ECO316: Applied game theory

Lecture 6

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University of Toronto

2017.10.12

Midterm

Date	Friday, October 20, 1:10pm to 3pm
Location	EX 100
Coverage	All the material in weeks 1–6
Weight	40%
Sample	Previous midterm exams, with solutions, are available on the course website

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Implications of rationality

Every player

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⇕

action is best response to belief about other players' actions

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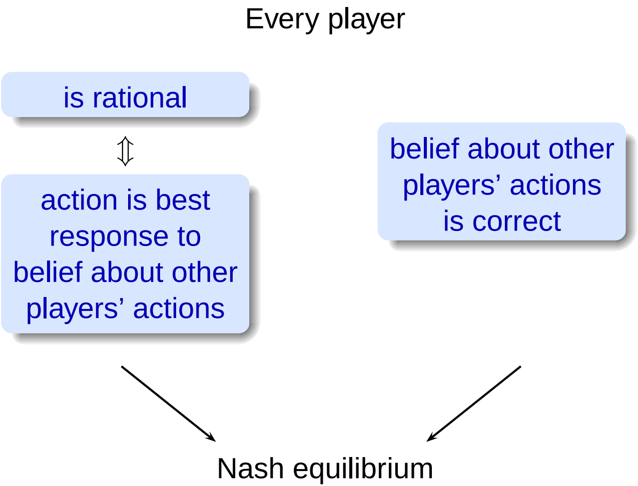
Implications of rationality



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Nash equilibrium

Implications of rationality

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Which actions are best responses to some belief?

?

Beliefs

Definition

A belief of a player in a strategic game about the other players' actions is a probability distribution over the set of lists of the other players' actions.

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Example

	H	L
H	$2, 2$	$0, 3$
L	$3, 0$	$1, 1$

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	q	$1-q$
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- A belief of player 1 is a probability distribution over $\{H, L\}$

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Example

	q	$1-q$
	H	L
H	$2, 2$	$0, 3$
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- A belief of player 1 is a probability distribution over $\{H, L\}$
- The probability the belief assigns to each action is the probability with which player 1 believes player 2 will choose that action

Beliefs

Example

Players	Three individuals
Actions	Each player can choose *A* or *B*
Preferences	Anything

Beliefs

Example

Players Three individuals

Actions Each player can choose A or B

Preferences Anything

- A belief of player 1 is a probability distribution over the set $\{(A, A), (A, B), (B, A), (B, B)\}$ of the pairs of actions of the other two players

Beliefs

Example

Players Three individuals

Actions Each player can choose A or B

Preferences Anything

- A belief of player 1 is a probability distribution over the set $\{(A, A), (A, B), (B, A), (B, B)\}$ of the pairs of actions of the other two players
- For example, one belief assigns
  - probability 0.25 to (A, A)
  - probability 0.25 to (A, B)
  - probability 0.5 to (B, A)
  - probability 0 to (B, B)

Best responses to a belief

In a strategic game, denote player *i*'s expected payoff when she uses the mixed strategy α_i and the other players' actions are a_{-i} by

$$U_i(\alpha_i, a_{-i})$$

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(p)	H	$2, 2$	$0, 3$
$(1-p)$	L	$3, 0$	$1, 1$

Best responses to a belief

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Example

		H	L
(p)	H	$2, 2$	$0, 3$
$(1-p)$	L	$3, 0$	$1, 1$

$$U_1((p, 1-p), H) = 2p + 3(1-p)$$

Best responses to a belief

In a strategic game, denote player *i*'s expected payoff when she uses the mixed strategy α_i and the other players' actions are a_{-i} by

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Example

		H	L
(p)	H	$2, 2$	$0, 3$
$(1-p)$	L	$3, 0$	$1, 1$

$$U_1((p, 1-p), H) = 2p + 3(1-p)$$

$$U_1((p, 1-p), L) = 1 - p$$

Best responses to a belief

Definition

The mixed strategy α_i of player i is a best response to i's belief β_i if α_i maximizes player i's expected payoff

$$\sum_{a_{-i} \in A_{-i}} \beta_i(a_{-i}) U_i(\alpha_i, a_{-i}).$$

That is,

$$\sum_{a_{-i} \in A_{-i}} \beta_i(a_{-i}) U_i(\alpha_i, a_{-i}) \geq \sum_{a_{-i} \in A_{-i}} \beta_i(a_{-i}) U_i(\alpha_i', a_{-i})$$

for every other mixed strategy α_i' of player i.

Rationality

Definition

A player is rational if her mixed strategy is a best response to *some* belief.

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A player is rational if her mixed strategy is a best response to *some* belief.

- How can we determine whether a mixed strategy is a best response to some belief?

Rationality

Definition

A player is rational if her mixed strategy is a best response to *some* belief.

- How can we determine whether a mixed strategy is a best response to some belief?
- Let's start with a simpler question: when is a player's *action* a best response to a list of *actions* of the other players?

Strictly dominated actions

Example

Other players’ actions

i’s actions

	a'_{-i}	a''_{-i}	a'''_{-i}
T	1	4	0
B	2	6	3

Player *i*’s payoffs

Strictly dominated actions

In general, other players have many possible lists of actions; in this example there are only three

Example

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Player *i*'s payoffs

- Is T a best response to any actions of the other players?
- No! Whatever actions *i* the other players take, B yields higher payoff than does T

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Player *i*'s payoffs

- Is *T* a best response to any actions of the other players?
- No! Whatever actions *i* the other players take, *B* yields higher payoff than does *T*

Strictly dominated actions

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Other players’ actions

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	a'_{-i}	a''_{-i}	a'''_{-i}
T	1	4	0
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Player *i*’s payoffs

- Is T a best response to any actions of the other players?
- No! Whatever actions i the other players take, B yields higher payoff than does T
- We say B *strictly dominates* T, or T is *strictly dominated* by B

Strictly dominated actions

Definition

Player i's action a_i'' strictly dominates her action a_i' if

a_i'' is better for i than a_i' *whatever* the other players do

Strictly dominated actions

Definition

Player i's action a_i'' strictly dominates her action a_i' if

> a_i'' is better for i than a_i' *whatever* the other players do

or, precisely,

$u_i(a_i'', a_{-i}) > u_i(a_i', a_{-i})$ for *every* list a_{-i} of other players' actions

Strict domination

- If a_i'' strictly dominates a_i' then a_i' is not best response to *any* list of the other players' actions

Strict domination

- If a''_i strictly dominates a'_i then a'_i is not best response to *any* list of the other players' actions
- Example

Other players' actions

i's actions

	a'_{-i}	a''_{-i}	a'''_{-i}
T	1	4	0
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Player *i*'s payoffs

T is not a best response to any list of other players' actions

Strict domination

- In fact, if a_i'' strictly dominates a_i' then a_i' is not a best response to any *probability distribution* over lists of the other players' actions — that is, to any belief

Strict domination

- In fact, if a''_i strictly dominates a'_i then a'_i is not a best response to any *probability distribution* over lists of the other players' actions — that is, to any belief
- Example

Other players' actions

		q_1	q_2	q_3
		a'_{-i}	a''_{-i}	a'''_{-i}
i's actions	T	1	4	0
	B	2	6	3

Player *i*'s payoffs

T is not a best response to any belief (q_1, q_2, q_3): for every (q_1, q_2, q_3),

$$2 \cdot q_1 + 6 \cdot q_2 + 3 \cdot q_3 > 1 \cdot q_1 + 4 \cdot q_2 + 0 \cdot q_3$$

Strict domination and best responses

Proposition

An action of player *i* in a strategic game that is strictly dominated by another action is not a best response to any belief of player *i* about the other players' actions.

