

Problems for Tutorial 9

1. An incumbent plays an entry game with two challengers sequentially. In the first period, challenger 1 chooses between *In* and *Out*; if it chooses *In*, then the incumbent chooses between *Acquiesce* and *Fight*; if it chooses *Out*, then the incumbent has no action in the first period. In the second period, challenger 2 chooses between *In* and *Out*; if it chooses *In*, then the incumbent chooses between *Acquiesce* and *Fight*; if it chooses *Out*, then the incumbent has no action in the second period. Thus two examples of terminal histories are $(In, Acquiesce, In, Fight)$ and $(In, Acquiesce, Out)$. Each challenger's payoff is its standard entry game payoff in the period in which it interacts with the incumbent. For example, the payoff of challenger 1 is 2 for any terminal history starting $(In, Acquiesce)$, 0 for any terminal history starting $(In, Fight)$, and 1 for any terminal history starting *Out*. The payoff of the incumbent is the sum of its payoffs in the two periods. For example, its payoff for the terminal history $(In, Acquiesce, Out)$ is $1 + 2 = 3$.
 - (a) Draw a diagram of the game.
 - (b) How many strategies does the incumbent have?
 - (c) Find the subgame perfect equilibrium (equilibria?) of the game.
 - (d) Can you extend your argument to the case of a game with one incumbent and n challengers (who act sequentially)?
2. Two players use the following procedure to divide a cake. Player 1 divides the cake into two pieces and then player 2 chooses one of the pieces, leaving the other piece for player 1. Both players like cake.
 - (a) Suppose that the cake is perfectly homogeneous. Describe a strategy for player 1 and a strategy for player 2 in the extensive games that models the procedure. Find the subgame perfect equilibria of the game. (Be sure to give a pair of *strategies*!)

- (b) Suppose that half of the cake is chocolate flavored and the other half is coffee flavored. Player 2 is indifferent between the two flavors—she cares only about the size of the piece she receives. Player 1 cares only about the size of the chocolate part of the piece she receives. Does the game have a subgame perfect equilibrium in which player 1 receives the chocolate half of the cake? Does it have a subgame perfect equilibrium in which the piece player 1 receives differs from the chocolate half?