Economics 316

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Solutions to Problem Set 6

1. The actions T and B are strictly dominated by M for player 1. No other action of either player is strictly dominated.

2. For any output q_2 of firm 2, firm 1's profit if its output is q_1 is $q_1(\alpha - q_1 - q_2 - c)$. Thus firm 1's optimal output is $\frac{1}{2}(\alpha - c - q_2)$. This output is equal to q_1' when $q_1' = \frac{1}{2}(\alpha - c - q_2)$, or $q_2 = \alpha - c - 2q_1'$.

3. (a) Neither action of player 2 is strictly dominated by the other action. The action M of player 1 is not strictly dominated by any mixed strategy because the payoff it yields when player 2 chooses L, namely 3, is larger than the payoff either of player 1's other actions yields in this case. Similarly, the action B of player 1 is not strictly dominated by any mixed strategy because the payoff it yields when player 2 chooses R, namely 5, is larger than the payoff either of player 1's other actions yields in this case. So the only action of either player that may be strictly dominated is T.

   The action T of player 1 is strictly dominated by the mixed strategy $(0, p, 1-p)$ if we have

   $$3p > 1$$
   $$p + 5(1-p) > 3,$$

   or if $p > \frac{1}{3}$ and $p < \frac{1}{2}$.

   (Another way to find a mixed strategy that strictly dominates T is to notice that T is not strictly dominated by the mixed strategy $(0, \frac{1}{2}, \frac{1}{2})$ because the expected payoff of player 1 if she uses this strategy and player 2 chooses R is 3, which is equal (not greater than) her payoff if she chooses T and player 2 chooses R. However, the expected payoff of player 1 if she uses this strategy and player 2 chooses L is 1.5, greater than her payoff if she chooses T and player 2 chooses L. So if player 1 puts a little more weight on B her expected payoff when player 2 chooses L is still greater than her expected payoff when she chooses T and also her expected payoff when player 2 chooses R is greater than her expected payoff when she chooses T. Thus the resulting mixed strategy strictly dominates T.)

(b) When T is eliminated, the resulting 2×2 game has three mixed strategy Nash equilibria, $((1,0),(1,0))$, $((0,1),(0,1))$, and $((\frac{2}{3},\frac{1}{3}),(\frac{4}{7},\frac{3}{7}))$. Thus the whole game has three mixed strategy Nash equilibria, $((0,1,0),(1,0))$, $((0,0,1),(0,1))$, and $((0,\frac{2}{3},\frac{1}{3}),(\frac{4}{7},\frac{3}{7}))$.

4. The sequence of eliminations is as follows.

   1. Y is strictly dominated by Z.
   2. A is strictly dominated by B (and by D).
   3. W is strictly dominated by Z.
   4. C is strictly dominated by D.

   No more actions are strictly dominated. Four action pairs remain: (X,B), (X,D), (Z,B), and (Z,D).

5. Yes. The following game is an example.

	A	B
X	3,3	2,1
Y	5,1	2,2

   Neither action of either player is strictly dominated, so all four action profiles survive iterated elimination of strictly dominated actions.

   The game has a unique Nash equilibrium, (Y,B), and both players' payoffs are lower in this equilibrium than they are in the outcome (X,A).

6. If the action profile a is eliminated at some stage, then for some player, say i, there is an action, say a_i', different from a_i, such that $u_i(a_i',a_{-i}) > u_i(a_i,a_{-i})$. But that means that a_i is not a best response to a_{-i}, and hence a is not a Nash equilibrium.

7. The action pairs that survive are those in which each firm's location is one of the two central locations.

8. The only Nash equilibrium of the game in Figure 1 is (T,L). The action T is weakly dominated by M and the action L is weakly dominated by

C. (There are of course many other games that satisfy the conditions.)

	L	C	R
T	1, 1	0, 1	0, 0
M	1, 0	2, 1	1, 2
B	0, 0	1, 1	2, 0

Figure 1. A game with a unique Nash equilibrium, in which both players' equilibrium actions are weakly dominated. (The unique Nash equilibrium is (T, L).)

9. No position is strictly dominated (see Figure 2). The positions 0 and 2 are weakly dominated (by position 1).

   The game has a unique Nash equilibrium, in which all three parties locate at position 1. (Any party that deviates to 0 or 1 loses. Note that a party cannot deviate slightly from 1—the only positions a party is allowed to take are 0, 1, and 2.)

	0, 0	0, 1	0, 2	1, 2	1, 1	2, 2
0	3-tie	lose	lose	lose	lose	win
1	win	2-tie	win	2-tie	3-tie	win
2	win	lose	lose	lose	lose	3-tie

Figure 2. The outcomes for a party as a function of the other parties' positions in Problem 8.

10. Let $p_1 > c$ with $D(p_1) > 0$. For p_1 to be weakly dominated, there must be another price p_1' that yields firm 1 a payoff at least as high as the payoff to p_1 regardless of firm 2's price, and higher than the payoff to p_1 for at least one value of firm 2's price.

    If $p_1 > p^m$ (the monopoly price), then p^m is such a price: if $p_2 < p_1$ then $\pi_1(p^m, p_2) \geq \pi_1(p_1, p_2) = 0$ and if $p_2 \geq p_1$ then $\pi_1(p^m, p_2) > \pi_1(p_1, p_2)$. Thus any price greater than p^m is weakly dominated (by p^m).

    If $p_1 \leq p^m$, then there is no such price. If $p_1' < p_1$ then $\pi_1(p_1', p_2) < \pi_1(p_1, p_2)$ if $p_2 > p_1$, and if $p_1' > p_1$ then $0 = \pi_1(p_1', p_2) < \pi_1(p_1, p_2)$ if $p_1 < p_2 < p_1'$. Thus no price at most equal to p^m is weakly dominated.