Strict domination and best responses

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An action of player i in a strategic game that is strictly dominated by another action is not a best response to any belief of player i about the other players' actions. That is, an action for player i that is strictly dominated by another action is not a rational action for player i.

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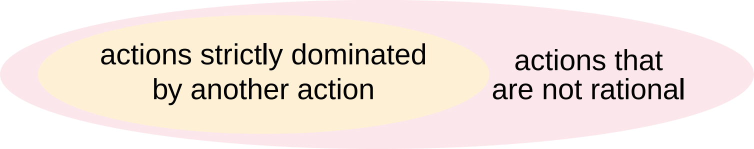

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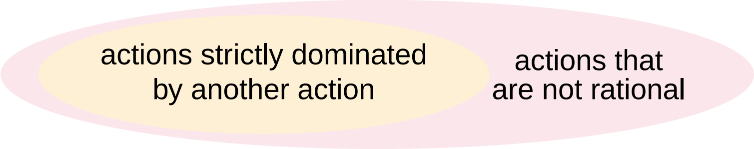

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- The result does not say that an action that is *not* strictly dominated is necessarily rational

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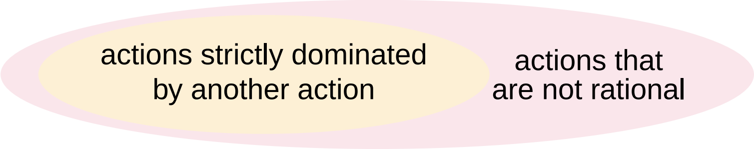

- The result does not say that an action that is *not* strictly dominated is necessarily rational
- In fact, we will see later than some actions that are not strictly dominated are *not* rational

Example: *Prisoner's Dilemma*

	Q	F
Q	2 , 2	0 , 3
F	3 , 0	1 , 1

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	Q	F
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- Q is worse than F for player 1 if player 2 chooses Q

Example: *Prisoner's Dilemma*

	Q	*F*
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- *Q* is worse than *F* for player 1 if player 2 chooses *Q*
- *Q* is worse than *F* for player 1 if player 2 chooses *F*

Example: *Prisoner's Dilemma*

	Q	F
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- Q is worse than F for player 1 if player 2 chooses Q
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- So Q is strictly dominated by F
- Rationality of player 1 ⇒ she chooses F

Example: *Prisoner's Dilemma*

	Q	F
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- Q is worse than F for player 1 if player 2 chooses Q
- Q is worse than F for player 1 if player 2 chooses F
- So Q is strictly dominated by F
- Rationality of player 1 $\Rightarrow$ she chooses F
- Same argument for player 2, so players' rationality alone implies outcome (F, F)

Examples of strict domination

Hotelling's game

- Consumers uniformly distributed along line segment $[0, 4]$

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Examples of strict domination

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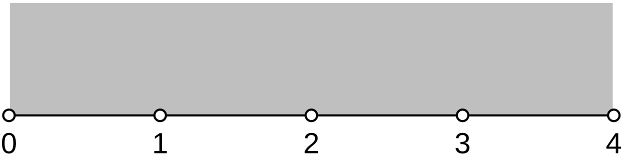
- Firms restricted to points 0, 1, 2, 3, 4



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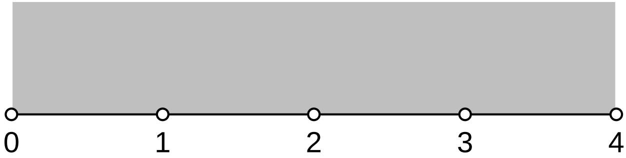

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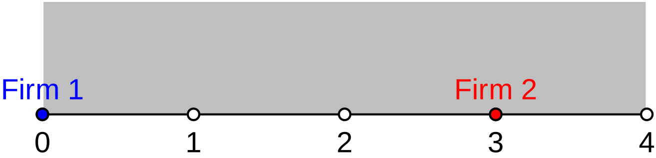

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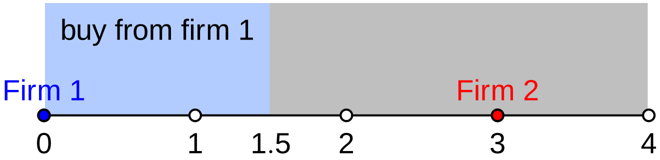

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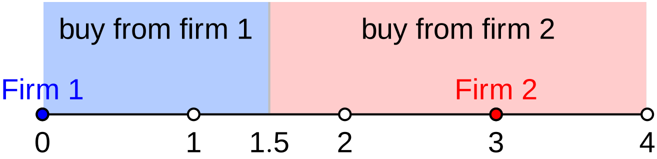


Examples of strict domination

Hotelling's game

- Is any action strictly dominated?

Examples of strict domination

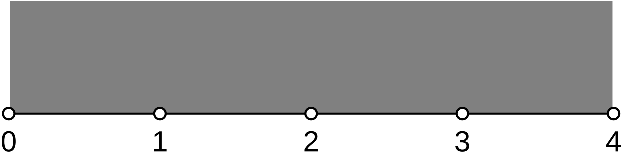
Hotelling's game

- Is any action strictly dominated?
- Compare actions 0 and 1 for firm 1

Firm 2

		0	1	2	3	4
Firm 1	0					
	1					

Firm 1's payoffs



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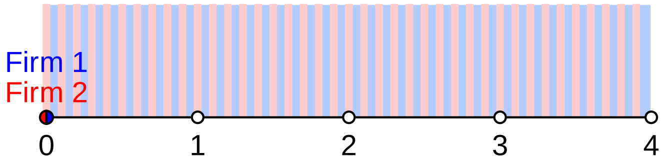

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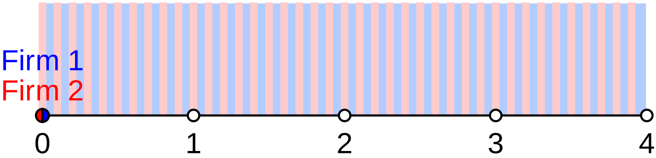

Examples of strict domination

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- Is any action strictly dominated?
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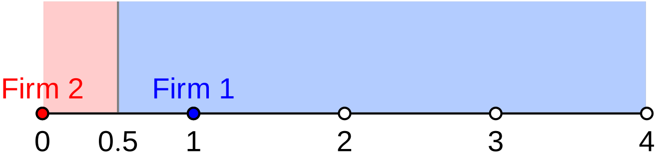
Firm 1		0	1	2	3	4
	0	2				
	1	3.5				

Firm 1's payoffs



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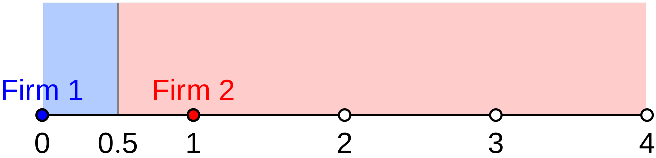
		0	1	2	3	4
Firm 1	0	2	0.5			
	1	3.5				

