Economics 316

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Problem Set 9

1. (a) Represent the following extensive game in a diagram like the one I used in class for the entry game.

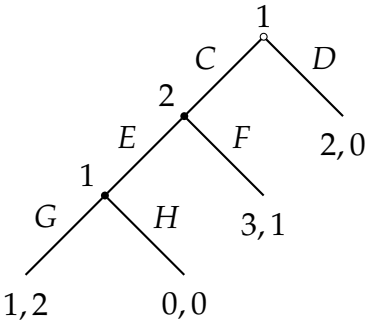
   Players $\{1, 2, 3\}$

   Terminal histories $\{(C, F), (C, G), (D, H), (D, I), E\}$

   Player function $P(\varnothing) = 1$, $P(C) = 2$, $P(D) = 3$

   Payoffs $u_1(C, F) = 0$, $u_2(C, F) = 2$, $u_3(C, F) = 3$; $u_1(C, G) = 3$, $u_2(C, G) = 1$, $u_3(C, G) = 0$; $u_1(D, H) = 2$, $u_2(D, H) = 3$, $u_3(D, H) = 1$; $u_1(D, I) = 1$, $u_2(D, I) = 2$, $u_3(D, I) = 2$; $u_1(E) = 2$, $u_2(E) = 0$, $u_3(E) = 0$.

   (b) Write down the set of players, set of terminal histories, and player function for the extensive game in the following diagram.



2. An extensive game has two players, 1 and 2. At the start of the game, player 1 moves and has three possible actions. After each of these actions, player 2 moves and has two actions. After each of player 2's actions, player 1 moves again and has two actions.

   (a) How many strategies does player 1 have?

   (b) How many strategies does player 2 have?

3. Calculate the number of strategies of each player in ticktacktoe assuming (to make the calculation a little easier) that the players continue putting x's and o's until all boxes are filled (rather than stopping as soon as one player wins).

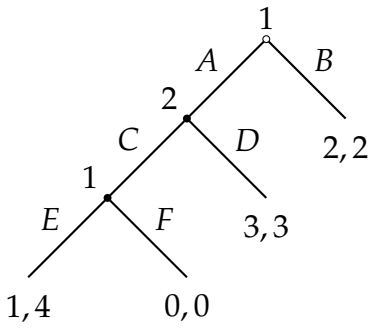
4. For the following extensive game,

   (a) Construct the strategic form of the game.

   (b) Find the set of (pure strategy) Nash equilibria of the game.

   (c) Find the set of subgame perfect equilibria of the game.

   (d) Compare the players payoffs in the Nash equilibria and the subgame perfect equilibria.



5. A firm's output is $L(100 - L)$ when it uses $L \leq 50$ units of labor, and 2500 when it uses $L > 50$ units of labor. The price of output is 1. A union that represents workers presents a wage demand (a nonnegative number w), which the firm either accepts or rejects. If the firm accepts the demand, it chooses the number L of workers to employ (which you should take to be a continuous variable, not an integer); if it rejects the demand, no production takes place ($L = 0$). The firm's preferences are represented by its profit; the union's preferences are represented by the value of wL.

   (a) Formulate this situation as an extensive game with perfect information.

   (b) Find the subgame perfect equilibrium (equilibria?) of the game.

   (c) Is there a feasible outcome of the game that both parties prefer to all subgame perfect equilibrium outcomes?

   (d) Find a Nash equilibrium for which the outcome differs from any subgame perfect equilibrium outcome.

6. This problem is optional. It is interesting, but also fairly difficult; you should be able to understand the main points, but the details are hard to pin down.

Consider a variant of the model of the sequential choice of positions by political parties considered in class to the case of three parties and each party has the option of staying out of the race. That is, first party 1 chooses either to stay out of the race or chooses a position (a number). Party 2 observes party 1's choice, and then chooses either to stay out of the race or chooses a position. Finally, party 3 observes the positions of parties 1 and 2 and either chooses to stay out of the race or chooses a position. Assume that the voters' favorite positions are distributed uniformly between 0 and 1.

Assume that each party prefers to stay out than to enter and lose, prefers to enter and tie with any number of parties than to stay out, and prefers to tie with as few other parties as possible.

Find the subgame perfect equilibrium outcomes of the extensive game that models this situation.

You need to first consider the best action for the third party given any pair of actions for the first two parties. The precise analysis of this case is fairly complicated, but you should be able to determine the basic structure of the third party's optimal strategy. Then you need to consider the best action of the third party, given the optimal reaction of the third party. Finally, you need to consider the optimal action of the first party, given the optimal reactions of the second and third parties.