ECO316: Applied game theory

Lecture 7

Martin J. Osborne

Department of Economics
University of Toronto

2017.10.19

Midterm

Date	Friday, October 20, 1:10pm to 3pm (tomorrow)
Location	EX 100
Coverage	All the material in weeks 1–6
Weight	40%
Sample	Previous exams (with solutions) are available on the course website
ID	You must bring your student ID to the exam
Writing	Please write using a PENCIL and use an ERASER as necessary. You will not be able to write any part of your answer outside the area designated for an answer on the test.

Table of contents

Collective choice

- Group of people has to choose one of several actions

Collective choice

- Group of people has to choose one of several actions
- Group members' preferences differ

Collective choice

- Group of people has to choose one of several actions
- Group members' preferences differ
- How should action of group be selected?

Collective choice

Two actions

$A \succ B$

$B \succ A$

$B \succ A$

$A \succ B$

$B \succ A$

$A \succ B$

$B \succ A$

$A \succ B$

$A \succ B$

$A \succ B$

$A \succ B$

Collective choice

Two actions

$A \succ B$

$B \succ A$

$B \succ A$

$A \succ B$

$B \succ A$

$A \succ B$

$B \succ A$

$A \succ B$

$A \succ B$

$A \succ B$

$A \succ B$

- Only information we have is whether each individual prefers A or B, not *intensity* of their preference

Collective choice

Two actions

$A \succ B$

$B \succ A$

$B \succ A$

$A \succ B$

$B \succ A$

$A \succ B$

$B \succ A$

$A \succ B$

$A \succ B$

$A \succ B$

$A \succ B$

- Only information we have is whether each individual prefers A or B, not *intensity* of their preference
- Knowing option an individual likes best tells us their entire preference relation (because only two alternatives)

Collective choice

Two actions

$A \succ B$

$B \succ A$

$B \succ A$

$A \succ B$

$B \succ A$

$A \succ B$

$B \succ A$

$A \succ B$

$A \succ B$

$A \succ B$

$A \succ B$

- If we know individuals' preferences and want to treat A and B symmetrically and individuals equally, natural to select action favored by majority

Collective choice

Two actions

$A \succ B$

$B \succ A$

$B \succ A$

$A \succ B$

$B \succ A$

$A \succ B$

$B \succ A$

$A \succ B$

$A \succ B$

$A \succ B$

$A \succ B$

- But typically we don't know individuals' preferences

Collective choice

Two actions

$A \succ B$

$B \succ A$

$B \succ A$

$A \succ B$

$B \succ A$

$A \succ B$

$B \succ A$

$A \succ B$

$A \succ B$

$A \succ B$

$A \succ B$

- But typically we don't know individuals' preferences
- Could ask each individual to name an option (vote for an option), and then select action with most votes

Collective choice

Two actions

$A \succ B$

$B \succ A$

$B \succ A$

$A \succ B$

$B \succ A$

$A \succ B$

$B \succ A$

$A \succ B$

$A \succ B$

$A \succ B$

$A \succ B$

- But typically we don't know individuals' preferences
- Could ask each individual to name an option (vote for an option), and then select action with most votes
- What are the *strategic* properties of this mechanism?

Majority rule: two candidates

- Two candidates, A and B

Majority rule: two candidates

- Two candidates, A and B
- k citizens, each of whom votes for A or votes for B

Majority rule: two candidates

- Two candidates, A and B
- k citizens, each of whom votes for A or votes for B
- Winner is candidate who obtains the most votes

Majority rule: two candidates

- Two candidates, A and B
- k citizens, each of whom votes for A or votes for B
- Winner is candidate who obtains the most votes

Strategic game

- Players: The k citizens

Majority rule: two candidates

- Two candidates, A and B
- k citizens, each of whom votes for A or votes for B
- Winner is candidate who obtains the most votes

Strategic game

- Players: The k citizens
- For each player,

Majority rule: two candidates

- Two candidates, A and B
- k citizens, each of whom votes for A or votes for B
- Winner is candidate who obtains the most votes

Strategic game

- Players: The k citizens
- For each player,
  - Actions: Vote for A, vote for B

Majority rule: two candidates

- Two candidates, A and B
- k citizens, each of whom votes for A or votes for B
- Winner is candidate who obtains the most votes

Strategic game

- Players: The k citizens
- For each player,
  - Actions: Vote for A, vote for B
  - Preferences: For player who prefers A to B,

$$A \text{ gets most votes} \succ A \text{ and } B \text{ tie} \succ B \text{ gets most votes}$$

Majority rule: two candidates

- Two candidates, A and B
- k citizens, each of whom votes for A or votes for B
- Winner is candidate who obtains the most votes

Strategic game

- Players: The k citizens
- For each player,
  - Actions: Vote for A, vote for B
  - Preferences: For player who prefers A to B,

$$A \text{ gets most votes} \succ A \text{ and } B \text{ tie} \succ B \text{ gets most votes}$$

and for player who prefers B to A,

$$B \text{ gets most votes} \succ A \text{ and } B \text{ tie} \succ A \text{ gets most votes}$$

Majority rule: two candidates

- Suppose citizen i prefers A to B
- How should she vote?

Majority rule: two candidates

- Suppose citizen i prefers A to B
- How should she vote?
- Compare outcome of i's voting for A and voting for B:

Among other citizens' votes:

		A wins by ≥ 2	A wins by 1	A and B tie	B wins by 1	B wins by ≥ 2
Citizen i	vote for A					
	vote for B					

Election outcomes

Majority rule: two candidates

- Suppose citizen i prefers A to B
- How should she vote?
- Compare outcome of i's voting for A and voting for B:

		Among other citizens' votes:				
		A wins by ≥ 2	A wins by 1	A and B tie	B wins by 1	B wins by ≥ 2
Citizen i	vote for A					
	vote for B					

Election outcomes

Majority rule: two candidates

- Suppose citizen i prefers A to B
- How should she vote?
- Compare outcome of i's voting for A and voting for B:

Among other citizens' votes:

		A wins by ≥ 2	A wins by 1	A and B tie	B wins by 1	B wins by ≥ 2
Citizen i	vote for A	A				
	vote for B					

Election outcomes

Majority rule: two candidates

- Suppose citizen i prefers A to B
- How should she vote?
- Compare outcome of i's voting for A and voting for B:

Among other citizens' votes:

		A wins by ≥ 2	A wins by 1	A and B tie	B wins by 1	B wins by ≥ 2
Citizen i	vote for A	A				
	vote for B					

Election outcomes

Majority rule: two candidates

- Suppose citizen i prefers A to B
- How should she vote?
- Compare outcome of i's voting for A and voting for B:

Among other citizens' votes:

		A wins by ≥ 2	A wins by 1	A and B tie	B wins by 1	B wins by ≥ 2
Citizen i	vote for A	A	A			
	vote for B					

Election outcomes

Majority rule: two candidates

- Suppose citizen i prefers A to B
- How should she vote?
- Compare outcome of i's voting for A and voting for B:

Among other citizens' votes:

		A wins by ≥ 2	A wins by 1	A and B tie	B wins by 1	B wins by ≥ 2
Citizen i	vote for A	A	A			
	vote for B					

Election outcomes

Majority rule: two candidates

- Suppose citizen i prefers A to B
- How should she vote?
- Compare outcome of i's voting for A and voting for B:

Among other citizens' votes:

		A wins by ≥ 2	A wins by 1	A and B tie	B wins by 1	B wins by ≥ 2
Citizen i	vote for A	A	A	A		
	vote for B					

Election outcomes

Majority rule: two candidates

- Suppose citizen i prefers A to B
- How should she vote?
- Compare outcome of i's voting for A and voting for B:

Among other citizens' votes:

Citizen i	A wins by ≥ 2	A wins by 1	A and B tie	B wins by 1	B wins by ≥ 2
vote for A	A	A	A		
vote for B					

Election outcomes

Majority rule: two candidates

- Suppose citizen i prefers A to B
- How should she vote?
- Compare outcome of i's voting for A and voting for B:

Among other citizens' votes:

Citizen i	A wins by ≥ 2	A wins by 1	A and B tie	B wins by 1	B wins by ≥ 2
vote for A	A	A	A	tie	
vote for B					

Election outcomes

Majority rule: two candidates

- Suppose citizen i prefers A to B
- How should she vote?
- Compare outcome of i's voting for A and voting for B:

Among other citizens' votes:

		A wins by ≥ 2	A wins by 1	A and B tie	B wins by 1	B wins by ≥ 2
Citizen i	vote for A	A	A	A	tie	
	vote for B					

Election outcomes

Majority rule: two candidates

- Suppose citizen i prefers A to B
- How should she vote?
- Compare outcome of i's voting for A and voting for B:

Among other citizens' votes:

		A wins by ≥ 2	A wins by 1	A and B tie	B wins by 1	B wins by ≥ 2
Citizen i	vote for A	A	A	A	tie	B
	vote for B					

Election outcomes

Majority rule: two candidates

- Suppose citizen i prefers A to B
- How should she vote?
- Compare outcome of i's voting for A and voting for B:

Among other citizens' votes:

Citizen i		A wins by ≥ 2	A wins by 1	A and B tie	B wins by 1	B wins by ≥ 2
	vote for A	A	A	A	tie	B
	vote for B					

Election outcomes

Majority rule: two candidates

- Suppose citizen i prefers A to B
- How should she vote?
- Compare outcome of i's voting for A and voting for B:

Among other citizens' votes:

Citizen i	A wins by ≥ 2	A wins by 1	A and B tie	B wins by 1	B wins by ≥ 2
vote for A	A	A	A	tie	B
vote for B	A	tie	B	B	B

Election outcomes

Majority rule: two candidates

- Suppose citizen i prefers A to B
- How should she vote?
- Compare outcome of i's voting for A and voting for B:

Among other citizens' votes:

		A wins by ≥ 2	A wins by 1	A and B tie	B wins by 1	B wins by ≥ 2
Citizen i	vote for A	A	A	A	tie	B
	vote for B	A	tie	B	B	B

Election outcomes

- For citizen i, voting for A weakly dominates voting for B

Majority rule: two candidates

- Suppose citizen i prefers A to B
- How should she vote?
- Compare outcome of i's voting for A and voting for B:

Among other citizens' votes:

		A wins by ≥ 2	A wins by 1	A and B tie	B wins by 1	B wins by ≥ 2
Citizen i	vote for A	A	A	A	tie	B
	vote for B	A	tie	B	B	B

Election outcomes

- For citizen i, voting for A weakly dominates voting for B
- Similarly, for citizen who prefers B to A, vote for B weakly dominates vote for A

Majority rule: two candidates

Nash equilibrium

Majority rule: two candidates

Nash equilibrium

- One Nash equilibrium:

Majority rule: two candidates

Nash equilibrium

- One Nash equilibrium: every citizen votes for her favorite candidate

Majority rule: two candidates

Nash equilibrium

- One Nash equilibrium: every citizen votes for her favorite candidate
  - Only Nash equilibrium in which no citizen uses a weakly dominated action

Majority rule: two candidates

Nash equilibrium

- One Nash equilibrium: every citizen votes for her favorite candidate
  - Only Nash equilibrium in which no citizen uses a weakly dominated action
- Other Nash equilibria?

Majority rule: two candidates

Nash equilibrium

- One Nash equilibrium: every citizen votes for her favorite candidate
  - Only Nash equilibrium in which no citizen uses a weakly dominated action
- Other Nash equilibria?
  - Yes: for example, *all* citizens vote for A

Majority rule: two candidates

Nash equilibrium

- One Nash equilibrium: every citizen votes for her favorite candidate
  - Only Nash equilibrium in which no citizen uses a weakly dominated action
- Other Nash equilibria?
  - Yes: for example, *all* citizens vote for A
- Problem asks you to find all Nash equilibria

Majority rule: three candidates

- Candidates A, B, and C

Majority rule: three candidates

- Candidates A, B, and C
- Consider citizen who ranks $A \succ B \succ C$

Majority rule: three candidates

- Candidates A, B, and C
- Consider citizen who ranks $A \succ B \succ C$
- Is her voting for C weakly dominated?