Firm 1's payoffs



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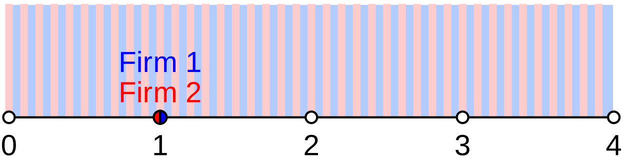
		0	1	2	3	4
Firm 1	0	2	0.5			
	1	3.5	2			

Firm 1's payoffs



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Hotelling's game

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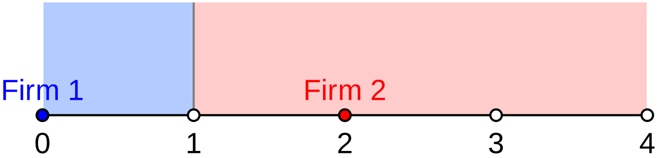
Firm 1	0	2	0.5	1		
	1	3.5	2			

Firm 1's payoffs



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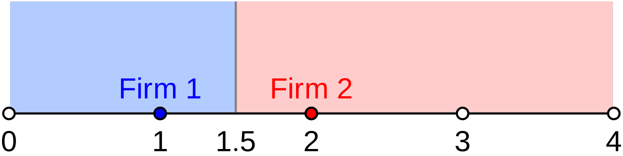
	1	3.5	2	1.5		

Firm 1's payoffs



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Firm 1	0	2	0.5	1	1.5	2
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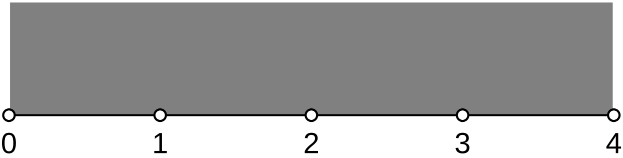
Firm 1's payoffs



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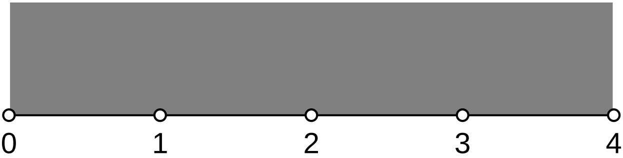
Firm 1's payoffs



- 1 strictly dominates 0

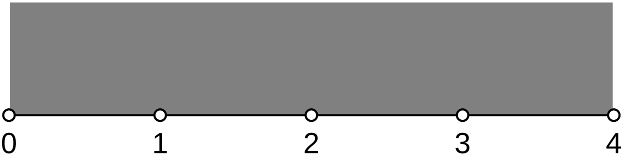
Examples of strict domination

Hotelling's game

- Is any action strictly dominated?
- Compare actions 0 and 1 for firm 1

		Firm 2				
		0	1	2	3	4
Firm 1	3	2.5	2	1.5	2	3.5
	4	2	1.5	1	0.5	2

Firm 1's payoffs



- 1 strictly dominates 0; symmetrically, 3 strictly dominates 4

Examples of strict domination

Hotelling’s game

- Is any action strictly dominated?
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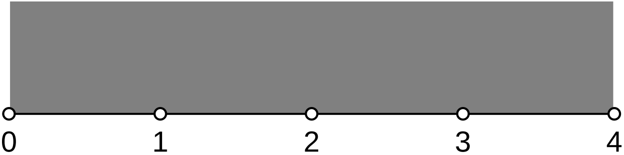
		0	1	2	3	4
Firm 1	3	2.5	2	1.5	2	3.5
	4	2	1.5	1	0.5	2

Firm 1’s payoffs



- 1 strictly dominates 0; symmetrically, 3 strictly dominates 4
- So no rational firm locates at 0 or 4

Examples of strict domination

Hotelling's game

- Is action 1 strictly dominated?

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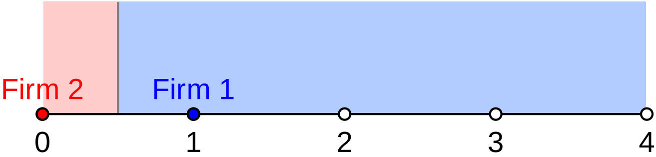

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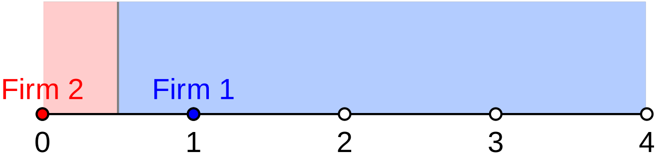

- No: if other firm is at 0, 1 is best place to be!

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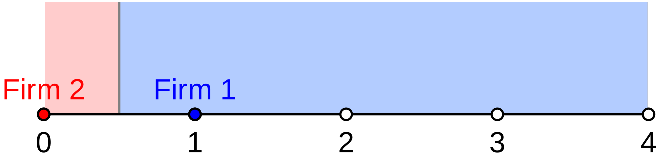

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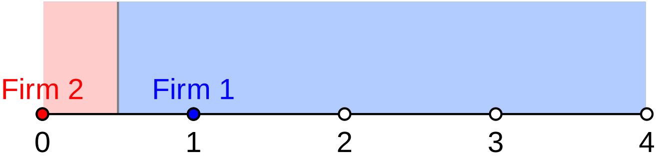

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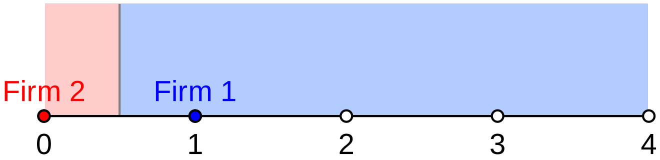
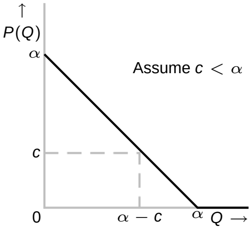


- No: if other firm is at 0, 1 is best place to be!
- Similarly locations 2 and 3 are not strictly dominated
- So only strictly dominated locations are 0 and 4
- Hence rationality of each firm $\Rightarrow$ firm does not choose extreme locations (0 and 4)

Examples of strict domination

Cournot’s duopoly game

- Consider example with linear inverse demand $P(Q) = \alpha - Q$ and constant unit cost $c > 0$



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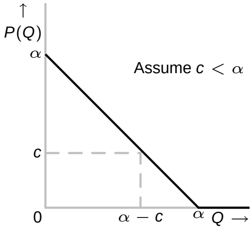

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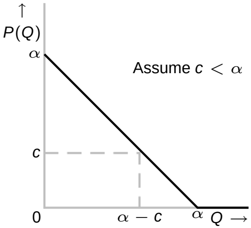
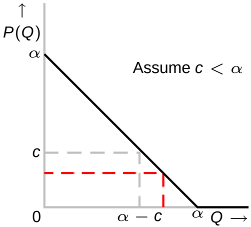


	any q_2
$q_1 > \alpha - c$	?

