Economics 316

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Problems for Tutorial 8

1. Consider an independent private values sealed-bid second-price auction. We saw in class that such an auction has a Nash equilibrium in which every player's bid is equal to her valuation. Does it also have a Nash equilibrium in which one of the players, say player i, bids $\overline{v}$ (the highest possible valuation) regardless of her valuation and all other players bid $\underline{v}$ (the lowest possible valuation) regardless of their valuations? Is the outcome of this strategy profile efficient? (I.e. is the object always allocated to the player with the highest valuation for it?)

2. Consider a variant of the independent private values sealed-bid auctions considered in class in which the players are risk-averse. Specifically, assume that the payoff of a player with valuation v who wins the object and pays the price p is $(v-p)^{1/m}$, where $m > 1$. (In class we considered the case $m = 1$, in which the bidders are "risk neutral". For $m > 1$, the bidders are "risk averse".)

   (a) Find an equilibrium of the second-price auction.

   (b) Suppose that there are two players and each player's valuation is drawn independently from a uniform distribution on $[0, 1]$ (as we assumed in class). Find an equilibrium of the first-price auction. (*Hint*: Assume that when player 2's valuation is v_2 she bids βv_2, where β is a constant. Find the best response of player 1 to this strategy of player 2 when player 1's valuation is v_1.)

   (c) For the environment of part (b) (two players, uniform distribution of valuations), compare the expected value of the price paid by a bidder with valuation v in the equilibrium of a second-price and a first-price auction. How does the auctioneer's revenue differ between the two auctions?