Majority rule: three candidates

- Candidates A, B, and C
- Consider citizen who ranks $A \succ B \succ C$
- Is her voting for C weakly dominated?
- Effects of switching vote from C to A:

Other citizens' votes

vote for A							
vote for C							

vote for A						
vote for C						

Winning candidate (X–Y means tie between X and Y)

Majority rule: three candidates

- Candidates A, B, and C
- Consider citizen who ranks $A \succ B \succ C$

A is 1 vote ahead of B and at least 2 votes ahead of C

C weakly dominated?

ng vote from C to A:

Other citizens' votes

vote for A							
vote for C							

vote for A						
vote for C						

Winning candidate (X–Y means tie between X and Y)

Majority rule: three candidates

- Candidates A, B, and C
- Consider citizen who ranks $A \succ B \succ C$
- Is her voting for C weakly dominated?
- Effects of switching vote from C to A:

Other citizens' votes

vote for A	A						
vote for C	A						

vote for A						
vote for C						

Winning candidate (X–Y means tie between X and Y)

Majority rule: three candidates

- Candidates *A*, *B*, and *C*
- Consider citizen who ranks $A \succ B \succ C$
- Is her voting for *C* weakly dominated?
- Effects of switching vote from *C* to *A*:

Other citizens' votes


vote for *A*	*A*						
vote for *C*	*A*						

vote for *A*						
vote for *C*						

Winning candidate (X–Y means tie between X and Y)

Majority rule: three candidates

- Candidates A, B, and C
- Consider citizen who ranks $A \succ B \succ C$
- Is her voting for C weakly dominated?
- Effects of switching vote from C to A:

Other citizens' votes

							
vote for A	A	A					
vote for C	A	C					

vote for A						
vote for C						

Winning candidate (X–Y means tie between X and Y)

Majority rule: three candidates

- Candidates A, B, and C
- Consider citizen who ranks $A \succ B \succ C$
- Is her voting for C weakly dominated?
- Effects of switching vote from C to A:

Other citizens' votes

							
vote for A	A	A					
vote for C	A	C					

vote for A						
vote for C						

Winning candidate (X–Y means tie between X and Y)

Majority rule: three candidates

- Candidates A, B, and C
- Consider citizen who ranks $A \succ B \succ C$
- Is her voting for C weakly dominated?
- Effects of switching vote from C to A:

Other citizens' votes

							
vote for A	A	A	A				
vote for C	A	C	A–B				

vote for A						
vote for C						

Winning candidate (X–Y means tie between X and Y)

Majority rule: three candidates

- Candidates *A*, *B*, and *C*
- Consider citizen who ranks $A \succ B \succ C$
- Is her voting for *C* weakly dominated?
- Effects of switching vote from *C* to *A*:

Other citizens' votes

vote for *A*	*A*	*A*	*A*	*A*	*A*	*B*	*B*
vote for *C*	*A*	*C*	*A*–*B*	*A*–*C*	*A*–*B*–*C*	*B*	*B*–*C*

vote for *A*	*C*	*A*–*B*	*A*–*B*	*A*–*C*	*B*–*C*	*A*–*B*–*C*
vote for *C*	*C*	*B*	*B*–*C*	*C*	*C*	*C*

Winning candidate (*X*–*Y* means tie between *X* and *Y*)

Majority rule: three candidates

- Candidates A, B, and C
- Consider citizen who ranks $A \succ B \succ C$
- Is her voting for C weakly dominated?
- Effects of switching vote from C to A:

Other citizens' votes

vote for A	A	A	A	A	A	B	B
vote for C	A	C	A–B	A–C	A–B–C	B	B–C

vote for A	C	A–B	A–B	A–C	B–C	A–B–C
vote for C	C	B	B–C	C	C	C

Winning candidate (X–Y means tie between X and Y)

- Voting for A weakly dominates voting for C

Majority rule: three candidates

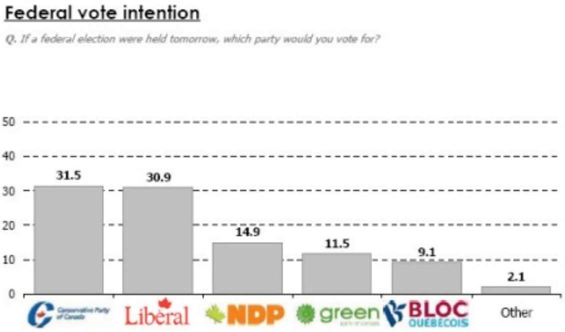
Dominated actions

- For citizen who ranks $A \succ B \succ C$, is voting for B weakly dominated?

Majority rule: three candidates

Dominated actions

- For citizen who ranks $A \succ B \succ C$, is voting for B weakly dominated?
- Suppose you rank NDP $\succ$ Liberal $\succ$ Conservative and the polls say:



January 21, 2010

Majority rule: three candidates

Dominated actions

- For citizen who ranks $A \succ B \succ C$, voting for B is *not* weakly dominated

Majority rule: three candidates

Dominated actions

- For citizen who ranks $A \succ B \succ C$, voting for B is *not* weakly dominated
- If votes among other citizens are, for example,

candidates	other citizens' votes
A	15
B	30
C	31

then

- vote for $A \Rightarrow$

Majority rule: three candidates

Dominated actions

- For citizen who ranks $A \succ B \succ C$, voting for B is *not* weakly dominated
- If votes among other citizens are, for example,

candidates	other citizens' votes
A	15
B	30
C	31

then

- vote for $A \Rightarrow C$ wins

Majority rule: three candidates

Dominated actions

- For citizen who ranks $A \succ B \succ C$, voting for B is *not* weakly dominated
- If votes among other citizens are, for example,

candidates	other citizens' votes
A	15
B	30
C	31

  then
  - vote for $A \Rightarrow C$ wins
  - vote for $B \Rightarrow$

Majority rule: three candidates

Dominated actions

- For citizen who ranks $A \succ B \succ C$, voting for B is *not* weakly dominated
- If votes among other citizens are, for example,

candidates	other citizens' votes
A	15
B	30
C	31

then

- vote for $A \Rightarrow C$ wins
- vote for $B \Rightarrow B$ and C tie

Majority rule: three candidates

Dominated actions

- For citizen who ranks $A \succ B \succ C$, voting for B is *not* weakly dominated
- If votes among other citizens are, for example,

candidates	other citizens' votes
A	15
B	30
C	31

  then
  - vote for $A \Rightarrow C$ wins
  - vote for $B \Rightarrow B$ and C tie
- That is, citizen is better off voting for B than for A

Majority rule: three candidates

Dominated actions

- For citizen who ranks $A \succ B \succ C$, voting for B is *not* weakly dominated
- If votes among other citizens are, for example,

candidates	other citizens' votes
A	15
B	30
C	31

  then
  - vote for $A \Rightarrow C$ wins
  - vote for $B \Rightarrow B$ and C tie
- That is, citizen is better off voting for B than for A
- "Strategic" voting is not "sincere"

Majority rule: three candidates

Nash equilibrium

- Voting game has many Nash equilibria

Majority rule: three candidates

Nash equilibrium

- Voting game has many Nash equilibria
- For example, any action profile in which the winning margin is 3 or more is a Nash equilibrium

Majority rule: three candidates

Nash equilibrium

- Voting game has many Nash equilibria
- For example, any action profile in which the winning margin is 3 or more is a Nash equilibrium
  - no change in any player's action changes the outcome

Majority rule: three candidates

Nash equilibrium

- Voting game has many Nash equilibria
- For example, any action profile in which the winning margin is 3 or more is a Nash equilibrium
  - no change in any player's action changes the outcome
- And, for any preferences, in some equilibria at least one player does not vote for her favorite candidate and does not use a weakly dominated action

Majority rule: summary

Two candidates

- Voting for less preferred candidate is weakly dominated

Majority rule: summary

Two candidates

- Voting for less preferred candidate is weakly dominated
- Many Nash equilibria, but only one in which no one uses a weakly dominated strategy: everyone votes for their favorite candidate

Majority rule: summary

Two candidates

- Voting for less preferred candidate is weakly dominated
- Many Nash equilibria, but only one in which no one uses a weakly dominated strategy: everyone votes for their favorite candidate

Three candidates

- Voting for least preferred candidate is weakly dominated

Majority rule: summary

Two candidates

- Voting for less preferred candidate is weakly dominated
- Many Nash equilibria, but only one in which no one uses a weakly dominated strategy: everyone votes for their favorite candidate

Three candidates

- Voting for least preferred candidate is weakly dominated
- But voting for any other candidate is not weakly dominated

Majority rule: summary

Two candidates

- Voting for less preferred candidate is weakly dominated
- Many Nash equilibria, but only one in which no one uses a weakly dominated strategy: everyone votes for their favorite candidate

Three candidates

- Voting for least preferred candidate is weakly dominated
- But voting for any other candidate is not weakly dominated
- Many Nash equilibria, including ones in which some citizens do not vote for their favorite candidate and do not use a weakly dominated action

Alternative voting systems

- Many schemes have been proposed with the aim of improving on plurality rule

Alternative voting systems

- Many schemes have been proposed with the aim of improving on plurality rule
- Two such schemes allow voters to express more information than simply naming a single candidate:

Alternative voting systems

- Many schemes have been proposed with the aim of improving on plurality rule
- Two such schemes allow voters to express more information than simply naming a single candidate:
  - approval voting

Alternative voting systems

- Many schemes have been proposed with the aim of improving on plurality rule
- Two such schemes allow voters to express more information than simply naming a single candidate:
  - approval voting
  - single-transferable vote

Alternative voting systems

Approval voting

- Each citizen votes for (“approves of”) a *set* of candidates rather than a single candidate

Alternative voting systems

Approval voting

- Each citizen votes for (“approves of”) a *set* of candidates rather than a single candidate
- The candidate with the most votes wins

Alternative voting systems

Approval voting

- Each citizen votes for (“approves of”) a *set* of candidates rather than a single candidate
- The candidate with the most votes wins
- Weakly dominated actions:

Alternative voting systems

Approval voting

- Each citizen votes for ("approves of") a *set* of candidates rather than a single candidate
- The candidate with the most votes wins
- Weakly dominated actions:
  - vote that includes least favorite candidate

Alternative voting systems

Approval voting

- Each citizen votes for ("approves of") a *set* of candidates rather than a single candidate
- The candidate with the most votes wins
- Weakly dominated actions:
  - vote that includes least favorite candidate
  - vote that does not include favorite candidate

Alternative voting systems

Approval voting

- Each citizen votes for ("approves of") a *set* of candidates rather than a single candidate
- The candidate with the most votes wins
- Weakly dominated actions:
    - vote that includes least favorite candidate
    - vote that does not include favorite candidate
- So vote that is not weakly dominated

Alternative voting systems

Approval voting

- Each citizen votes for (“approves of”) a *set* of candidates rather than a single candidate
- The candidate with the most votes wins
- Weakly dominated actions:
  - vote that includes least favorite candidate
  - vote that does not include favorite candidate
- So vote that is not weakly dominated
  - does not include least favorite candidate

Alternative voting systems

Approval voting

- Each citizen votes for ("approves of") a *set* of candidates rather than a single candidate
- The candidate with the most votes wins
- Weakly dominated actions:
  - vote that includes least favorite candidate
  - vote that does not include favorite candidate
- So vote that is not weakly dominated
  - does not include least favorite candidate
  - includes favorite candidate

Alternative voting systems

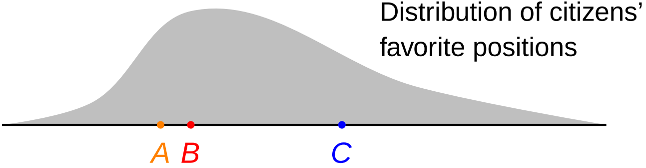
Approval voting

- Each citizen votes for ("approves of") a *set* of candidates rather than a single candidate
- The candidate with the most votes wins
- Weakly dominated actions:
  - vote that includes least favorite candidate
  - vote that does not include favorite candidate
- So vote that is not weakly dominated
  - does not include least favorite candidate
  - includes favorite candidate
- Three candidates: if $A \succ B \succ C$ then only strategies $\{A\}$ and $\{A, B\}$ are not weakly dominated

Alternative voting systems

Approval voting

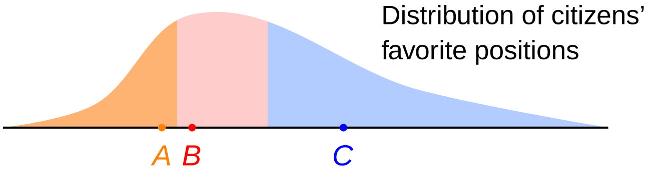
- One situation in which system is intended to work well



Alternative voting systems

Approval voting

- One situation in which system is intended to work well

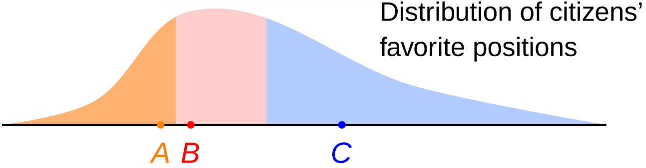


- If every citizen votes for her favorite candidate, *C* wins

Alternative voting systems

Approval voting

- One situation in which system is intended to work well

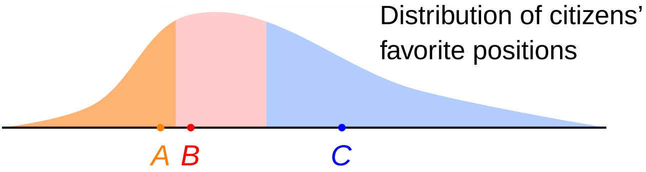


- If every citizen votes for her favorite candidate, *C* wins
- But citizens on left are almost indifferent between *A* and *B*, and together are more numerous than supporters of *C*

Alternative voting systems

Approval voting

- One situation in which system is intended to work well

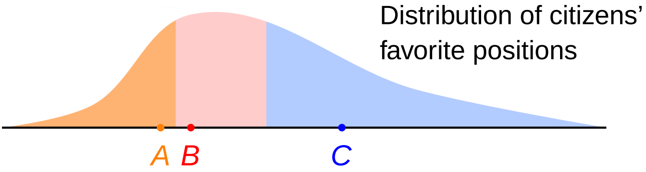


- If every citizen votes for her favorite candidate, C wins
- But citizens on left are almost indifferent between A and B, and together are more numerous than supporters of C
- Idea: all citizens on left approve $\{A, B\}$ and citizens on right approve $\{C\}$, so that A and B tie for first place

