Economics 316

Fall 2017 Martin J. Osborne

Solutions to problems for Tutorial 8

1. Yes, the strategy profile in which player i bids $\overline{v}$ regardless of her valuation and all the other players bid $\underline{v}$ regardless of her valuation is a Nash equilibrium. Player i wins and pays the price $\underline{v}$. Thus she obtains a nonnegative payoff regardless of her valuation, and given the other players' strategies cannot do better. The other players lose; if any of them changes her bid to one that wins, then her payoff is nonpositive, because her bid must be at least $\overline{v}$ and her valuation is at most $\overline{v}$.

   The outcome is efficient only if player i's valuation is the highest valuation; the probability of this event is $1/n$, where n is the number of bidders.

2. (a) A player's bidding her valuation weakly dominates all other bids, by the same argument as for the case of risk-neutrality.

   (b) The analysis follows the lines of the analysis for the risk-neutral case in class. If player 2's bid is βv_2, then her bids are uniformly distributed on $[0, \beta]$. Suppose that player 1 bids b_1. Then her payoff is

   $$\begin{cases} (b_1/\beta)(v_1 - b_1)^{1/m} & \text{if } b_1 < \beta \\ (v_1 - b_1)^{1/m} & \text{if } b_1 > \beta. \end{cases}$$

   The maximizer of this function is

   $$\tfrac{m}{m+1} v_1$$

   if this amount is at most β. The argument for player 2 is symmetric. Thus the auction has an equilibrium in which each player i bids

   $$\tfrac{m}{m+1} v_i$$

   when her valuation is v_i.

(c) If the winner of the second-price auction has valuation v, then she pays the expected value of the other player's valuation conditional on that valuation being less than v, which is $\frac{1}{2}v$.

If the winner of the first-price auction has valuation v, then she pays her bid, which is $(m/(m+1))v$.

If $m = 1$ (risk-neutral bidders), then the two prices are the same. But if $m > 1$ (risk-averse bidders), then the price paid in the first-price auction is higher.

The distribution of valuations is the same in the two auctions, so the probability that a bidder with any given valuation wins is the same in both auctions. Hence the auctioneer's expected revenue is greater in a first-price auction than it is in a second-price auction.