Economics 316

Fall 2017 Martin J. Osborne

Solutions for Problem Set 7

1. Because the number of citizens is odd, the winning margin (the difference between the number of votes for the winner and the number of votes for the loser) is odd.

   First consider an action profile in which the winning margin is one and at least one citizen who votes for the winner prefers the loser to the winner. Any citizen who votes for the winner and prefers the loser to the winner can, by switching her vote, cause her favorite candidate to win rather than lose. Thus no such action profile is a Nash equilibrium.

   Next consider an action profile in which the winning margin is one and all citizens who vote for the winner prefer the winner to the loser. Because a majority of citizens prefer A to B, the winner in any such case must be A. No citizen who prefers A to B can induce a better outcome by changing her vote, since her favorite candidate wins. Now consider a citizen who prefers B to A. By assumption, every such citizen votes for B; a change in her vote has no effect on the outcome (A still wins). Thus every such action profile is a Nash equilibrium.

   Finally consider an action profile in which the winning margin is at least three. In this case no change in any citizen's vote has any effect on the outcome. Thus every such profile is a Nash equilibrium.

   In summary, the Nash equilibria are: any action profile in which A receives one more vote than B and all the citizens who vote for A prefer A to B, and any action profile in which the winner receives at least three more votes than the loser.

   The only equilibrium in which no citizen uses a weakly dominated action is that in which every citizen votes for her favorite candidate.

2. Fix some players, say i, and consider the candidate she ranks in the middle, say candidate B. The action profile in which all citizens vote for B is a Nash equilibrium. (No citizen's changing her vote affects the

outcome.) In this equilibrium, player i does not vote for her favorite candidate, but the action she takes is not weakly dominated. (Other Nash equilibria also satisfy the conditions in the problem.)

3. (a) I argue that any action a_i of player i that includes a vote for i's least preferred candidate, say candidate k, is weakly dominated by the action a_i' that differs from a_i only in that candidate k does not receive a vote in a_i'. For any list a_{-i} of the other players' actions, the outcome of (a_i', a_{-i}) differs from that of (a_i, a_{-i}) only in that the total number of votes received by candidate k is one less in (a_i', a_{-i}) than it is in (a_i, a_{-i}). There are three possible implications for the winners of the election, depending on a_{-i}: (i) the set of winners is the same in (a_i, a_{-i}) as it is in (a_i', a_{-i}); (ii) candidate k wins outright in (a_i, a_{-i}) but ties for first place in (a_i', a_{-i}); (iii) candidate k ties for first place in (a_i, a_{-i}) but loses in (a_i', a_{-i}). Because candidate k is player i's least preferred candidate, a_i' thus weakly dominates a_i.

   (b) I argue that any action a_i of player i that excludes a vote for i's most preferred candidate, say candidate 1, is weakly dominated by the action a_i' that differs from a_i only in that candidate 1 receives a vote in a_i'. The argument is symmetric with the one for (a). For any list a_{-i} of the other players' actions, the outcome of (a_i', a_{-i}) differs from that of (a_i, a_{-i}) only in that the total number of votes received by candidate 1 is one more in (a_i', a_{-i}) than it is in (a_i, a_{-i}). There are three possible implications for the winners of the election, depending on a_{-i}: (i) the set of winners is the same in (a_i, a_{-i}) as it is in (a_i', a_{-i}); (ii) candidate 1 is the outright winner in (a_i', a_{-i}) but ties for first place in (a_i, a_{-i}); (iii) candidate 1 ties for first place in (a_i', a_{-i}) but loses in (a_i, a_{-i}). Because candidate 1 is player i's most preferred candidate, a_i' thus weakly dominates a_i.

4. (a) The only action for a player that is weakly dominated is a vote for the player's least-preferred candidate. Thus in an action profile in which no player uses a weakly dominated action, persons 1, 2, and 3 can vote for either A or B and person 4 can vote for either A or C. For all of these possible action profiles, A wins, A and B tie, or B wins. The action profile in which all players vote for A is a Nash equilibrium and the action profile in which players 1, 2, and 3 vote for B and player 4 votes for C is also a

Nash equilibrium. (In neither case does a change in any player's vote affect the outcome.) Thus there is a Nash equilibrium in which A is the winner and also a Nash equilibrium in which B is the winner, but no Nash equilibrium in which C is the winner.

(b) An action for a player that is not weakly dominated includes the player's favorite candidate and does not include the candidate's least-preferred candidate. Thus in any action profile in which no player's action is weakly dominated, A gets four votes and B gets at most three votes. Thus in every Nash equilibrium, A wins.

5. (a) On the first round, A and C get 3 votes, and B gets 2 votes. Thus B is eliminated. On the second round, A gets 3 votes and C gets 5 votes, so C wins.

(b) Suppose that person 1 submits the ranking B, A, C rather than her preferences. Then on the first round B and C get 3 votes and A gets 2 votes. So A is eliminated and B then gets 5 votes on the second round, and wins. Person 1 prefers B to C, so the action profile in which everyone submits their preferences is not a Nash equilibrium.

6. When the policy chosen is the mean of the announced policies, player i's announcing her favorite policy does not weakly dominate all her other actions. For example, if there are three players, the favorite policy of player 1 is 0.3, and the other players both announce the policy 0, then the best policy for player 1 to announce is 0.9, which leads to the policy 0.3 ($= (0 + 0 + 0.9)/3$) being chosen, rather than 0.3, which leads to the policy 0.1.