Alternative voting systems

Approval voting

- One situation in which system is intended to work well



- If every citizen votes for her favorite candidate, C wins
- But citizens on left are almost indifferent between A and B, and together are more numerous than supporters of C
- Idea: all citizens on left approve $\{A, B\}$ and citizens on right approve $\{C\}$, so that A and B tie for first place
- But as for majority rule, game has many Nash equilibria

Alternative voting systems

Single transferable vote

- Each voter ranks candidates

Alternative voting systems

Single transferable vote

- Each voter ranks candidates
- Method for determining winner

Alternative voting systems

Single transferable vote

- Each voter ranks candidates
- Method for determining winner

Determine candidate with highest number of first-place votes

Alternative voting systems

Single transferable vote

- Each voter ranks candidates
- Method for determining winner

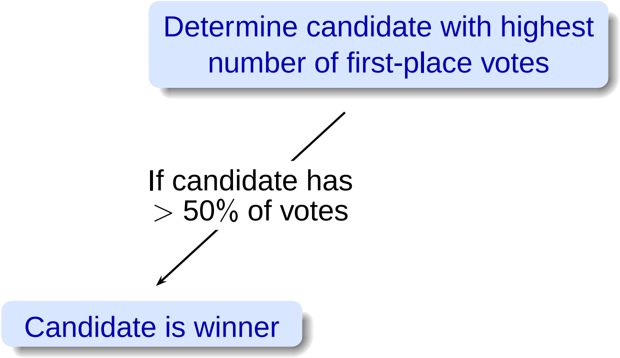
Determine candidate with highest number of first-place votes

If candidate has $>$ 50% of votes

Alternative voting systems

Single transferable vote

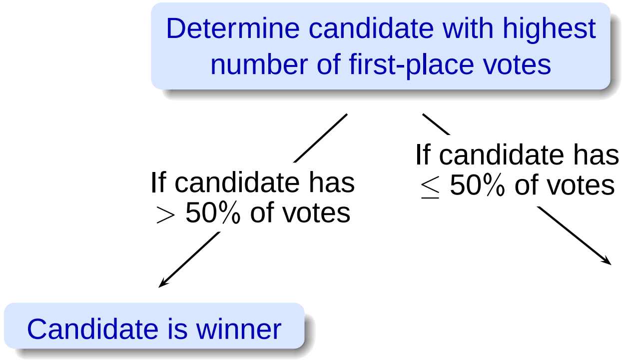
- Each voter ranks candidates
- Method for determining winner



Alternative voting systems

Single transferable vote

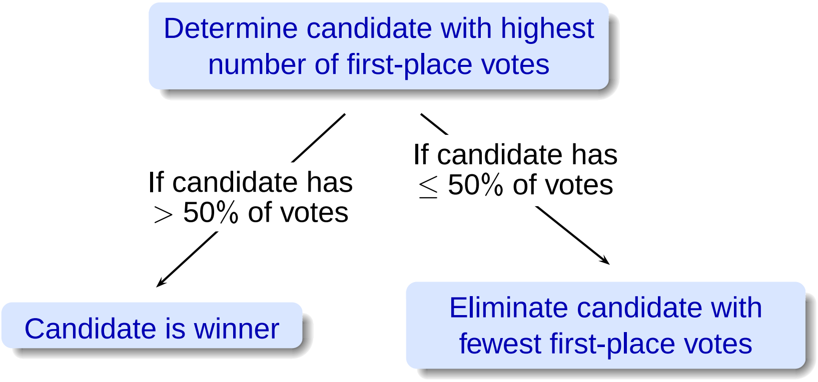
- Each voter ranks candidates
- Method for determining winner



Alternative voting systems

Single transferable vote

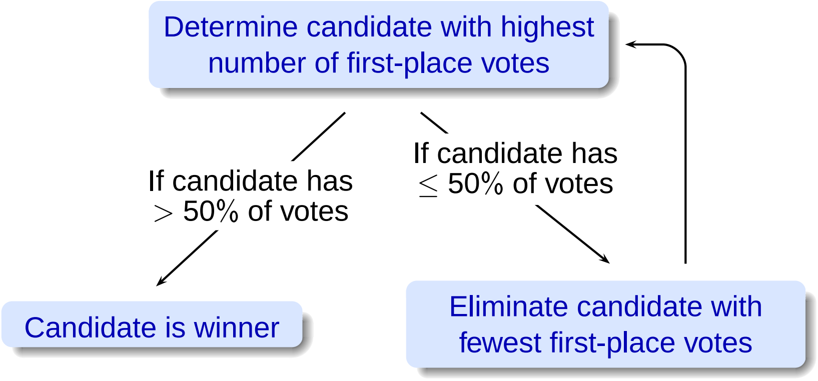
- Each voter ranks candidates
- Method for determining winner



Alternative voting systems

Single transferable vote

- Each voter ranks candidates
- Method for determining winner



Alternative voting systems

Single transferable vote

Example:

# of voters	rankings
6	A, B, C, D
5	B, A, C, D
3	C, B, A, D
1	D, C, A, B

Alternative voting systems

Single transferable vote

Example:

# of voters	rankings
6	*A*, *B*, *C*, *D*
5	*B*, *A*, *C*, *D*
3	*C*, *B*, *A*, *D*
1	*D*, *C*, *A*, *B*

Procedure:

	votes for			
round	*A*	*B*	*C*	*D*

Alternative voting systems

Single transferable vote

Example:

# of voters	rankings
6	A, B, C, D
5	B, A, C, D
3	C, B, A, D
1	D, C, A, B

Procedure:

	votes for			
round	A	B	C	D
1	6	5	3	1

Alternative voting systems

Single transferable vote

Example:

# of voters	rankings
6	A, B, C, D
5	B, A, C, D
3	C, B, A, D
1	D, C, A, B

Procedure:

	votes for				
round	A	B	C	D	
1	6	5	3	1	Eliminate D

Alternative voting systems

Single transferable vote

Example:

# of voters	rankings
6	*A*, *B*, *C*, *D*
5	*B*, *A*, *C*, *D*
3	*C*, *B*, *A*, *D*
1	*D*, *C*, *A*, *B*

Procedure:

	votes for				
round	*A*	*B*	*C*	*D*	
1	6	5	3	1	Eliminate *D*
2	6	5	4		

Alternative voting systems

Single transferable vote

Example:

# of voters	rankings
6	*A*, *B*, *C*, *D*
5	*B*, *A*, *C*, *D*
3	*C*, *B*, *A*, *D*
1	*D*, *C*, *A*, *B*

Procedure:

	votes for				
round	*A*	*B*	*C*	*D*	
1	6	5	3	1	Eliminate *D*
2	6	5	4		Eliminate *C*

Alternative voting systems

Single transferable vote

Example:

# of voters	rankings
6	A, B, C, D
5	B, A, C, D
3	C, B, A, D
1	D, C, A, B

Procedure:

	votes for				
round	A	B	C	D	
1	6	5	3	1	Eliminate D
2	6	5	4		Eliminate C
3	7	8			

Alternative voting systems

Single transferable vote

Example:

# of voters	rankings
6	*A*, *B*, *C*, *D*
5	*B*, *A*, *C*, *D*
3	*C*, *B*, *A*, *D*
1	*D*, *C*, *A*, *B*

Procedure:

	votes for				
round	*A*	*B*	*C*	*D*	
1	6	5	3	1	Eliminate *D*
2	6	5	4		Eliminate *C*
3	7	8			Winner is *B*

Alternative voting systems

Single transferable vote

Example:

# of voters	rankings
6	*A*, *B*, *C*, *D*
5	*B*, *A*, *C*, *D*
3	*C*, *B*, *A*, *D*
1	*D*, *C*, *A*, *B*

Procedure:

	votes for				
round	*A*	*B*	*C*	*D*	
1	6	5	3	1	Eliminate *D*
2	6	5	4		Eliminate *C*
3	7	8			Winner is *B*

By contrast, under plurality rule, if each player votes for her favorite candidate, *A* wins

Alternative voting systems

Single transferable vote

- Can formulate as strategic game: action of voter is ranking

Alternative voting systems

Single transferable vote

- Can formulate as strategic game: action of voter is ranking
- Analysis of Nash equilibria is difficult

Alternative voting systems

Single transferable vote

- Can formulate as strategic game: action of voter is ranking
- Analysis of Nash equilibria is difficult
- No simple general results (Problem Set has example)

Alternative voting systems

Summary

- Many alternatives to plurality voting exist

Alternative voting systems

Summary

- Many alternatives to plurality voting exist
- No system is perfect

Alternative voting systems

Summary

- Many alternatives to plurality voting exist
- No system is perfect
- For some configurations of preferences, systems like approval voting or the single transferable vote generate outcomes that reflect voters' preferences better than outcome of plurality rule, at least for some type of "sincere voting"

Committee decision-making

- Group of n people; assume n is odd

Committee decision-making

- Group of n people; assume n is odd
- Have to choose a policy—a number

Committee decision-making

- Group of n people; assume n is odd
- Have to choose a policy—a number
- Each person i has favorite policy, x_i^*

Committee decision-making

- Group of n people; assume n is odd
- Have to choose a policy—a number
- Each person i has favorite policy, x_i^*
- Each person i prefers y to z if $|x_i^* - y| < |x_i^* - z|$

Committee decision-making

- Group of n people; assume n is odd
- Have to choose a policy—a number
- Each person i has favorite policy, x_i^*
- Each person i prefers y to z if $|x_i^* - y| < |x_i^* - z|$

Procedure

- Each person names a policy

Committee decision-making

- Group of n people; assume n is odd
- Have to choose a policy—a number
- Each person i has favorite policy, x_i^*
- Each person i prefers y to z if $|x_i^* - y| < |x_i^* - z|$

Procedure

- Each person names a policy
- The *median* named policy is selected

Committee decision-making

- Group of n people; assume n is odd
- Have to choose a policy—a number
- Each person i has favorite policy, x_i^*
- Each person i prefers y to z if $|x_i^* - y| < |x_i^* - z|$

Procedure

- Each person names a policy
- The *median* named policy is selected

Committee decision-making

- Group of n people; assume n is odd
- Have to choose a policy—a number
- Each person i has favorite policy, x_i^*
- Each person i prefers y to z if $|x_i^* - y| < |x_i^* - z|$

Procedure

- Each person names a policy
- The *median* named policy is selected

Committee decision-making

Strategic game

- Players: the n people

Committee decision-making

Strategic game

- Players: the n people
- For each player i

Committee decision-making

Strategic game

- Players: the n people
- For each player i
  - actions: set of possible policies (numbers)

Committee decision-making

Strategic game

- Players: the n people
- For each player i
  - actions: set of possible policies (numbers)
  - payoffs: $-|x_i^* - m(a_1, \ldots, a_n)|$, where $m(a_1, \ldots, a_n)$ is the median policy named by the players

Committee decision-making

Domination

- Compare outcome of i's naming x_i^* with her naming some other policy $y > x_i^*$

Committee decision-making

Domination

- Compare outcome of i's naming x_i^* with her naming some other policy $y > x_i^*$
- Suppose $n = 3$, denote other players' actions a and b, and assume $a < b$

Committee decision-making

Domination

- Compare outcome of i's naming x_i^* with her naming some other policy $y > x_i^*$
- Suppose $n = 3$, denote other players' actions a and b, and assume $a < b$
- Effect of naming y rather than x_i^*:

Committee decision-making

Domination

- Compare outcome of i's naming x_i^* with her naming some other policy $y > x_i^*$
- Suppose $n = 3$, denote other players' actions a and b, and assume $a < b$
- Effect of naming y rather than x_i^*:

Committee decision-making

Domination

- Compare outcome of i's naming x_i^* with her naming some other policy $y > x_i^*$
- Suppose $n = 3$, denote other players' actions a and b, and assume $a < b$
- Effect of naming y rather than x_i^*:

Committee decision-making

Domination

- Compare outcome of i's naming x_i^* with her naming some other policy $y > x_i^*$
- Suppose $n = 3$, denote other players' actions a and b, and assume $a < b$
- Effect of naming y rather than x_i^*:

Committee decision-making

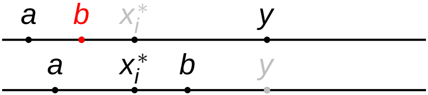
Domination

- Compare outcome of i's naming x_i^* with her naming some other policy $y > x_i^*$
- Suppose $n = 3$, denote other players' actions a and b, and assume $a < b$
- Effect of naming y rather than x_i^*:

Committee decision-making

Domination

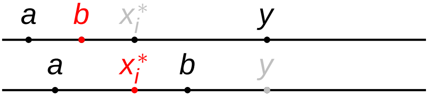
- Compare outcome of i's naming x_i^* with her naming some other policy $y > x_i^*$
- Suppose $n = 3$, denote other players' actions a and b, and assume $a < b$
- Effect of naming y rather than x_i^*:



Committee decision-making

Domination

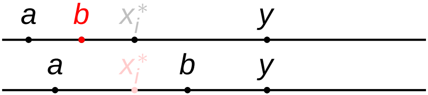
- Compare outcome of i's naming x_i^* with her naming some other policy $y > x_i^*$
- Suppose $n = 3$, denote other players' actions a and b, and assume $a < b$
- Effect of naming y rather than x_i^*:



