Economics 316

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Solutions to problems for Tutorial 6

1. (a) No action of either player is strictly dominated, by the following argument.

   If $3 \leq a_i \leq 99$ then player i's payoffs to $(a_i, 2)$ and $(a_i + 1, 2)$ are both 0, so no action a_i with $3 \leq a_i \leq 100$ is strictly dominated by any action in the same range.

   Player i's payoff to $(2, 2)$ is 2, so no action a_i with $3 \leq a_i \leq 100$ strictly dominates 2.

   Finally, the action 2 does not strictly dominate any other action a_i because player i's payoff to $(2, 100)$ is 4 whereas her payoff to $(a_i, 100)$ is $a_i + 2 > 4$ if $a_i \leq 99$ and 100 if $a_i = 100$.

   (b) In the modified game, the action 100 for each player is strictly dominated by the action 99. If $2 \leq a_j \leq 98$ then $u_i(99, a_j) = a_j - 1 - 0.01(99 - a_j) = 1.01a_j - 1.99$ and $u_i(100, a_j) = a_j - 1 - 0.01(100 - a_j) = 1.01a_j - 2$. Also, $u_i(100, 99) = 97.9$ and $u_i(99, 99) = 99$; and $u_i(100, 100) = 100$ and $u_i(99, 100) = 101$.

2. (a) The action T of player 1 is strictly dominated by the mixed strategy $(0, 0.7, 0.3)$. (Notice that it is not strictly dominated by the mixed strategy $(0, \frac{1}{2}, \frac{1}{2})$, because the expected payoff of player 1 if she uses this strategy and player 2 chooses L is 1.5, which is less than her payoff if she chooses T and player 2 chooses L. However, the expected payoff of player 1 if she uses this strategy and player 2 chooses R is 3.5, greater than her payoff if she chooses T and player 2 chooses R. So if player 1 puts a little more weight on T her expected payoff when player 2 chooses R is still greater than her expected payoff when she chooses T and also her expected payoff when player 2 chooses L is greater than her expected payoff when she chooses T. Increasing the weight on T to 0.7 results in a mixed strategy that strictly dominates T.)

(b) Because T is strictly dominated, it is used with probability 0 in every mixed strategy Nash equilibrium. So we can find all the mixed strategy Nash equilibria of the game by finding the mixed strategy Nash equilibria of the 2×2 game that results when T is eliminated. This game has three mixed strategy Nash equilibria: $((1,0),(1,0))$, $((0,1),(0,1))$, and $((\frac{2}{3},\frac{1}{3}),(\frac{7}{10},\frac{3}{10}))$. (Draw a picture of the best response functions.) Thus the whole game has three mixed strategy Nash equilibria, $((0,1,0),(1,0))$, $((0,0,1),(0,1))$, and $((0,\frac{2}{3},\frac{1}{3}),(\frac{7}{10},\frac{3}{10}))$.

3. No, the price $c+1$ is not weakly dominated. If the other firm's price is $c+2$, any price above $c+2$ yields a profit of zero, the price $c+2$ yields the profit $\frac{1}{2} \cdot 2D(c+2) = D(c+2)$, the price $c+1$ yields the profit $D(c+1)$, the price c yields a profit of zero, and any lower price yields a negative profit. Thus no other price yields a profit as high as the profit from $c+1$ if the other firm's price is $c+2$, and hence no price weakly dominates the price $c+1$.