Examples of strict domination



Assume $c < \alpha$

Cournot's duopoly game

- Consider example with linear inverse demand $P(Q) = \alpha - Q$ and constant unit cost $c > 0$
- Is any output strictly dominated?
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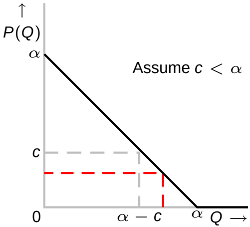

	any q_2
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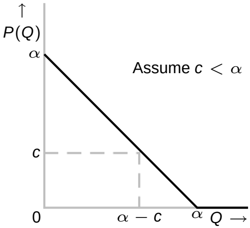

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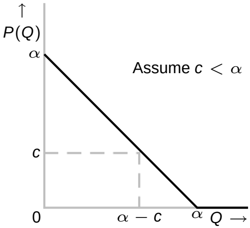

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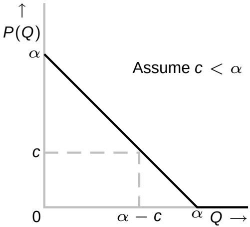

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- Every $q_1 > \alpha - c$ is strictly dominated by $q_1 = 0$

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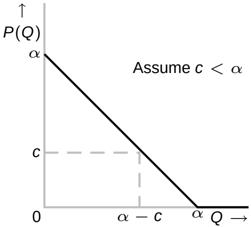

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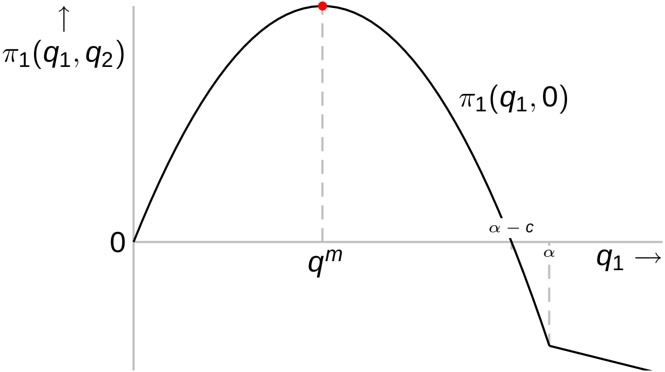
- Every $q_1 > \alpha - c$ is strictly dominated by $q_1 = 0$
- Are any outputs smaller than $\alpha - c$ strictly dominated?

Cournot's duopoly game

- Does it ever make sense for a firm to produce more than the monopoly output ($q^m = \frac{1}{2}(\alpha - c)$)?



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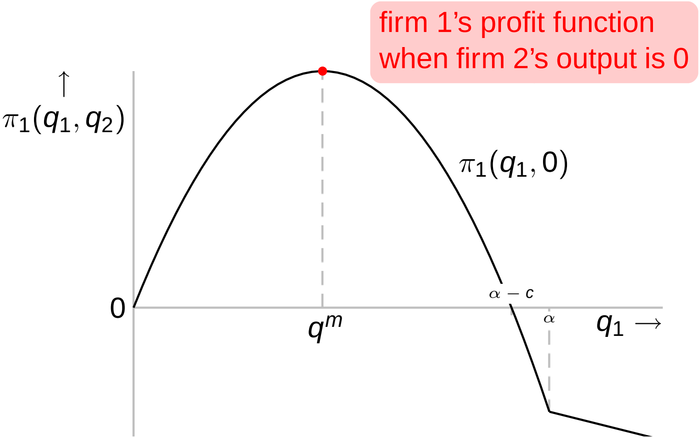
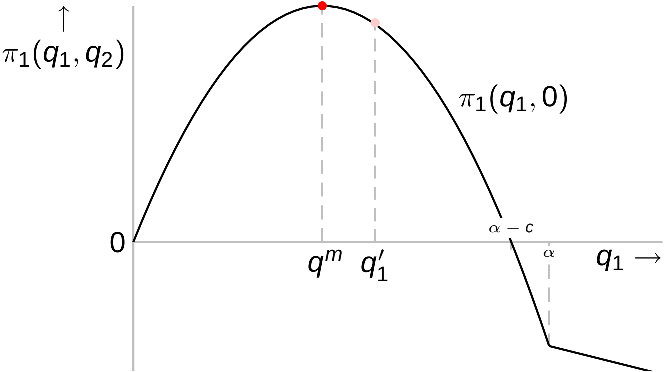

firm 1's profit function when firm 2's output is 0

Cournot's duopoly game

- Does it ever make sense for a firm to produce more than the monopoly output ($q^m = \frac{1}{2}(\alpha - c)$)?
- Fix $q_1' > q^m$

when firm 2's output is 0, firm 1's profit is higher when it produces q^m than when it produces q_1'



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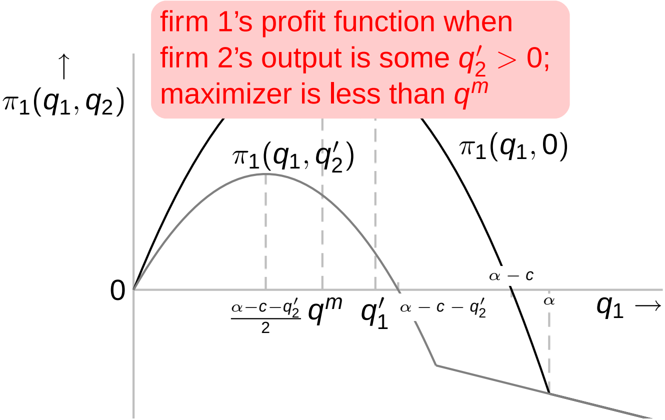

firm 1's profit function when firm 2's output is some $q_2' > 0$; maximizer is less than q^m

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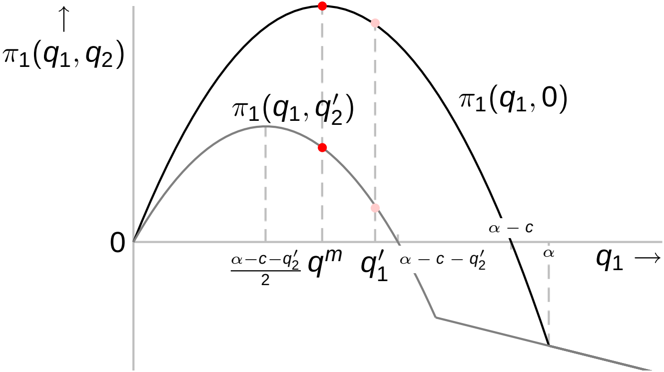
when firm 2’s output is q_2', firm 1’s profit is also higher when it produces q^m than when it produces q_1'



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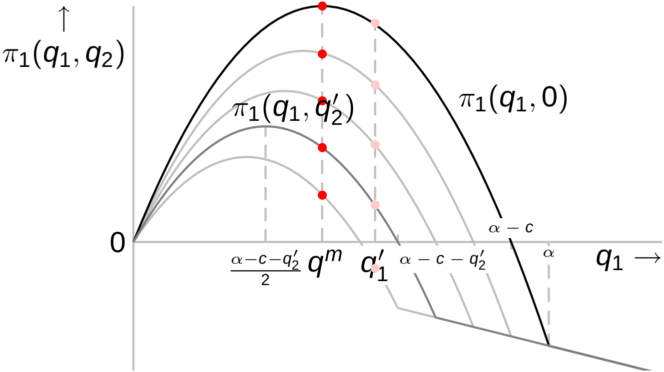