Committee decision-making

Domination

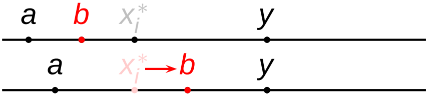
- Compare outcome of i's naming x_i^* with her naming some other policy $y > x_i^*$
- Suppose $n = 3$, denote other players' actions a and b, and assume $a < b$
- Effect of naming y rather than x_i^*:



Committee decision-making

Domination

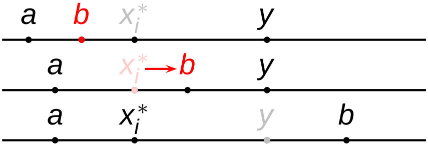
- Compare outcome of i's naming x_i^* with her naming some other policy $y > x_i^*$
- Suppose $n = 3$, denote other players' actions a and b, and assume $a < b$
- Effect of naming y rather than x_i^*:



Committee decision-making

Domination

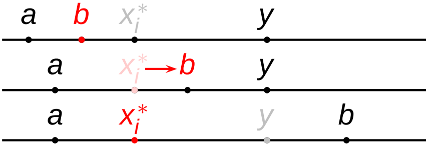
- Compare outcome of i's naming x_i^* with her naming some other policy $y > x_i^*$
- Suppose $n = 3$, denote other players' actions a and b, and assume $a < b$
- Effect of naming y rather than x_i^*:



Committee decision-making

Domination

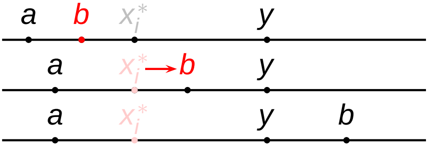
- Compare outcome of i's naming x_i^* with her naming some other policy $y > x_i^*$
- Suppose $n = 3$, denote other players' actions a and b, and assume $a < b$
- Effect of naming y rather than x_i^*:



Committee decision-making

Domination

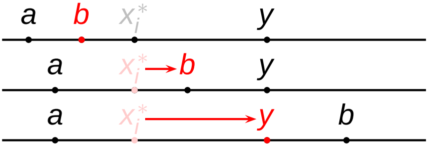
- Compare outcome of i's naming x_i^* with her naming some other policy $y > x_i^*$
- Suppose $n = 3$, denote other players' actions a and b, and assume $a < b$
- Effect of naming y rather than x_i^*:



Committee decision-making

Domination

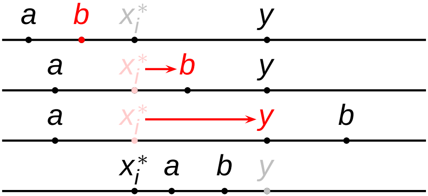
- Compare outcome of i's naming x_i^* with her naming some other policy $y > x_i^*$
- Suppose $n = 3$, denote other players' actions a and b, and assume $a < b$
- Effect of naming y rather than x_i^*:



Committee decision-making

Domination

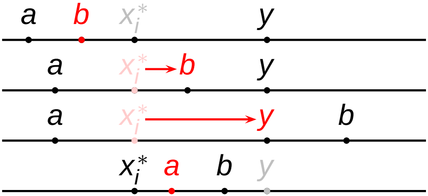
- Compare outcome of i's naming x_i^* with her naming some other policy $y > x_i^*$
- Suppose $n = 3$, denote other players' actions a and b, and assume $a < b$
- Effect of naming y rather than x_i^*:



Committee decision-making

Domination

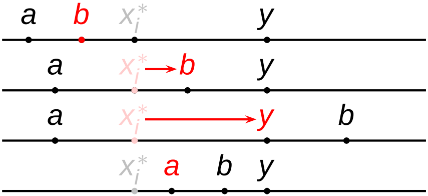
- Compare outcome of i's naming x_i^* with her naming some other policy $y > x_i^*$
- Suppose $n = 3$, denote other players' actions a and b, and assume $a < b$
- Effect of naming y rather than x_i^*:



Committee decision-making

Domination

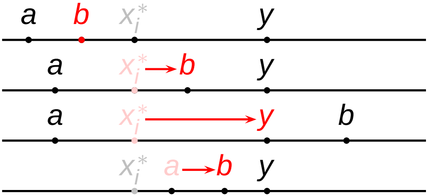
- Compare outcome of i's naming x_i^* with her naming some other policy $y > x_i^*$
- Suppose $n = 3$, denote other players' actions a and b, and assume $a < b$
- Effect of naming y rather than x_i^*:



Committee decision-making

Domination

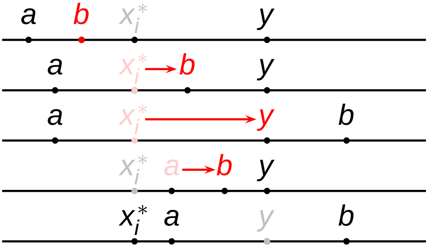
- Compare outcome of i's naming x_i^* with her naming some other policy $y > x_i^*$
- Suppose $n = 3$, denote other players' actions a and b, and assume $a < b$
- Effect of naming y rather than x_i^*:



Committee decision-making

Domination

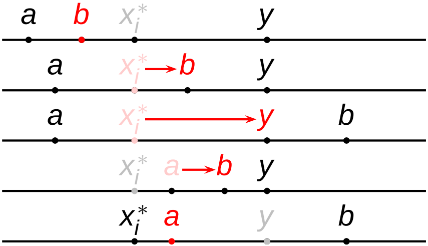
- Compare outcome of i's naming x_i^* with her naming some other policy $y > x_i^*$
- Suppose $n = 3$, denote other players' actions a and b, and assume $a < b$
- Effect of naming y rather than x_i^*:



Committee decision-making

Domination

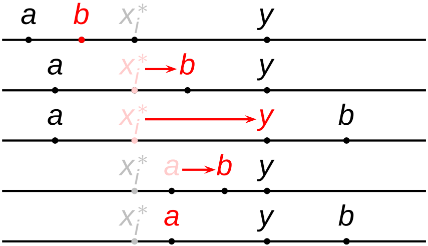
- Compare outcome of i's naming x_i^* with her naming some other policy $y > x_i^*$
- Suppose $n = 3$, denote other players' actions a and b, and assume $a < b$
- Effect of naming y rather than x_i^*:



Committee decision-making

Domination

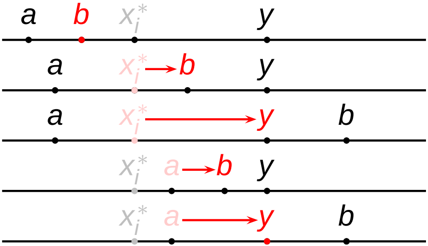
- Compare outcome of i's naming x_i^* with her naming some other policy $y > x_i^*$
- Suppose $n = 3$, denote other players' actions a and b, and assume $a < b$
- Effect of naming y rather than x_i^*:



Committee decision-making

Domination

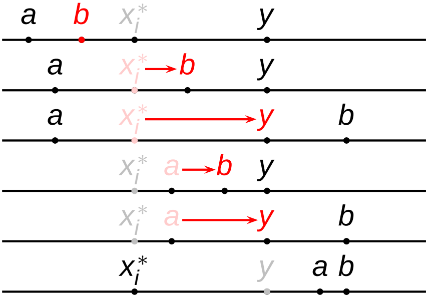
- Compare outcome of i's naming x_i^* with her naming some other policy $y > x_i^*$
- Suppose $n = 3$, denote other players' actions a and b, and assume $a < b$
- Effect of naming y rather than x_i^*:



Committee decision-making

Domination

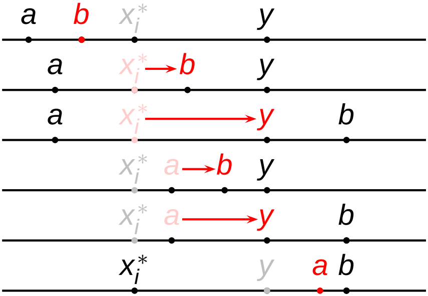
- Compare outcome of i's naming x_i^* with her naming some other policy $y > x_i^*$
- Suppose $n = 3$, denote other players' actions a and b, and assume $a < b$
- Effect of naming y rather than x_i^*:



Committee decision-making

Domination

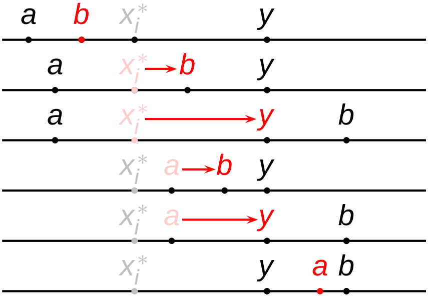
- Compare outcome of i's naming x_i^* with her naming some other policy $y > x_i^*$
- Suppose $n = 3$, denote other players' actions a and b, and assume $a < b$
- Effect of naming y rather than x_i^*:



Committee decision-making

Domination

- Compare outcome of i's naming x_i^* with her naming some other policy $y > x_i^*$
- Suppose $n = 3$, denote other players' actions a and b, and assume $a < b$
- Effect of naming y rather than x_i^*:



Committee decision-making

Domination

- In every case, naming y rather than x_i^* either does not affect the outcome or makes the outcome worse for i

Committee decision-making

Domination

- In every case, naming y rather than x_i^* either does not affect the outcome or makes the outcome worse for i
  - There is no benefit from exaggeration (why?)

Committee decision-making

Domination

- In every case, naming y rather than x_i^* either does not affect the outcome or makes the outcome worse for i
  - There is no benefit from exaggeration (why?)
- So i's naming x_i^* weakly dominates her naming y

Committee decision-making

Domination

- In every case, naming y rather than x_i^* either does not affect the outcome or makes the outcome worse for i
  - There is no benefit from exaggeration (why?)
- So i's naming x_i^* weakly dominates her naming y
- Same argument applies if $y < x_i^*$, so i's action x_i^* weakly dominates all her other actions

Committee decision-making

Domination

- In every case, naming y rather than x_i^* either does not affect the outcome or makes the outcome worse for i
  - There is no benefit from exaggeration (why?)
- So i's naming x_i^* weakly dominates her naming y
- Same argument applies if $y < x_i^*$, so i's action x_i^* weakly dominates all her other actions
- Same argument applies also if $n > 3$

Committee decision-making

Domination

- In every case, naming y rather than x_i^* either does not affect the outcome or makes the outcome worse for i
  - There is no benefit from exaggeration (why?)
- So i's naming x_i^* weakly dominates her naming y
- Same argument applies if $y < x_i^*$, so i's action x_i^* weakly dominates all her other actions
- Same argument applies also if $n > 3$
  - in that case, take a and b to be the two central positions among the other players' actions

Committee decision-making

Domination

- In every case, naming y rather than x_i^* either does not affect the outcome or makes the outcome worse for i
  - There is no benefit from exaggeration (why?)
- So i's naming x_i^* weakly dominates her naming y
- Same argument applies if $y < x_i^*$, so i's action x_i^* weakly dominates all her other actions
- Same argument applies also if $n > 3$
  - in that case, take a and b to be the two central positions among the other players' actions
- Thus in all cases i's action x_i^* weakly dominates all her other actions

Committee decision-making

Nash equilibrium

- x_i^* weakly dominates every other action for every player i

Committee decision-making

Nash equilibrium

- x_i^* weakly dominates every other action for every player i

⇒ one Nash equilibrium is for every player i to name her favorite action

Committee decision-making

Nash equilibrium

- x_i^* weakly dominates every other action for every player i

⇒ one Nash equilibrium is for every player i to name her favorite action

⇒ this Nash equilibrium is the only equilibrium in which no player uses a weakly dominated action

Committee decision-making

Nash equilibrium

- x_i^* weakly dominates every other action for every player i

⇒ one Nash equilibrium is for every player i to name her favorite action

⇒ this Nash equilibrium is the only equilibrium in which no player uses a weakly dominated action

- If we think of the game as one in which we ask the players to report their favorite policies, *truth-telling* is an equilibrium

Committee decision-making

Nash equilibrium

- x_i^* weakly dominates every other action for every player i

⇒ one Nash equilibrium is for every player i to name her favorite action

⇒ this Nash equilibrium is the only equilibrium in which no player uses a weakly dominated action

- If we think of the game as one in which we ask the players to report their favorite policies, *truth-telling* is an equilibrium
- Do the same conclusions hold if the *mean* rather than the *median* of the actions is the outcome? [problem]

Voting under imperfect information

- So far, voters differ in their preferences

Voting under imperfect information

- So far, voters differ in their preferences
- Suppose instead that the differ in their *information*

Voting under imperfect information

- So far, voters differ in their preferences
- Suppose instead that the differ in their *information*
- Economy is either in state a or state b

Voting under imperfect information

- So far, voters differ in their preferences
- Suppose instead that the differ in their *information*
- Economy is either in state a or state b
- All voters agree that candidate A is best in state a and candidate B is best in state b

Voting under imperfect information

- So far, voters differ in their preferences
- Suppose instead that the differ in their *information*
- Economy is either in state a or state b
- All voters agree that candidate A is best in state a and candidate B is best in state b
- Assume payoffs

Voting under imperfect information

- So far, voters differ in their preferences
- Suppose instead that the differ in their *information*
- Economy is either in state a or state b
- All voters agree that candidate A is best in state a and candidate B is best in state b
- Assume payoffs
  - in state a: 1 if A elected, 0 if B elected

Voting under imperfect information

- So far, voters differ in their preferences
- Suppose instead that the differ in their *information*
- Economy is either in state *a* or state *b*
- All voters agree that candidate *A* is best in state *a* and candidate *B* is best in state *b*
- Assume payoffs
  - in state *a*: 1 if *A* elected, 0 if *B* elected
  - in state *b*: 0 if *A* elected, 1 if *B* elected

Voting under imperfect information

- So far, voters differ in their preferences
- Suppose instead that the differ in their *information*
- Economy is either in state a or state b
- All voters agree that candidate A is best in state a and candidate B is best in state b
- Assume payoffs
  - in state a: 1 if A elected, 0 if B elected
  - in state b: 0 if A elected, 1 if B elected
- Some voters know the state, others do not

Voting under imperfect information

- So far, voters differ in their preferences
- Suppose instead that the differ in their *information*
- Economy is either in state a or state b
- All voters agree that candidate A is best in state a and candidate B is best in state b
- Assume payoffs
  - in state a: 1 if A elected, 0 if B elected
  - in state b: 0 if A elected, 1 if B elected
- Some voters know the state, others do not
- How do citizens vote in equilibrium?

