

	A	B	C
A	$1, 5$	$3, 6$	$2, 4$
B	$5, 3$	$2, 1$	$0, 4$
C	$2, 3$	$4, 2$	$3, 0$

2. Each of two players chooses a number from the set $\{0, 1, 2, \ldots, M\}$, where $M \geq 3$. If the sum of the numbers chosen by the players is less than or equal to M, each player's payoff is the number she chose. Otherwise, each player's payoff is zero.

   (a) [5] Is the action of choosing 0 strictly dominated for player 1?

Question continues on next page

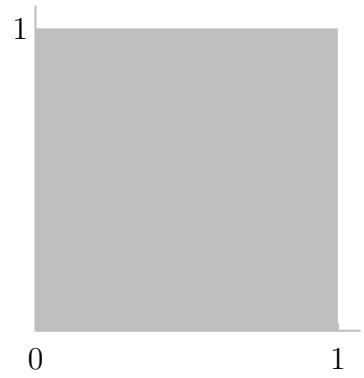
(b) [10] Find all the pure strategy Nash equilibria of this game.

3. [20] Consider a variant of the example of Cournot's duopoly game considered in class in which the payoff of firm 1 is its profit if this profit is negative, but otherwise (that is, if firm 1's profit is nonnegative) is firm 1's *output* (that is, sales), and the payoff of firm 2 is always its profit (as assumed in class).

   (Each firm's cost of producing q units of output is cq and the price when the total output of the firms is Q is $\alpha - Q$, with $\alpha > c$.)

   Find the set of Nash equilibria of the game.

4. Consider Hotelling's model of electoral competition with *four* candidates. Assume that each candidate wants to maximize her probability of winning and that no candidate has the option of exiting the race. (If k candidates tie for first place, each of them wins the probability $1/k$.) Suppose that the distribution of the voters' favorite positions is uniform from 0 to 1. That is, the distribution looks like this:



(a) [7] Does the game have a Nash equilibrium in which the positions of all four candidates are equal to $\frac{1}{2}$?

Question continues on next page

(b) [8] Does the game have a Nash equilibrium in which the positions of two candidates are equal to $\frac{1}{4}$ and the positions of the other two candidates are equal to $\frac{3}{4}$?

5. [20] Consider a variant of the crime-reporting model in which each individual independently observes the crime with probability q, which may be less than 1 (instead of observing the crime for sure). Only individuals who observe the crime have the option to report it. Suppose that every individual reports the crime with probability p in the event she observes it. Then the probability that an individual reports the crime is qp (the probability that she observes it times the probability she reports it in the event she observes it).

   As in the model discussed in class, an individual who reports the crime incurs the cost $c > 0$, and if the crime is reported then every individual obtains the benefit $v > c$. Denote the number of individuals by n.

   Find a ("symmetric") mixed strategy equilibrium of the strategic game that models this situation in which the probability that each individual reports the crime, conditional on observing it, is the same.

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End of examination
Total pages: 12
Total marks: 100