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- Fix $q_1' > q^m$
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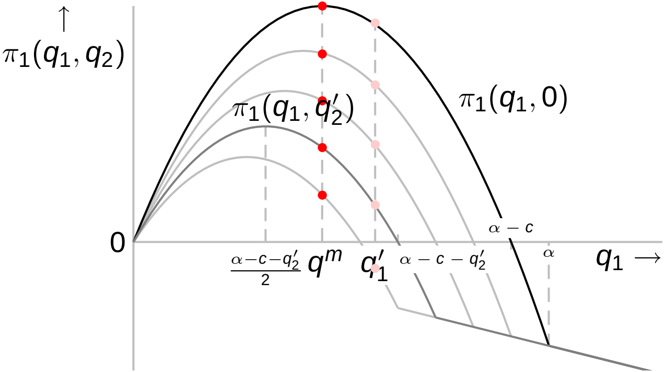
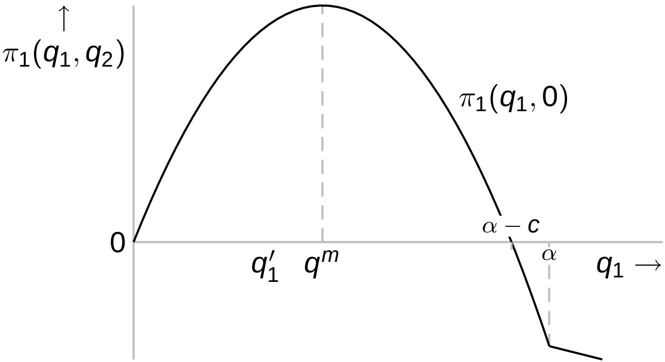
- So $q_1' > q^m$ is strictly dominated by q^m for firm 1



Cournot's duopoly game

- Now consider $q_1' < q^m$



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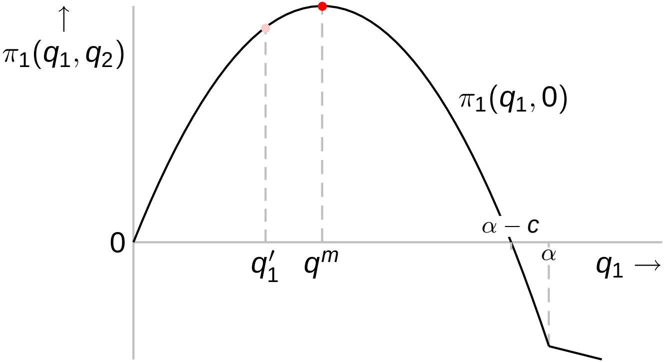
- Now consider $q_1' < q^m$
- For q_2 small, $\pi(q^m, q_2) > \pi(q_1', q_2)$



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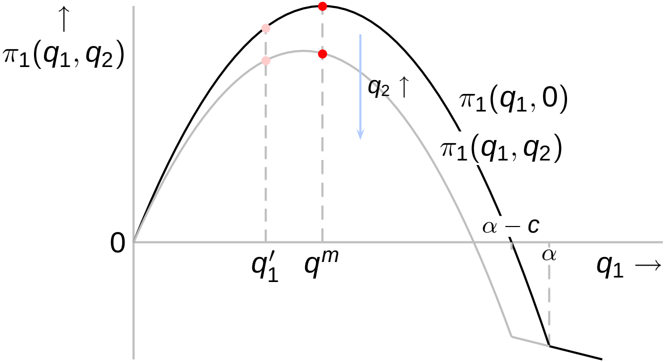

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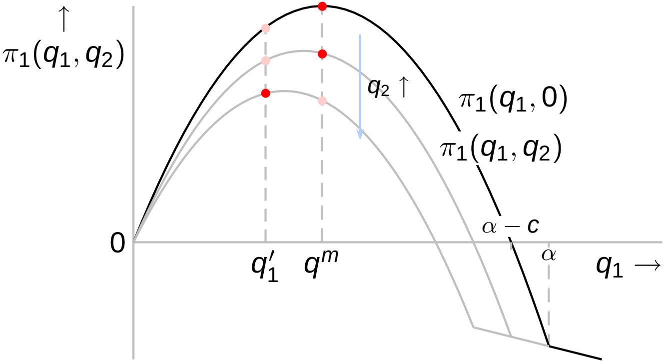

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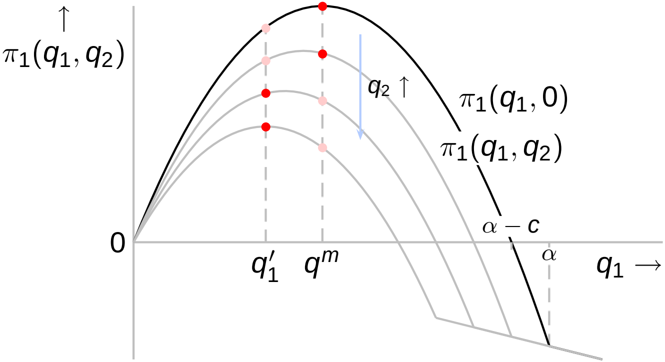

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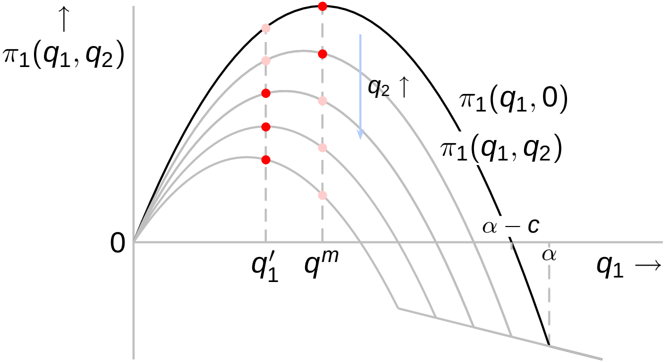

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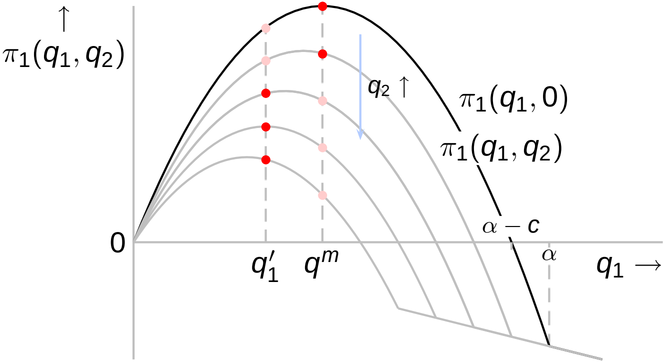
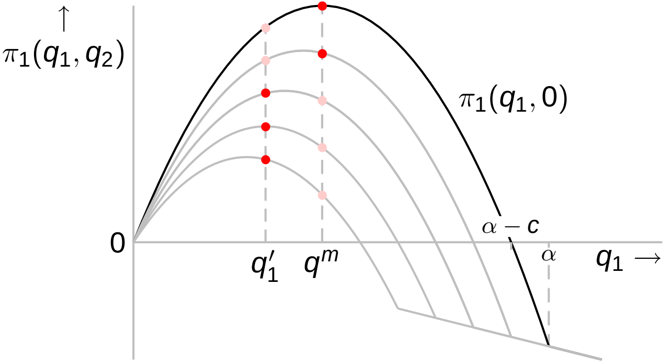


Cournot's duopoly game

- Is q_1' strictly dominated by *any* other output?