Voting under imperfect information

- Assume *two* citizens:

Voting under imperfect information

- Assume *two* citizens:
  - citizen 1 knows the state

Voting under imperfect information

- Assume *two* citizens:
  - citizen 1 knows the state
  - citizen 2 does not know the state; she believes the state is *a* with probability 0.9 and *b* with probability 0.1

Voting under imperfect information

- Assume *two* citizens:
  - citizen 1 knows the state
  - citizen 2 does not know the state; she believes the state is *a* with probability 0.9 and *b* with probability 0.1

Note: citizen 2 thinks state *a* is very likely

Voting under imperfect information

- Assume *two* citizens:
  - citizen 1 knows the state
  - citizen 2 does not know the state; she believes the state is *a* with probability 0.9 and *b* with probability 0.1
- Each citizen can
  - vote for candidate *A*
  - vote for candidate *B*
  - abstain

Voting under imperfect information

- Assume *two* citizens:
  - citizen 1 knows the state
  - citizen 2 does not know the state; she believes the state is a with probability 0.9 and b with probability 0.1
- Each citizen can
  - vote for candidate A
  - vote for candidate B
  - abstain
- To model this situation, we need to generalize the notion of a strategic game to allow for imperfect information

Voting under imperfect information

Strategic game with imperfect information

Voting under imperfect information

Strategic game with imperfect information

- Players: the two citizens

Voting under imperfect information

Strategic game with imperfect information

- Players: the two citizens
- Citizen 1 has two possible *types*: she may know state is *a* *or* she may know state is *b*

Voting under imperfect information

Strategic game with imperfect information

- Players: the two citizens
- Citizen 1 has two possible *types*: she may know state is *a* *or* she may know state is *b*
- Citizen 2 has only one possible type: she doesn't know state; she believes state is *a* with probability 0.9

Voting under imperfect information

Strategic game with imperfect information

- Players: the two citizens
- Citizen 1 has two possible *types*: she may know state is *a* *or* she may know state is *b*
- Citizen 2 has only one possible type: she doesn't know state; she believes state is *a* with probability 0.9
- Each citizen has three actions, vote for *A*, vote for *B*, and abstain

Voting under imperfect information

Strategic game with imperfect information

- Players: the two citizens
- Citizen 1 has two possible *types*: she may know state is *a* *or* she may know state is *b*
- Citizen 2 has only one possible type: she doesn't know state; she believes state is *a* with probability 0.9
- Each citizen has three actions, vote for *A*, vote for *B*, and abstain
- Payoffs of each player:

	A	*B*	abs.
A	1	$\frac{1}{2}$	1
B	$\frac{1}{2}$	0	0
abs.	1	0	$\frac{1}{2}$

State *a*

Voting under imperfect information

Strategic game with imperfect information

- Players: the two citizens
- Citizen 1 has two possible *types*: she may know state is *a* *or* she may know state is *b*
- Citizen 2 has only one possible type: she doesn't know state; she believes state is *a* with probability 0.9
- Each citizen has three actions, vote for *A*, vote for *B*, and abstain
- Payoffs of each player:

	A	*B*	abs.
A	1	$\frac{1}{2}$	1
B	$\frac{1}{2}$	0	0
abs.	1	0	$\frac{1}{2}$

State *a*

	A	*B*	abs.
A	0	$\frac{1}{2}$	0
B	$\frac{1}{2}$	1	1
abs.	0	1	$\frac{1}{2}$

State *b*

Voting under imperfect information

Nash equilibrium

- The action of each type of citizen 1 is a best response to the action of citizen 2

Voting under imperfect information

Nash equilibrium

- The action of each type of citizen 1 is a best response to the action of citizen 2
- The action of citizen 2 is a best response to the actions of the two types of citizen 1, given citizen 2's belief about the probabilities of states *a* and *b*

Voting under imperfect information: Nash equilibrium

Citizen 1

Voting under imperfect information: Nash equilibrium

Citizen 1

- Knows state

Voting under imperfect information: Nash equilibrium

Citizen 1

- Knows state
- If state is *a*, payoffs

	A	B	abs.
A	1	$\frac{1}{2}$	1
B	$\frac{1}{2}$	0	0
abs.	1	0	$\frac{1}{2}$

State a

Voting under imperfect information: Nash equilibrium

Citizen 1

- Knows state
- If state is *a*, payoffs

	A	*B*	abs.
A	1	$\frac{1}{2}$	1
B	$\frac{1}{2}$	0	0
abs.	1	0	$\frac{1}{2}$

State *a*

- So voting for *A* strictly dominates voting for *B* and weakly dominates abstention

Voting under imperfect information: Nash equilibrium

Citizen 1

- Knows state
- If state is *a*, payoffs

	A	*B*	abs.
A	1	$\frac{1}{2}$	1
B	$\frac{1}{2}$	0	0
abs.	1	0	$\frac{1}{2}$

State *a*

- So voting for *A* strictly dominates voting for *B* and weakly dominates abstention
- Similarly, if state is *b*, voting for *B* strictly dominates voting for *A* and weakly dominates abstention

Voting under imperfect information: Nash equilibrium

Citizen 1

- Knows state
- If state is a, payoffs

	A	B	abs.
A	1	$\frac{1}{2}$	1
B	$\frac{1}{2}$	0	0
abs.	1	0	$\frac{1}{2}$

State a

- So voting for A strictly dominates voting for B and weakly dominates abstention
- Similarly, if state is b, voting for B strictly dominates voting for A and weakly dominates abstention
- So if no player uses weakly dominated strategy, citizen 1 votes for A in state a and for B in state b

Voting under imperfect information: Nash equilibrium

Citizen 2

- Given that citizen 1 votes for A in state a and for B in state b, what is citizen 2's optimal action?

Voting under imperfect information: Nash equilibrium

Citizen 2

- Given that citizen 1 votes for A in state a and for B in state b, what is citizen 2's optimal action?
- Payoffs:

Vote for A

Vote for B

Abstain

Voting under imperfect information: Nash equilibrium

Citizen 2

Believes that state is *a* with probability 0.9, and in state *a* citizen 1 votes for *A*

- Give[…] for *A* in state *a* and for *B* in state *b*, w[…]al action?
- Payoffs:

Vote for *A*	$0.9 \cdot 1$
Vote for *B*	
Abstain	

Voting under imperfect information: Nash equilibrium

Citizen 2

- Given that citiz… …ate *a* and for *B* in state *b*, what is citiz…
- Payoffs:

Vote for A	$0.9 \cdot 1 + 0.1 \cdot 0.5$
Vote for B	
Abstain	

Believes that state is *b* with probability 0.1, and in state *b* citizen 1 votes for *B*

Voting under imperfect information: Nash equilibrium

Citizen 2

- Given that citizen 1 votes for *A* in state *a* and for *B* in state *b*, what is citizen 2's optimal action?
- Payoffs:

Vote for *A*	$0.9 \cdot 1 + 0.1 \cdot 0.5 = 0.95$
Vote for *B*	
Abstain	

Voting under imperfect information: Nash equilibrium

Citizen 2

- Given ... or A in state a and for B in state b, wh... l action?
- Payo...

Believes that state is a with probability 0.9, and in state a citizen 1 votes for A

Vote for A	$0.9 \cdot 1 + 0.1 \cdot 0.5 = 0.95$
Vote for B	$0.9 \cdot 0.5$
Abstain	

Voting under imperfect information: Nash equilibrium

Citizen 2

- Given that citize[…]e a and for B in state b, what is citize[…]
- Payoffs:

Believes that state is b with probability 0.1, and in state b citizen 1 votes for B

Vote for A	$0.9 \cdot 1 + 0.1 \cdot 0.5 = 0.95$
Vote for B	$0.9 \cdot 0.5 + 0.1 \cdot 1$
Abstain	

Voting under imperfect information: Nash equilibrium

Citizen 2

- Given that citizen 1 votes for *A* in state *a* and for *B* in state *b*, what is citizen 2's optimal action?
- Payoffs:

Vote for *A*	$0.9 \cdot 1 + 0.1 \cdot 0.5 = 0.95$
Vote for *B*	$0.9 \cdot 0.5 + 0.1 \cdot 1 = 0.46$
Abstain	

Voting under imperfect information: Nash equilibrium

Citizen 2

- Given that citizen 1 votes for A in state a and for B in state b, what is citizen 2's optimal action?
- Payoffs:

$$\begin{aligned} \text{Vote for } A \quad & 0.9 \cdot 1 + 0.1 \cdot 0.5 = 0.95 \\ \text{Vote for } B \quad & 0.9 \cdot 0.5 + 0.1 \cdot 1 = 0.46 \\ \text{Abstain} \quad & 0.9 \cdot 1 \end{aligned}$$

Voting under imperfect information: Nash equilibrium

Citizen 2

- Given that citizen 1 votes for *A* in state *a* and for *B* in state *b*, what is citizen 2's optimal action?
- Payoffs:

Vote for *A*	$0.9 \cdot 1 + 0.1 \cdot 0.5 = 0.95$
Vote for *B*	$0.9 \cdot 0.5 + 0.1 \cdot 1 = 0.46$
Abstain	$0.9 \cdot 1 + 0.1 \cdot 1 = 1$

Voting under imperfect information: Nash equilibrium

Citizen 2

- Given that citizen 1 votes for A in state a and for B in state b, what is citizen 2's optimal action?
- Payoffs:

Vote for A	$0.9 \cdot 1 + 0.1 \cdot 0.5 = 0.95$
Vote for B	$0.9 \cdot 0.5 + 0.1 \cdot 1 = 0.46$
Abstain	$0.9 \cdot 1 + 0.1 \cdot 1 = 1$

- So optimal action is to abstain—even though she is quite sure state is a

Voting under imperfect information

Conclusion

- Game has only one equilibrium in which no player uses a weakly dominated action:

  Citizen 1 votes for *A* when state is *a* and votes for *B* when state is *b*

  Citizen 2 abstains

Voting under imperfect information

Conclusion

- Game has only one equilibrium in which no player uses a weakly dominated action:

  Citizen 1 votes for *A* when state is *a* and votes for *B* when state is *b*

  Citzen 2 abstains

- Citizen 2, uninformed of state, suffers from *swing voter's curse*: if she votes, whenever her vote affects the outcome it affects it adversely

Voting under imperfect information

Conclusion

- Game has only one equilibrium in which no player uses a weakly dominated action:

  Citizen 1 votes for *A* when state is *a* and votes for *B* when state is *b*

  Citzen 2 abstains

- Citizen 2, uninformed of state, suffers from *swing voter's curse*: if she votes, whenever her vote affects the outcome it affects it adversely
  - if she votes for *A* then her vote makes no difference if state is *a* (citizen 1 votes for *A* in any case)

Voting under imperfect information

Conclusion

- Game has only one equilibrium in which no player uses a weakly dominated action:

Citizen 1	votes for *A* when state is *a* and votes for *B* when state is *b*
Citzen 2	abstains

- Citizen 2, uninformed of state, suffers from *swing voter's curse*: if she votes, whenever her vote affects the outcome it affects it adversely
  - if she votes for *A* then her vote makes no difference if state is *a* (citizen 1 votes for *A* in any case) and induces a worse outcome if state is *b* (in which best candidate is *B*)

Voting under imperfect information

Conclusion

- Game has only one equilibrium in which no player uses a weakly dominated action:

  Citizen 1 votes for A when state is a and votes for B when state is b

  Citizen 2 abstains

- Citizen 2, uninformed of state, suffers from *swing voter's curse*: if she votes, whenever her vote affects the outcome it affects it adversely
  - if she votes for A then her vote makes no difference if state is a (citizen 1 votes for A in any case) and induces a worse outcome if state is b (in which best candidate is B)
  - if she votes for B then her vote makes no difference if state is b (citizen 1 votes for B in any case)

Voting under imperfect information

Conclusion

- Game has only one equilibrium in which no player uses a weakly dominated action:

  Citizen 1 votes for *A* when state is *a* and votes for *B* when state is *b*

  Citzen 2 abstains

- Citizen 2, uninformed of state, suffers from *swing voter's curse*: if she votes, whenever her vote affects the outcome it affects it adversely
  - if she votes for *A* then her vote makes no difference if state is *a* (citizen 1 votes for *A* in any case) and induces a worse outcome if state is *b* (in which best candidate is *B*)
  - if she votes for *B* then her vote makes no difference if state is *b* (citizen 1 votes for *B* in any case) and induces a worse outcome if state is *a* (in which best candidate is *A*)

