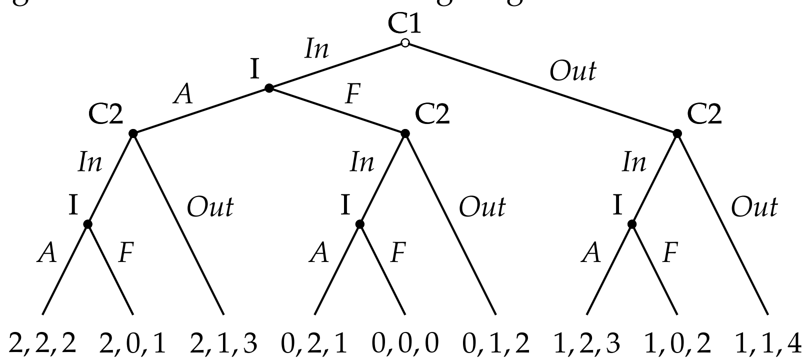
Economics 316

Fall 2017 — Martin J. Osborne

Solutions to problems for Tutorial 9

1. (a) The game is shown in the following diagram.



(b) The incumbent moves after the history *In* and after any history that starts with one of the three outcomes in the first period and is followed by *In*. In each case it has two actions. There are four such histories, so it has $2^4 = 16$ strategies.

(c) In every subgame perfect equilibrium, challenger 2 chooses *In* whenever it moves (after the histories $(In, Acquiesce)$, $(In, Fight)$, and *Out* and the incumbent chooses *Acquiesce* after any history in which challenger 2 chooses *In*. Thus the outcome in the second period is independent of the outcome in the first period, so that in any subgame perfect equilibrium the incumbent chooses *Acquiesce* in the first period and challenger 1 chooses *In*.

Thus the game has a unique subgame perfect equilibrium, in which challenger 1's strategy is *In*, challenger 2's strategy specifies *In* after the histories $(In, Acquiesce)$, $(In, Fight)$, and *Out*, and the incumbent's strategy specifies *Acquiesce* after the histories *In*, $(In, Acquiesce, In)$, $(In, Fight, In)$, and (Out, In).

(d) The game in which there are n challengers also has a unique subgame perfect equilibrium, in which every challenger enters after every history and the incumbent chooses *Acquiesce* whenever it is its turn to move.

2. (a) A strategy for player 1 is a division of the cake into two parts. A strategy for player 2 is a rule saying which part she chooses for *every* possible division.

   For any division in which the sizes of the parts are not the same, player 2 chooses the larger piece. For any division in which the sizes are the same, she chooses either piece. So in every subgame perfect equilibrium, player 1 divides the cake into two equal parts and player 2 chooses one of the parts.

   (b) The game has a subgame perfect equilibrium in which player 1 receives the chocolate half of the cake. Here is one such equilibrium.

   - player 1 divides the cake into the chocolate part and the coffee part
   - player 2 chooses the larger part if the parts are not the same size, the part with more coffee if the parts have the same size but one has more coffee, and either part if the parts have the same size and the same amount of coffee.

   The game does not have a subgame perfect equilibrium in which player 1 receives a different piece.

   First suppose that player 1 divides the cake into pieces of different sizes. Then player 2 chooses the larger piece, and player 1 can increase her payoff by increasing the size of the smaller piece a little by adding to it a small piece of the chocolate part in such a way that the piece that was originally smaller remains smaller.

   So in any subgame perfect equilibrium, player 1 divides the cake into two pieces of the same size.

   Now suppose that each of the two pieces into which player 1 divides the cake has some chocolate and some coffee. Player 2 is then indifferent between the two pieces—choosing either of them is a best response. Whichever piece she chooses, I claim that player 1 can choose a different division and do better. Here's one possible different division: one piece consists of the chocolate part minus tiny bit and the other piece consists of the coffee part plus the tiny bit of chocolate. Player 2 chooses the part that is mainly coffee (because it is bigger) and player 1 gets the other part, which is almost all the chocolate part.

   The conclusion is that in any subgame perfect equilibrium, player 1 divides the cake into the chocolate part and the coffee part, and player 2 chooses the coffee part.