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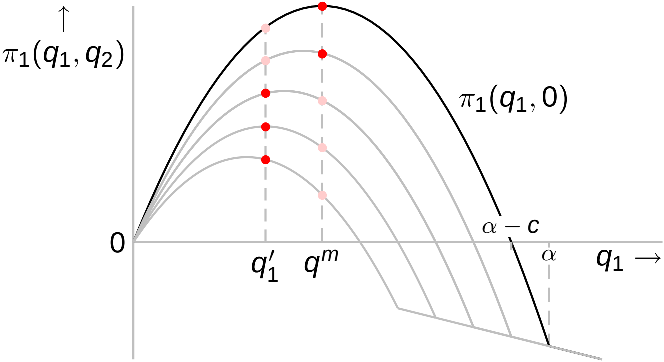

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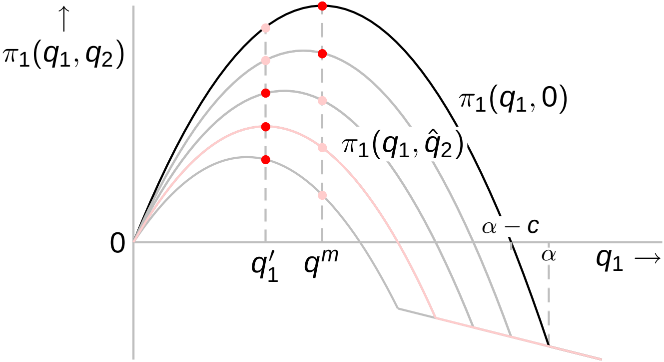

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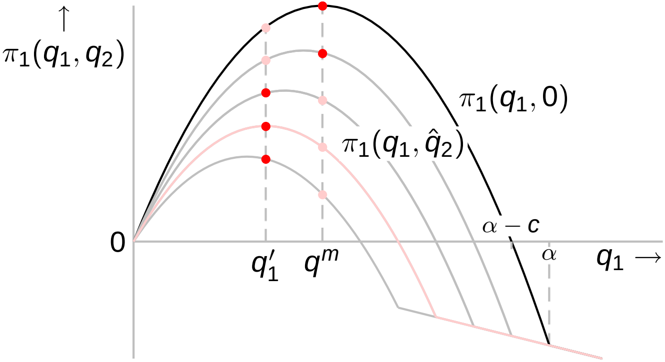

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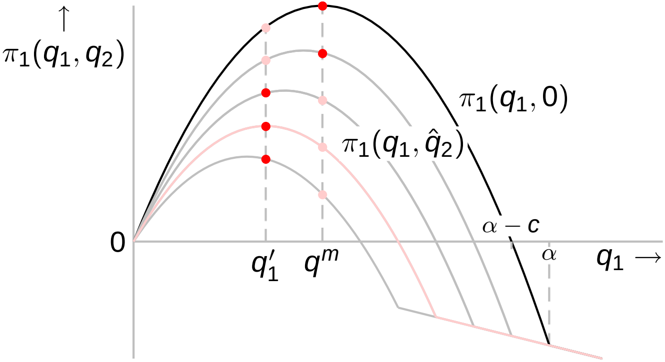
- So $q_1' < q^m$ is not strictly dominated by *any* q_1



Cournot's duopoly game

Conclusion

- Every output $> q^m$ is strictly dominated by q^m

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Cournot's duopoly game

Conclusion

- Every output $> q^m$ is strictly dominated by q^m
- No output $< q^m$ is strictly dominated by q^m
- In fact, no output $< q^m$ is strictly dominated by *any* output
- So rationality of each firm $\Rightarrow$ each firm produces at most monopoly output

Rationality and strict domination

- Rationality $\Leftrightarrow$ player's action is best response to *some* belief

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An action of player i in a strategic game that is strictly dominated by another action is not a best response to any belief of player i about the other players' actions.

Rationality and strict domination

- Rationality $\Leftrightarrow$ player's action is best response to *some* belief
- Earlier result:

Proposition

An action of player i in a strategic game that is strictly dominated by another action is not a best response to any belief of player i about the other players' actions.

- Are any *other* actions not best responses to any belief?

Never-best responses

	L (q)	R $(1-q)$
T	1	1
M	4	0
B	0	4

Player 1’s payoffs

Never-best responses

	L (q)	R $(1-q)$
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Player 1's payoffs

- Is T a best response to any belief of player 1?

Never-best responses

	L (q)	R $(1-q)$	Exp. payoff
T	1	1	1
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B	0	4	$4(1-q)$

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Player 1’s payoffs

- Is T a best response to any belief of player 1?
  - If $q < \frac{3}{4}$, payoff to $B >$ payoff to T
    if $q > \frac{1}{4}$, payoff to $M >$ payoff to T

Never-best responses

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  - So T is not a best response to any belief
- Is T strictly dominated by another action?
  - No: B yields lower payoff if player 2 chooses L and M yields lower payoff if player 2 chooses R

Never-best responses

	L (q)	R $(1-q)$	Exp. payoff
$(0)\ T$	1	1	1
$(\frac{1}{2})\ M$	4	0	$4q$
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Player 1's payoffs

- Suppose player 1 uses mixed strategy $(0, \frac{1}{2}, \frac{1}{2})$:

Never-best responses

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  - expected payoff if player 2 uses L: $\frac{1}{2} \cdot 4 = 2$
  - expected payoff if player 2 uses R: $\frac{1}{2} \cdot 4 = 2$

Never-best responses

	L (q)	R $(1-q)$	Exp. payoff
(0) T	1	1	1
$(\frac{1}{2})$ M	4	0	$4q$
$(\frac{1}{2})$ B	0	4	$4(1-q)$

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- So mixed strategy $(0, \frac{1}{2}, \frac{1}{2})$ is better for player 1 than T regardless of player 2’s action
- That, is, the *mixed strategy* $(0, \frac{1}{2}, \frac{1}{2})$ strictly dominates T
  - (Other mixed strategies also strictly dominate T)

Never-best responses and strict domination

Definition

Player *i*'s mixed strategy α_i strictly dominates her action a_i if

$$i\text{'s expected payoff to } (\alpha_i, a_{-i}) > i\text{'s payoff to } (a_i, a_{-i})$$

for *every* list a_{-i} of the other players' actions.

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Necessary and sufficient condition for an action to be a best response to a belief:

Proposition

An action is not a best response to any belief if and only if it is strictly dominated by a mixed strategy

Never-best responses and strict domination

Summary

- Rationality $\Leftrightarrow$ player chooses best response to *some* belief

Never-best responses and strict domination

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Relation with Nash equilibrium

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Relation with Nash equilibrium

- Can a strictly dominated action be used in a Nash equilibrium?
- No! A strictly dominated action isn't a best response to any list of the other players' actions
- Is it possible for a player to assign positive probability to a never-best response in a mixed strategy Nash equilibrium?
- No! Every action used with positive probability in a mixed strategy equilibrium must be a best response to the other players' mixed strategies

Relation with Nash equilibrium

Proposition

An action that is not a best response to any belief is not used with positive probability in any mixed strategy Nash equilibrium.

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Relation with Nash equilibrium

Proposition

An action that is not a best response to any belief is not used with positive probability in any mixed strategy Nash equilibrium.

- So when looking for a mixed strategy equilibrium, we can eliminate from consideration actions that are strictly dominated by mixed strategies
- However, determining whether an action is strictly dominated by a mixed strategy can be difficult
- Mostly we will consider only strict domination by an action

Rationality and equilibrium

Every player

is rational

⇕

action is best response to belief about other players' actions

believes that other players are rational

. . . and believes that other players believe she is rational

. . . and believes that other players believe she believes they are rational

. . . and so on

?