Voting under imperfect information

Conclusion

- If citizen 2 were the only voter, she would definitely vote for A

Voting under imperfect information

Conclusion

- If citizen 2 were the only voter, she would definitely vote for A
- But in the presence of citizen 1, she has to consider when her vote makes a difference

Voting under imperfect information

Conclusion

- If citizen 2 were the only voter, she would definitely vote for A
- But in the presence of citizen 1, she has to consider when her vote makes a difference
- Similar considerations affect analysis of other situations in which some players are imperfectly informed

Juries

- n jurors

Juries

- n jurors
- Before seeing evidence, each juror believes defendant is guilty with probability π

Juries

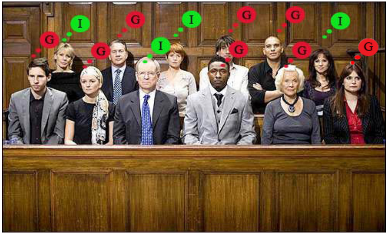
- n jurors
- Before seeing evidence, each juror believes defendant is guilty with probability π
- Assume $\pi = \frac{1}{2}$

Juries

- n jurors
- Before seeing evidence, each juror believes defendant is guilty with probability π
- Assume $\pi = \frac{1}{2}$
- All jurors share same goal: convict guilty person, acquit innocent one

Juries

- n jurors
- Before seeing evidence, each juror believes defendant is guilty with probability π
- Assume $\pi = \frac{1}{2}$
- All jurors share same goal: convict guilty person, acquit innocent one
- But jurors may interpret evidence differently



Juries

Information structure

- Model each juror as receiving a *signal* from the evidence

Juries

Information structure

- Model each juror as receiving a *signal* from the evidence
- Signals determined by random process

Juries

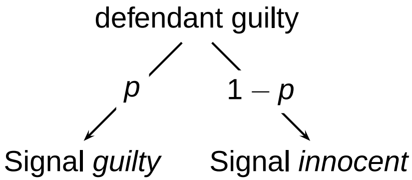
Information structure

- Model each juror as receiving a *signal* from the evidence
- Signals determined by random process
- If defendant guilty, more likely to get *guilty* signal; if defendant innocent, more likely to get *innocent* signal

Juries

Information structure

- Model each juror as receiving a *signal* from the evidence
- Signals determined by random process
- If defendant guilty, more likely to get *guilty* signal; if defendant innocent, more likely to get *innocent* signal

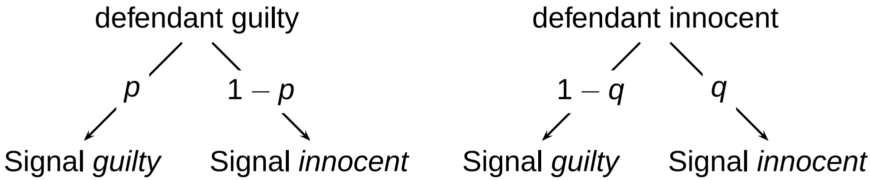


$$p > \frac{1}{2}$$

Juries

Information structure

- Model each juror as receiving a *signal* from the evidence
- Signals determined by random process
- If defendant guilty, more likely to get *guilty* signal; if defendant innocent, more likely to get *innocent* signal

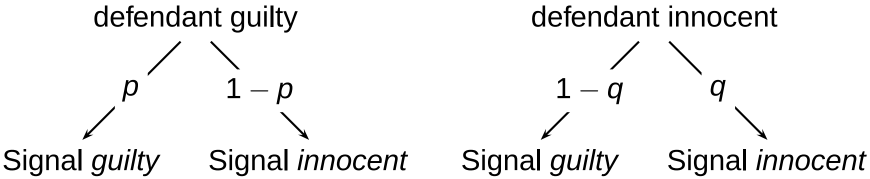


$p > \frac{1}{2}$ $q > \frac{1}{2}$

Juries

Information structure

- Model each juror as receiving a *signal* from the evidence
- Signals determined by random process
- If defendant guilty, more likely to get *guilty* signal; if defendant innocent, more likely to get *innocent* signal



$p > \frac{1}{2}$ $q > \frac{1}{2}$

- Given initial belief that defendant is guilty with probability $\frac{1}{2}$, juror who gets signal *guilty* believes defendant is more likely to be guilty than innocent

Juries

Decision-making

- After all jurors have received their signals, each juror votes to *acquit* or *convict*

Juries

Decision-making

- After all jurors have received their signals, each juror votes to *acquit* or *convict*
- Jurors do not share signals; they do not deliberate

Juries

Decision-making

- After all jurors have received their signals, each juror votes to *acquit* or *convict*
- Jurors do not share signals; they do not deliberate
- Defendant is convicted only if *all* jurors vote to *convict*

Juries

Strategic game with imperfect information

Players The n jurors

Juries

Strategic game with imperfect information

Players The n jurors

Information Each juror has two possible *types*: she may receive a *guilty* signal or an *innocent* signal

Juries

Strategic game with imperfect information

Players The n jurors

Information Each juror has two possible *types*: she may receive a *guilty* signal or an *innocent* signal

- if defendant is guilty, she receives a *guilty* signal with probability p ($> \frac{1}{2}$) and an *innocent* signal with probability $1 - p$

Juries

Strategic game with imperfect information

Players The n jurors

Information Each juror has two possible *types*: she may receive a *guilty* signal or an *innocent* signal

- if defendant is guilty, she receives a *guilty* signal with probability p ($> \frac{1}{2}$) and an *innocent* signal with probability $1 - p$
- if defendant is innocent, she receives a *guilty* signal with probability $1 - q$ ($< \frac{1}{2}$) and an *innocent* signal with probability q

Juries

Strategic game with imperfect information

Players The n jurors

Information Each juror has two possible *types*: she may receive a *guilty* signal or an *innocent* signal

- if defendant is guilty, she receives a *guilty* signal with probability p ($> \frac{1}{2}$) and an *innocent* signal with probability $1 - p$
- if defendant is innocent, she receives a *guilty* signal with probability $1 - q$ ($< \frac{1}{2}$) and an *innocent* signal with probability q

Before receiving a signal, each juror believes the defendant is guilty with probability $\frac{1}{2}$

Juries

Strategic game with imperfect information

Players The n jurors

Information Each juror has two possible *types*: she may receive a *guilty* signal or an *innocent* signal

- if defendant is guilty, she receives a *guilty* signal with probability p ($> \frac{1}{2}$) and an *innocent* signal with probability $1 - p$
- if defendant is innocent, she receives a *guilty* signal with probability $1 - q$ ($< \frac{1}{2}$) and an *innocent* signal with probability q

Before receiving a signal, each juror believes the defendant is guilty with probability $\frac{1}{2}$

Actions Each citizen has two actions, vote to *acquit* and vote to *convict*

Juries

Strategic game with imperfect information

Payoffs Each player's payoff is 0 if either

Juries

Strategic game with imperfect information

Payoffs Each player's payoff is 0 if either

- the defendant is innocent and at least one juror votes to acquit (so that the defendant is acquitted)

Juries

Strategic game with imperfect information

Payoffs Each player's payoff is 0 if either

- the defendant is innocent and at least one juror votes to acquit (so that the defendant is acquitted)
- the defendant is guilty and every juror votes to convict (so that the defendant is convicted)

Juries

Strategic game with imperfect information

Payoffs Each player's payoff is 0 if either

- the defendant is innocent and at least one juror votes to acquit (so that the defendant is acquitted)
- the defendant is guilty and every juror votes to convict (so that the defendant is convicted)

Each player's payoff is $-w$ if the defendant is innocent and every juror votes to convict (so that the defendant is wrongly convicted)

Juries

Strategic game with imperfect information

Payoffs Each player's payoff is 0 if either

- the defendant is innocent and at least one juror votes to acquit (so that the defendant is acquitted)
- the defendant is guilty and every juror votes to convict (so that the defendant is convicted)

Each player's payoff is $-w$ if the defendant is innocent and every juror votes to convict (so that the defendant is wrongly convicted)

Each player's payoff is $-e$ if the defendant is guilty and at least one juror votes to acquit (so that the defendant is erroneously acquitted)

Juries

Questions

- What are Nash equilibria of game?

Juries

Questions

- What are Nash equilibria of game?
  - Difficult to determine

Juries

Questions

- What are Nash equilibria of game?
  - Difficult to determine
- Easier question: Is it a Nash equilibrium for every juror who gets a *guilty* signal votes to convict and every juror who gets *innocent* signal votes to acquit?

Juries

Juror's decision

- Consider juror i

Juries

Juror's decision

- Consider juror i
- Suppose that every *other* juror votes according to her signal

Juries

Juror's decision

- Consider juror i
- Suppose that every *other* juror votes according to her signal

		other jurors' signals				
		all *innocent*	$n-2$ *innocent*	. . .	1 *innocent*	all *guilty*
juror i	Acquit					
	Convict					

Outcome (A = acquittal, C = conviction)

Juries

Juror's decision

- Consider juror i
- Su *ther* juror votes according to her sig

⇒ all other jurors vote to Acquit

		other jurors' signals				
		all *innocent*	$n-2$ *innocent*	. . .	1 *innocent*	all *guilty*
juror i	Acquit					
	Convict					

Outcome (A = acquittal, C = conviction)

Juries

Juror's decision

- Consider juror i
- Su... *ther* juror votes according to her sig...

⇒ all other jurors vote to Acquit

other jurors' signals

		all *innocent*	$n-2$ *innocent*	...	1 *innocent*	all *guilty*
juror i	Acquit	A				
	Convict					

Outcome (A = acquittal, C = conviction)

Juries

Juror’s decision

- Consider juror i
- Su ... *ther* juror votes according to her sig ...

$\Rightarrow$ all other jurors vote to Acquit

		other jurors’ signals				
		all *innocent*	$n-2$ *innocent*	...	1 *innocent*	all *guilty*
juror i	Acquit	A				
	Convict	A				

Outcome (A = acquittal, C = conviction)

Juries

Juror's decision

- Consider juror i
- Suppose that … otes according to her signal

$\Rightarrow$ all but one of the other jurors vote to Acquit

other jurors' signals

		all *innocent*	$n-2$ *innocent*	...	1 *innocent*	all *guilty*
juror i	Acquit	A				
	Convict	A				

Outcome (A = acquittal, C = conviction)

Juries

Juror's decision

- Consider juror i
- Suppose that … otes according to her signal

$\Rightarrow$ all but one of the other jurors vote to Acquit

other jurors' signals

		all *innocent*	$n-2$ *innocent*	…	1 *innocent*	all *guilty*
juror i	Acquit	A	A			
	Convict	A	A			

Outcome (A = acquittal, C = conviction)

Juries

Juror's decision

- Consider juror i
- Suppose that every *other* juror votes according to her signal

		other jurors' signals				
		all *innocent*	$n-2$ *innocent*	...	1 *innocent*	all *guilty*
juror i	Acquit	A	A	...		
	Convict	A	A	...		

Outcome (A = acquittal, C = conviction)

Juries

Juror's decision

- Consider juror i
- Suppose that every *other* juror votes [obscured by callout] signal

⇒ all but one of the other jurors vote to Convict

other jurors' signals

		all *innocent*	$n-2$ *innocent*	. . .	1 *innocent*	all *guilty*
juror i	Acquit	A	A	. . .		
	Convict	A	A	. . .		

Outcome (A = acquittal, C = conviction)

Juries

Juror's decision

- Consider juror i
- Suppose that every *other* juror votes signal

$\Rightarrow$ all but one of the other jurors vote to Convict

other jurors' signals

		all *innocent*	$n-2$ *innocent*	...	1 *innocent*	all *guilty*
juror i	Acquit	A	A	...	A	
	Convict	A	A	...	A	

Outcome (A = acquittal, C = conviction)

Juries

Juror's decision

- Consider juror i
- Suppose that every *other* juror votes according to her signal

⇒ all other jurors vote to Convict

other jurors' signals

		all *innocent*	$n-2$ *innocent*	...	1 *innocent*	all *guilty*
juror i	Acquit	A	A	...	A	
	Convict	A	A	...	A	

Outcome (A = acquittal, C = conviction)

Juries

Juror's decision

- Consider juror i
- Suppose that every *other* juror votes according to her signal

⇒ all other jurors vote to Convict

other jurors' signals

		all *innocent*	$n-2$ *innocent*	...	1 *innocent*	all *guilty*
juror i	Acquit	A	A	...	A	A
	Convict	A	A	...	A	

Outcome (A = acquittal, C = conviction)

Juries

Juror's decision

- Consider juror i
- Suppose that every *other* juror votes according to her signal

$\Rightarrow$ all other jurors vote to Convict

other jurors' signals

		all *innocent*	$n-2$ *innocent*	...	1 *innocent*	all *guilty*
juror i	Acquit	A	A	...	A	A
	Convict	A	A	...	A	C

Outcome (A = acquittal, C = conviction)

Juries: juror's decision

		other jurors' signals				
		all *innocent*	$n-2$ *innocent*	...	1 *innocent*	all *guilty*
juror i	Acquit	A	A	...	A	A
	Convict	A	A	...	A	C

Outcome (A = acquittal, C = conviction)

- How should juror i vote?

Juries: juror's decision

		other jurors' signals				
		all *innocent*	$n-2$ *innocent*	...	1 *innocent*	all *guilty*
juror i	Acquit	A	A	...	A	A
	Convict	A	A	...	A	C

Outcome (A = acquittal, C = conviction)

- How should juror i vote?
- Her action makes a difference to the outcome only if all the other jurors' signals are *guilty*

Juries: juror's decision

		other jurors' signals				
		all *innocent*	$n-2$ *innocent*	...	1 *innocent*	all *guilty*
juror i	Acquit	A	A	...	A	A
	Convict	A	A	...	A	C

Outcome (A = acquittal, C = conviction)

- Suppose her signal is *innocent*

Juries: juror's decision

		other jurors' signals				
		all *innocent*	$n-2$ *innocent*	...	1 *innocent*	all *guilty*
juror i	Acquit	A	A	...	A	A
	Convict	A	A	...	A	C

Outcome (A = acquittal, C = conviction)

- Suppose her signal is *innocent*
- Her expected payoff if she votes *Acquit* is

Juries: juror's decision

other jurors' signals

		all *innocent*	$n-2$ *innocent*	...	1 *innocent*	all *guilty*
juror i	Acquit	A	A	...	A	A
	Convict	A	A	...	A	C

Outcome (A = acquittal, C = conviction)

- Suppose her signal i
- Her expected payoff

juror i and all other jurors vote get *innocent* signals

$$\Pr(\text{defendant innocent} \mid n \textit{ innocent} \text{ signals}) \cdot 0$$

outcome is defendant is acquitted

Juries: juror's decision

other jurors' signals

		all *innocent*	$n-2$ *innocent*	...	1 *innocent*	all *guilty*
juror i	Acquit	A	A	...	A	A
	Convict	A	A	...	A	C

Outcome (A = acquittal, C = conviction)

- Suppose her signal is *innocent*
- Her expected payoff if she votes *Acquit* is

$$\Pr(\text{defendant innocent} \mid n \textit{ innocent} \text{ signals}) \cdot 0$$
$$+ \Pr(\text{defendant guilty} \mid n \textit{ innocent} \text{ signals}) \cdot -e$$

Juries: juror's decision

other jurors' signals

		all *innocent*	$n-2$ *innocent*	...	1 *innocent*	all *guilty*
juror i	Acquit	A	A	...	A	A
	Convict	A	A	...	A	C

Outcome (A = acquittal, C = conviction)

- Suppose her signal is *innocent*
- Her expected payoff if she votes *Acquit* is

$$\Pr(\text{defendant innocent} \mid n \textit{ innocent} \text{ signals}) \cdot 0$$
$$+ \Pr(\text{defendant guilty} \mid n \textit{ innocent} \text{ signals}) \cdot -e$$
$$+ \Pr(\text{defendant innocent} \mid n-1 \textit{ innocent} \text{ signals}) \cdot 0$$

Juries: juror's decision

other jurors' signals

		all *innocent*	$n-2$ *innocent*	...	1 *innocent*	all *guilty*
juror i	Acquit	A	A	...	A	A
	Convict	A	A	...	A	C

Outcome (A = acquittal, C = conviction)

- Suppose her signal is *innocent*
- Her expected payoff if she votes *Acquit* is

$$\Pr(\text{defendant innocent} \mid n \text{ } \textit{innocent} \text{ signals}) \cdot 0$$
$$+ \Pr(\text{defendant guilty} \mid n \text{ } \textit{innocent} \text{ signals}) \cdot -e$$
$$+ \Pr(\text{defendant innocent} \mid n-1 \text{ } \textit{innocent} \text{ signals}) \cdot 0$$
$$+ \Pr(\text{defendant guilty} \mid n-1 \text{ } \textit{innocent} \text{ signals}) \cdot -e$$

Juries: juror's decision

other jurors' signals

		all *innocent*	$n-2$ *innocent*	...	1 *innocent*	all *guilty*
juror i	Acquit	A	A	...	A	A
	Convict	A	A	...	A	C

Outcome (A = acquittal, C = conviction)

- Suppose her signal is *innocent*
- Her expected payoff if she votes *Acquit* is

$$\Pr(\text{defendant innocent} \mid n \textit{ innocent} \text{ signals}) \cdot 0$$
$$+ \Pr(\text{defendant guilty} \mid n \textit{ innocent} \text{ signals}) \cdot -e$$
$$+ \Pr(\text{defendant innocent} \mid n-1 \textit{ innocent} \text{ signals}) \cdot 0$$
$$+ \Pr(\text{defendant guilty} \mid n-1 \textit{ innocent} \text{ signals}) \cdot -e$$
$$+ \ldots$$

Juries: juror's decision

		other jurors' signals				
		all *innocent*	$n-2$ *innocent*	...	1 *innocent*	all *guilty*
juror i	Acquit	A	A	...	A	A
	Convict	A	A	...	A	C

Outcome (A = acquittal, C = conviction)

- Suppose her signal is *innocent*
- Her expected payoff if she votes *Acquit* is

$$\begin{aligned}
&\Pr(\text{defendant innocent} \mid n \textit{ innocent} \text{ signals}) \cdot 0 \\
&+ \Pr(\text{defendant guilty} \mid n \textit{ innocent} \text{ signals}) \cdot -e \\
&+ \Pr(\text{defendant innocent} \mid n-1 \textit{ innocent} \text{ signals}) \cdot 0 \\
&+ \Pr(\text{defendant guilty} \mid n-1 \textit{ innocent} \text{ signals}) \cdot -e \\
&+ \ldots \\
&+ \Pr(\text{defendant innocent} \mid 2 \textit{ innocent} \text{ signals}) \cdot 0
\end{aligned}$$

Juries: juror's decision

other jurors' signals

		all *innocent*	$n-2$ *innocent*	...	1 *innocent*	all *guilty*
juror i	Acquit	A	A	...	A	A
	Convict	A	A	...	A	C

Outcome (A = acquittal, C = conviction)

- Suppose her signal is *innocent*
- Her expected payoff if she votes *Acquit* is

$$
\begin{aligned}
&\Pr(\text{defendant innocent} \mid n \textit{ innocent} \text{ signals}) \cdot 0 \\
&+ \Pr(\text{defendant guilty} \mid n \textit{ innocent} \text{ signals}) \cdot -e \\
&+ \Pr(\text{defendant innocent} \mid n-1 \textit{ innocent} \text{ signals}) \cdot 0 \\
&+ \Pr(\text{defendant guilty} \mid n-1 \textit{ innocent} \text{ signals}) \cdot -e \\
&+ \dots \\
&+ \Pr(\text{defendant innocent} \mid 2 \textit{ innocent} \text{ signals}) \cdot 0 \\
&+ \Pr(\text{defendant guilty} \mid 2 \textit{ innocent} \text{ signals}) \cdot -e
\end{aligned}
$$

Juries: juror's decision

		other jurors' signals				
		all *innocent*	$n-2$ *innocent*	...	1 *innocent*	all *guilty*
juror i	Acquit	A	A	...	A	A
	Convict	A	A	...	A	C

Outcome (A = acquittal, C = conviction)

- Suppose her signal is *innocent*
- Her expected payoff if she votes *Acquit* is

$$\Pr(\text{defendant innocent} \mid n \textit{ innocent} \text{ signals}) \cdot 0$$
$$+ \Pr(\text{defendant guilty} \mid n \textit{ innocent} \text{ signals}) \cdot -e$$
$$+ \Pr(\text{defendant innocent} \mid n-1 \textit{ innocent} \text{ signals}) \cdot 0$$
$$+ \Pr(\text{defendant guilty} \mid n-1 \textit{ innocent} \text{ signals}) \cdot -e$$
$$+ \ldots$$
$$+ \Pr(\text{defendant innocent} \mid 2 \textit{ innocent} \text{ signals}) \cdot 0$$
$$+ \Pr(\text{defendant guilty} \mid 2 \textit{ innocent} \text{ signals}) \cdot -e$$
$$+ \Pr(\text{defendant innocent} \mid 1 \textit{ innocent} \text{ signal}) \cdot 0$$

Juries: juror's decision

		other jurors' signals				
		all *innocent*	$n-2$ *innocent*	...	1 *innocent*	all *guilty*
juror i	Acquit	A	A	...	A	A
	Convict	A	A	...	A	C

Outcome (A = acquittal, C = conviction)

- Suppose her signal is *innocent*
- Her expected payoff if she votes *Acquit* is

$$
\begin{aligned}
&\Pr(\text{defendant innocent} \mid n \textit{ innocent} \text{ signals}) \cdot 0 \\
&+ \Pr(\text{defendant guilty} \mid n \textit{ innocent} \text{ signals}) \cdot -e \\
&+ \Pr(\text{defendant innocent} \mid n-1 \textit{ innocent} \text{ signals}) \cdot 0 \\
&+ \Pr(\text{defendant guilty} \mid n-1 \textit{ innocent} \text{ signals}) \cdot -e \\
&+ \dots \\
&+ \Pr(\text{defendant innocent} \mid 2 \textit{ innocent} \text{ signals}) \cdot 0 \\
&+ \Pr(\text{defendant guilty} \mid 2 \textit{ innocent} \text{ signals}) \cdot -e \\
&+ \Pr(\text{defendant innocent} \mid 1 \textit{ innocent} \text{ signal}) \cdot 0 \\
&+ \Pr(\text{defendant guilty} \mid 1 \textit{ innocent} \text{ signal}) \cdot -e
\end{aligned}
$$

Juries: juror's decision

other jurors' signals

		all *innocent*	$n-2$ *innocent*	...	1 *innocent*	all *guilty*
juror i	Acquit	A	A	...	A	A
	Convict	A	A	...	A	C

Outcome (A = acquittal, C = conviction)

- Suppose her signal is *innocent*
- Her expected payoff if she votes *Convict* is almost the same:

$$
\begin{aligned}
&\Pr(\text{defendant innocent} \mid n \textit{ innocent} \text{ signals}) \cdot 0 \\
&+ \Pr(\text{defendant guilty} \mid n \textit{ innocent} \text{ signals}) \cdot -e \\
&+ \Pr(\text{defendant innocent} \mid n-1 \textit{ innocent} \text{ signals}) \cdot 0 \\
&+ \Pr(\text{defendant guilty} \mid n-1 \textit{ innocent} \text{ signals}) \cdot -e \\
&+ \dots \\
&+ \Pr(\text{defendant innocent} \mid 2 \textit{ innocent} \text{ signals}) \cdot 0 \\
&+ \Pr(\text{defendant guilty} \mid 2 \textit{ innocent} \text{ signals}) \cdot -e \\
&+ \Pr(\text{defendant innocent} \mid 1 \textit{ innocent} \text{ signal}) \cdot -w \\
&+ \Pr(\text{defendant guilty} \mid 1 \textit{ innocent} \text{ signal}) \cdot 0
\end{aligned}
$$

Juries: juror's decision

		other jurors' signals				
		all *innocent*	$n-2$ *innocent*	...	1 *innocent*	all *guilty*
juror i	Acquit	A	A	...	A	A
	Convict	A	A	...	A	C

Outcome (A = acquittal, C = conviction)

- Suppose her signal is *innocent*
- Her expected payoff if she votes *Convict* is almost the same:

$$
\begin{aligned}
&\Pr(\text{defendant innocent} \mid n \textit{ innocent} \text{ signals}) \cdot 0\\
&+ \Pr(\text{defendant guilty} \mid n \textit{ innocent} \text{ signals}) \cdot -e\\
&+ \Pr(\text{defendant innocent} \mid n-1 \textit{ innocent} \text{ signals}) \cdot 0\\
&+ \Pr(\text{defendant guilty} \mid n-1 \textit{ innocent} \text{ signals}) \cdot -e\\
&+ \ldots\\
&+ \Pr(\text{defendant innocent} \mid 2 \textit{ innocent} \text{ signals}) \cdot 0\\
&+ \Pr(\text{defendant guilty} \mid 2 \textit{ innocent} \text{ signals}) \cdot -e\\
&+ \Pr(\text{defendant innocent} \mid 1 \textit{ innocent} \text{ signal}) \cdot -w\\
&+ \Pr(\text{defendant guilty} \mid 1 \textit{ innocent} \text{ signal}) \cdot 0
\end{aligned}
$$

Juries: juror's decision

other jurors' signals

		all *innocent*	$n-2$ *innocent*	...	1 *innocent*	all *guilty*
juror i	Acquit	A	A	...	A	A
	Convict	A	A	...	A	C

Outcome (A = acquittal, C = conviction)

- Suppose her signal is *innocent*
- Then her expected payoff to voting *Convict* exceeds her expected payoff to voting *Acquit* if

$$\Pr(\text{defendant innocent} \mid 1 \textit{ innocent } \text{signal}) \cdot -w$$
$$+ \Pr(\text{defendant guilty} \mid 1 \textit{ innocent } \text{signal}) \cdot 0$$
$$> \Pr(\text{defendant innocent} \mid 1 \textit{ innocent } \text{signal}) \cdot 0$$
$$+ \Pr(\text{defendant guilty} \mid 1 \textit{ innocent } \text{signal}) \cdot -e$$

Juries: juror's decision

		other jurors' signals				
		all *innocent*	$n-2$ *innocent*	...	1 *innocent*	all *guilty*
juror i	Acquit	A	A	...	A	A
	Convict	A	A	...	A	C