Drawing conclusions from other players’ rationality

Example

	L	C	R
T	$0, 4$	$4, 0$	$2, 1$
M	$1, 0$	$3, 1$	$3, 2$
B	$0, 2$	$2, 3$	$1, 1$

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- Player 1 is rational $\Rightarrow$ does not choose B (strictly dominated by M)

Drawing conclusions from other players' rationality

Example

	L	*C*	*R*
T	0, 4	4, 0	2, 1
M	1, 0	3, 1	3, 2
~~*B*~~	~~0, 2~~	~~2, 3~~	~~1, 1~~

- Player 1 is rational $\Rightarrow$ does not choose *B* (strictly dominated by *M*)
- Player 2 believes player 1 is rational $\Rightarrow$

Drawing conclusions from other players' rationality

Example

	L	C	R
T	$0, 4$	$4, 0$	$2, 1$
M	$1, 0$	$3, 1$	$3, 2$
~~B~~	~~$0, 2$~~	~~$2, 3$~~	~~$1, 1$~~

- Player 1 is rational $\Rightarrow$ does not choose B (strictly dominated by M)
- Player 2 believes player 1 is rational $\Rightarrow$ player 2 believes player 1 does not choose B

Drawing conclusions from other players’ rationality

Example

	L	~~*C*~~	*R*
T	0, 4	~~4, 0~~	2, 1
M	1, 0	~~3, 1~~	3, 2
~~*B*~~	~~0, 2~~	~~2, 3~~	~~1, 1~~

- Player 1 is rational ⇒ does not choose *B* (strictly dominated by *M*)
- Player 2 believes player 1 is rational ⇒ player 2 believes player 1 does not choose *B*
  ⇒ if player 2 is rational she does not choose *C*

Drawing conclusions from other players’ rationality

Example

	L	~~*C*~~	*R*
T	$0, 4$	~~$4, 0$~~	$2, 1$
M	$1, 0$	~~$3, 1$~~	$3, 2$
~~*B*~~	~~$0, 2$~~	~~$2, 3$~~	~~$1, 1$~~

- Player 1 is rational ⇒ does not choose *B* (strictly dominated by *M*)
- Player 2 believes player 1 is rational ⇒ player 2 believes player 1 does not choose *B*
  ⇒ if player 2 is rational she does not choose *C*
- Player 1 believes that player 2 is rational *and* that player 2 believes player 1 is rational ⇒

Drawing conclusions from other players' rationality

Example

	L	~~C~~	R
T	$0, 4$	~~$4, 0$~~	$2, 1$
M	$1, 0$	~~$3, 1$~~	$3, 2$
~~B~~	~~$0, 2$~~	~~$2, 3$~~	~~$1, 1$~~

- Player 1 is rational $\Rightarrow$ does not choose B (strictly dominated by M)
- Player 2 believes player 1 is rational $\Rightarrow$ player 2 believes player 1 does not choose B
  $\Rightarrow$ if player 2 is rational she does not choose C
- Player 1 believes that player 2 is rational *and* that player 2 believes player 1 is rational $\Rightarrow$ player 1 believes player 2 believes player 1 does not choose B and that player 2 therefore does not choose C

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	L	~~C~~	R
~~T~~	~~0, 4~~	~~4, 0~~	~~2, 1~~
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  $\Rightarrow$ if player 1 is rational, she does not choose T

Drawing conclusions from other players' rationality

Example

	L	~~*C*~~	*R*
~~*T*~~	~~0, 4~~	~~4, 0~~	~~2, 1~~
M	1, 0	~~3, 1~~	3, 2
~~*B*~~	~~0, 2~~	~~2, 3~~	~~1, 1~~

- Player 1 is rational $\Rightarrow$ does not choose *B* (strictly dominated by *M*)
- Player 2 believes player 1 is rational $\Rightarrow$ player 2 believes player 1 does not choose *B*
  $\Rightarrow$ if player 2 is rational she does not choose *C*
- Player 1 believes that player 2 is rational *and* that player 2 believes player 1 is rational $\Rightarrow$ player 1 believes player 2 believes player 1 does not choose *B* and that player 2 therefore does not choose *C*
  $\Rightarrow$ if player 1 is rational, she does not choose *T*
- In one more step . . .

Drawing conclusions from other players' rationality

Example

	~~L~~	~~C~~	R
~~T~~	~~0, 4~~	~~4, 0~~	~~2, 1~~
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- Player 2 believes player 1 is rational $\Rightarrow$ player 2 believes player 1 does not choose B
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- Player 1 believes that player 2 is rational *and* that player 2 believes player 1 is rational $\Rightarrow$ player 1 believes player 2 believes player 1 does not choose B and that player 2 therefore does not choose C
  $\Rightarrow$ if player 1 is rational, she does not choose T
- In one more step . . . player 2 does not choose L

Drawing conclusions from other players' rationality

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  $\Rightarrow$ if player 1 is rational, she does not choose T
- In one more step ... player 2 does not choose L
  $\Rightarrow$ only action pair that remains is (M, R)

Iterated elimination of strictly dominated actions

- Process of sequentially deleting strictly dominated actions: iterated elimination of strictly dominated actions

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- *If* every player is rational
  and every player believes every other player is rational
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  and so forth . . .

  then the action profile that the players choose *survives iterated elimination of strictly dominated actions*

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- Change one payoff in game we just looked at:

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	L	*C*	*R*
T	0, 4	4, 0	2, 1
M	1, 0	3, 1	3, 2
B	2, 2	2, 3	1, 1

- Now *no* action of either player is strictly dominated
  $\Rightarrow$ *all* action profiles survive IESDA

Iterated elimination of strictly dominated actions

		Vizzini	
		Goblet *A*	Goblet *B*
Man in black	Poison in goblet *A*	$1, -10$	$-10, 1$
	Poison in goblet *B*	$-10, 1$	$1, -10$

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Vizzini:

> *Now, a clever man would put the poison into his own goblet, because he would know that only a great fool would reach for what he was given. I'm not a great fool, so I can clearly not choose the wine in front of you. But you must have known I was not a great fool [. . .] so I can clearly not choose the wine in front of me.*

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- Neither action of either player is strictly dominated

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- Neither action of either player is strictly dominated
- So neither action can be ruled out as not rational

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Man in black	Poison in goblet *A*	$1, -10$	$-10, 1$
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- In fact, in a random game, it is likely that no action of any player is strictly dominated, so that all action profiles survive IESDA

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- In fact, in a random game, it is likely that no action of any player is strictly dominated, so that all action profiles survive IESDA
- However, in several games of economic interest, only Nash equilibria survive IESDA

Hotelling's game

- For each player, extreme locations are strictly dominated

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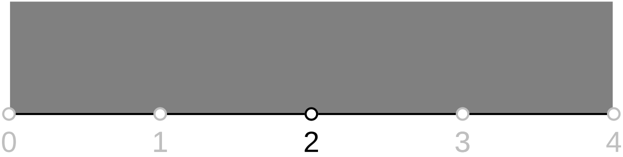
⇒ only action pair that survives iterated elimination of strictly dominated actions is $(2, 2)$: both firms choose the median position