Outcome (A = acquittal, C = conviction)

- Suppose her signal is *innocent*
- Then her expected payoff to voting *Convict* exceeds her expected payoff to voting *Acquit* if

$$\begin{aligned}&\Pr(\text{defendant innocent} \mid 1 \textit{ innocent} \text{ signal}) \cdot -w \\ &+ \Pr(\text{defendant guilty} \mid 1 \textit{ innocent} \text{ signal}) \cdot 0 \\ &> \Pr(\text{defendant innocent} \mid 1 \textit{ innocent} \text{ signal}) \cdot 0 \\ &+ \Pr(\text{defendant guilty} \mid 1 \textit{ innocent} \text{ signal}) \cdot -e\end{aligned}$$

or

$$\Pr(\text{defendant innocent} \mid 1 \textit{ innocent} \text{ signal}) \cdot -w > \Pr(\text{defendant guilty} \mid 1 \textit{ innocent} \text{ signal}) \cdot -e$$

Juries: juror's decision

		other jurors' signals				
		all *innocent*	$n-2$ *innocent*	...	1 *innocent*	all *guilty*
juror i	Acquit	A	A	...	A	A
	Convict	A	A	...	A	C

Outcome (A = acquittal, C = conviction)

- Suppose her signal is *innocent*
- Then her expected payoff to voting *Convict* exceeds her expected payoff to voting *Acquit* if

$$\Pr(\text{defendant innocent} \mid 1 \textit{ innocent} \text{ signal}) \cdot -w$$
$$> \Pr(\text{defendant guilty} \mid 1 \textit{ innocent} \text{ signal}) \cdot -e$$

Juries: juror's decision

other jurors' signals

		all *innocent*	$n-2$ *innocent*	...	1 *innocent*	all *guilty*
juror i	Acquit	A	A	...	A	A
	Convict	A	A	...	A	C

Outcome (A = acquittal, C = conviction)

- Suppose her signal is *innocent*
- Then her expected payoff to voting *Convict* exceeds her expected payoff to voting *Acquit* if

$$\Pr(\text{defendant innocent} \mid 1 \textit{ innocent} \text{ signal}) \cdot -w > \Pr(\text{defendant guilty} \mid 1 \textit{ innocent} \text{ signal}) \cdot -e$$

$\Leftrightarrow$

$$\Pr(\text{defendant innocent} \mid 1 \textit{ innocent} \text{ signal}) \cdot -w > (1 - \Pr(\text{defendant innocent} \mid 1 \textit{ innocent} \text{ signal})) \cdot -e$$

Juries: juror's decision

		other jurors' signals				
		all *innocent*	$n-2$ *innocent*	...	1 *innocent*	all *guilty*
juror i	Acquit	A	A	...	A	A
	Convict	A	A	...	A	C

Outcome (A = acquittal, C = conviction)

- Suppose her signal is *innocent*
- Then her expected payoff to voting *Convict* exceeds her expected payoff to voting *Acquit* if

$$\Pr(\text{defendant innocent} \mid 1 \textit{ innocent} \text{ signal}) \cdot -w > \Pr(\text{defendant guilty} \mid 1 \textit{ innocent} \text{ signal}) \cdot -e$$

$$\Leftrightarrow$$

$$\Pr(\text{defendant innocent} \mid 1 \textit{ innocent} \text{ signal}) \cdot -w > (1 - \Pr(\text{defendant innocent} \mid 1 \textit{ innocent} \text{ signal})) \cdot -e$$

$$\Leftrightarrow$$

$$\frac{w}{e} < \frac{1 - \Pr(\text{defendant innocent} \mid 1 \textit{ innocent} \text{ signal})}{\Pr(\text{defendant innocent} \mid 1 \textit{ innocent} \text{ signal})}$$

Juries: juror's decision

		other jurors' signals				
		all *innocent*	$n-2$ *innocent*	...	1 *innocent*	all *guilty*
juror i	Acquit	A	A	...	A	A
	Convict	A	A	...	A	C

Outcome (A = acquittal, C = conviction)

- Suppose her signal is *innocent*
- Then her expected payoff to voting *Convict* exceeds her expected payoff to voting *Acquit* if

$$-w\,\Pr(\text{defendant innocent} \mid 1\ \textit{innocent}\ \text{signal}) > -e(1 - \Pr(\text{defendant innocent} \mid 1\ \textit{innocent}\ \text{signal}))$$

Juries: juror's decision

other jurors' signals

		all *innocent*	$n-2$ *innocent*	...	1 *innocent*	all *guilty*
juror i	Acquit	A	A	...	A	A
	Convict	A	A	...	A	C

Outcome (A = acquittal, C = conviction)

- Suppose her signal is *innocent*
- Then her expected payoff to voting *Convict* exceeds her expected payoff to voting *Acquit* if

$$-w\,\Pr(\text{defendant innocent} \mid 1\ \textit{innocent}\ \text{signal})$$
$$> -e(1 - \Pr(\text{defendant innocent} \mid 1\ \textit{innocent}\ \text{signal}))$$
$$\Leftrightarrow$$
$$\frac{w}{e} < \frac{1 - \Pr(\text{defendant innocent} \mid 1\ \textit{innocent}\ \text{signal})}{\Pr(\text{defendant innocent} \mid 1\ \textit{innocent}\ \text{signal})}$$

Juries: juror's decision

other jurors' signals

		all *innocent*	$n-2$ *innocent*	...	1 *innocent*	all *guilty*
juror i	Acquit	A	A	...	A	A
	Convict	A	A	...	A	C

Outcome (A = acquittal, C = conviction)

- Suppose her signal is *innocent*
- Then her expected payoff to voting *Convict* exceeds her expected payoff to voting *Acquit* if

$$\frac{w}{e} < \frac{1 - \Pr(\text{defendant innocent} \mid 1 \textit{ innocent} \text{ signal})}{\Pr(\text{defendant innocent} \mid 1 \textit{ innocent} \text{ signal})}$$

Juries: juror's decision

other jurors' signals

		all *innocent*	$n-2$ *innocent*	...	1 *innocent*	all *guilty*
juror i	Acquit	A	A	...	A	A
	Convict	A	A	...	A	C

Outcome (A = acquittal, C = conviction)

- Suppose her signal is *innocent*
- Then her expected payoff to voting *Convict* exceeds her expected payoff to voting *Acquit* if

$$\frac{w}{e} < \frac{1 - \Pr(\text{defendant innocent} \mid 1\ \textit{innocent}\ \text{signal})}{\Pr(\text{defendant innocent} \mid 1\ \textit{innocent}\ \text{signal})}$$

- 1 *innocent* signal $\Rightarrow$ 11 *guilty* signals $\Rightarrow$ Pr(defendant innocent $\mid$ 1 *innocent* signal) is close to zero

Juries: juror's decision

other jurors' signals

		all *innocent*	$n-2$ *innocent*	...	1 *innocent*	all *guilty*
juror i	Acquit	A	A	...	A	A
	Convict	A	A	...	A	C

Outcome (A = acquittal, C = conviction)

- Suppose her signal is *innocent*
- Then her expected payoff to voting *Convict* exceeds her expected payoff to voting *Acquit* if

$$\frac{w}{e} < \frac{1 - \Pr(\text{defendant innocent} \mid 1 \textit{ innocent} \text{ signal})}{\Pr(\text{defendant innocent} \mid 1 \textit{ innocent} \text{ signal})}$$

- 1 *innocent* signal $\Rightarrow$ 11 *guilty* signals $\Rightarrow$ Pr(defendant innocent | 1 *innocent* signal) is close to zero
  - For example, if Pr(*guilty* signal | defendant guilty) = Pr(*innocent* signal | defendant innocent) = 0.8 then

$$\Pr(\text{defendant innocent} \mid 1 \textit{ innocent} \text{ sig.}) = \frac{0.8(0.2)^{11}}{0.8(0.2)^{11} + 0.2(0.8)^{11}} \approx 0.00000095$$

Juries: juror's decision

		other jurors' signals				
		all *innocent*	$n-2$ *innocent*	...	1 *innocent*	all *guilty*
juror i	Acquit	A	A	...	A	A
	Convict	A	A	...	A	C

Outcome (A = acquittal, C = conviction)

- Suppose her signal is *innocent*
- Then her expected payoff to voting *Convict* exceeds her expected payoff to voting *Acquit* if

$$\frac{w}{e} < \frac{1 - \Pr(\text{defendant innocent} \mid 1\ \textit{innocent}\ \text{signal})}{\Pr(\text{defendant innocent} \mid 1\ \textit{innocent}\ \text{signal})}$$

Juries: juror's decision

		other jurors' signals				
		all *innocent*	$n-2$ *innocent*	...	1 *innocent*	all *guilty*
juror i	Acquit	A	A	...	A	A
	Convict	A	A	...	A	C

Outcome (A = acquittal, C = conviction)

- Suppose her signal is *innocent*
- Then her expected payoff to voting *Convict* exceeds her expected payoff to voting *Acquit* if

$$\frac{w}{e} < \frac{1 - \Pr(\text{defendant innocent} \mid 1 \textit{ innocent} \text{ signal})}{\Pr(\text{defendant innocent} \mid 1 \textit{ innocent} \text{ signal})}$$

- So her expected payoff to voting *Convict* exceeds her expected payoff to voting *Acquit* unless w is very large compared with e

Juries: juror's decision

		other jurors' signals				
		all *innocent*	$n-2$ *innocent*	. . .	1 *innocent*	all *guilty*
juror i	Acquit	A	A	. . .	A	A
	Convict	A	A	. . .	A	C

Outcome (A = acquittal, C = conviction)

- Suppose her signal is *innocent*
- Then her expected payoff to voting *Convict* exceeds her expected payoff to voting *Acquit* if

$$\frac{w}{e} < \frac{1 - \Pr(\text{defendant innocent} \mid 1\ \textit{innocent}\ \text{signal})}{\Pr(\text{defendant innocent} \mid 1\ \textit{innocent}\ \text{signal})}$$

- So her expected payoff to voting *Convict* exceeds her expected payoff to voting *Acquit* unless w is very large compared with e
- So *if* every other juror votes according to her signal, a juror whose signal is *innocent* should vote *Convict* unless w is very large compared with e

Juries: juror's decision

other jurors' signals

		all *innocent*	$n-2$ *innocent*	...	1 *innocent*	all *guilty*
juror i	Acquit	A	A	...	A	A
	Convict	A	A	...	A	C

Outcome (A = acquittal, C = conviction)

- If her signal is *guilty*, a similar calculation leads to the conclusion that sher expected payoff to voting *Convict* exceeds her expected payoff to voting *Acquit* if

$$\frac{w}{e} < \frac{1 - \Pr(\text{defendant innocent} \mid 0 \textit{ innocent} \text{ signals})}{\Pr(\text{defendant innocent} \mid 0 \textit{ innocent} \text{ signals})}$$

which is even closer to 0

Juries: juror's decision

other jurors' signals

		all *innocent*	$n-2$ *innocent*	...	1 *innocent*	all *guilty*
juror i	Acquit	A	A	...	A	A
	Convict	A	A	...	A	C

Outcome (A = acquittal, C = conviction)

- If her signal is *guilty*, a similar calculation leads to the conclusion that sher expected payoff to voting *Convict* exceeds her expected payoff to voting *Acquit* if

$$\frac{w}{e} < \frac{1 - \Pr(\text{defendant innocent} \mid 0 \textit{ innocent} \text{ signals})}{\Pr(\text{defendant innocent} \mid 0 \textit{ innocent} \text{ signals})}$$

  which is even closer to 0

- So *if* all other jurors vote according to their signal, the remaining a juror should vote *Convict* regardless of her signal

Juries

Conclusion

- If all other jurors vote according to their signals, the remaining juror should vote for *conviction* regardless of her signal

Juries

Conclusion

- If all other jurors vote according to their signals, the remaining juror should vote for *conviction* regardless of her signal

Juries

Conclusion

- If all other jurors vote according to their signals, the remaining juror should vote for *conviction* regardless of her signal
- So there is no equilibrium in which all jurors vote according to their signals

Juries

Conclusion

- If all other jurors vote according to their signals, the remaining juror should vote for *conviction* regardless of her signal
- So there is no equilibrium in which all jurors vote according to their signals
  - Note that we have not determined what *is* an equilibrium

Juries

Conclusion

- If all other jurors vote according to their signals, the remaining juror should vote for *conviction* regardless of her signal
- So there is no equilibrium in which all jurors vote according to their signals
  - Note that we have not determined what *is* an equilibrium
  - Under some conditions, the game has a mixed strategy equilibrium in which every juror who gets a *guilty* signal votes for conviction and every juror who gets an *innocent* signal votes for conviction with positive probability and acquittal with positive probability

Juries

Conclusion

- If all other jurors vote according to their signals, the remaining juror should vote for *conviction* regardless of her signal
- So there is no equilibrium in which all jurors vote according to their signals
  - Note that we have not determined what *is* an equilibrium
  - Under some conditions, the game has a mixed strategy equilibrium in which every juror who gets a *guilty* signal votes for conviction and every juror who gets an *innocent* signal votes for conviction with positive probability and acquittal with positive probability
  - An interesting feature of this equilibrium is that the probability that an innocent defendant is convicted *increases* with the size of the jury