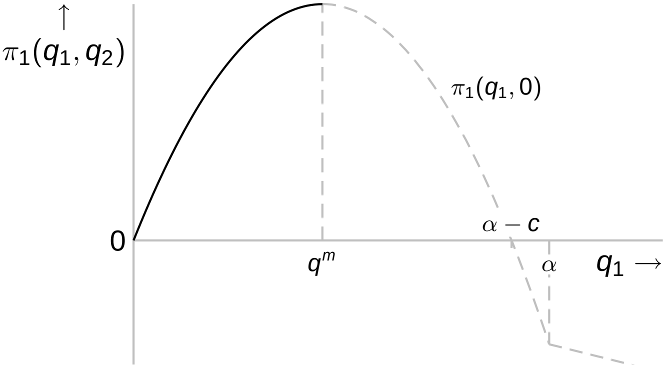
Hotelling's game

Summary

- Firms' rationality and their reasoning about each other's rationality $\Rightarrow$ outcome is $(2, 2)$, Nash equilibrium
- Argument generalizes to any finite number of possible positions for firms

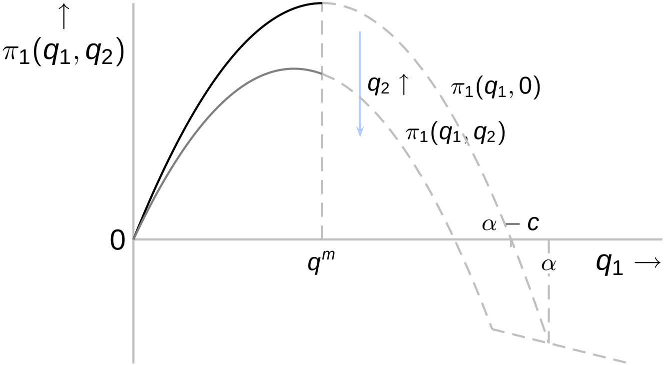
Cournot's duopoly game

- We know that all outputs $> q^m$ $(= \frac{1}{2}(\alpha - c))$ are strictly dominated



Cournot's duopoly game

- We know that all outputs $> q^m$ $(= \frac{1}{2}(\alpha - c))$ are strictly dominated
- So eliminate all outputs $> q^m$ for firm 2



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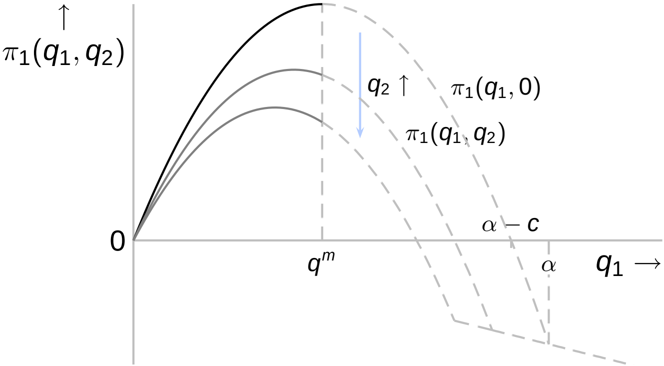

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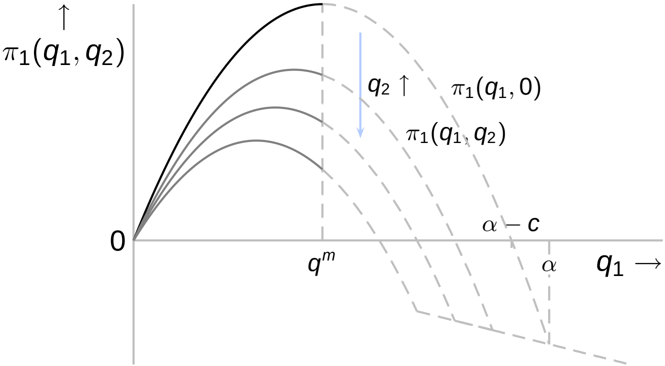

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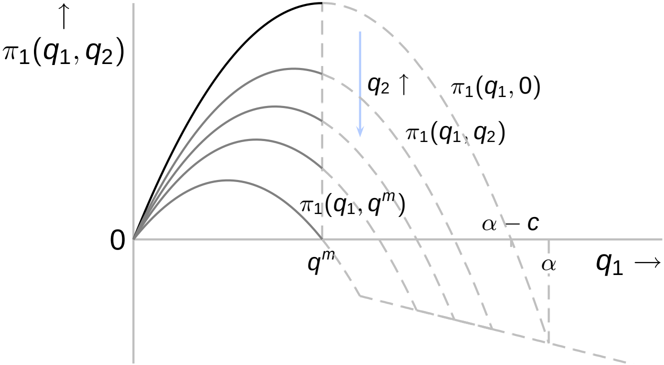

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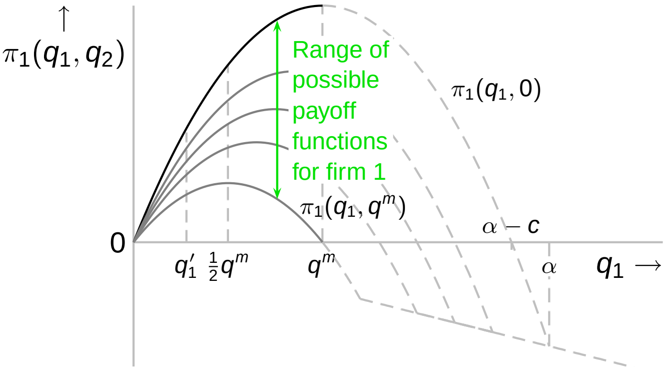

Cournot's duopoly game

- We know that all outputs $> q^m$ $(= \frac{1}{2}(\alpha - c))$ are strictly dominated
- So eliminate all outputs $> q^m$ for firm 2
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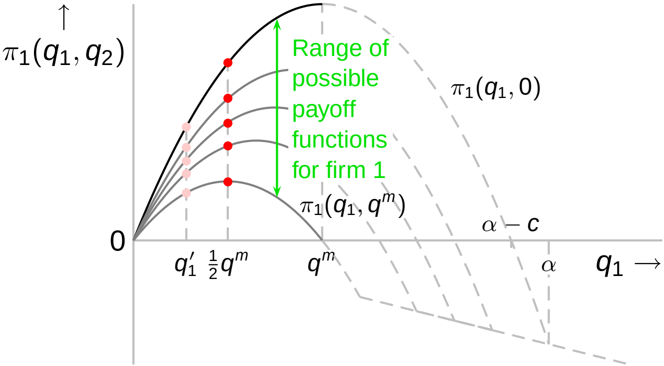

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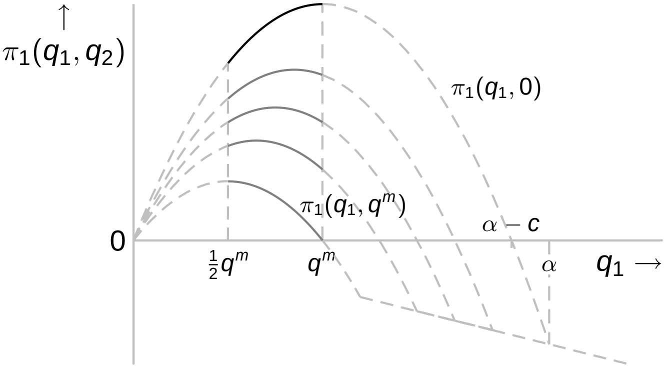

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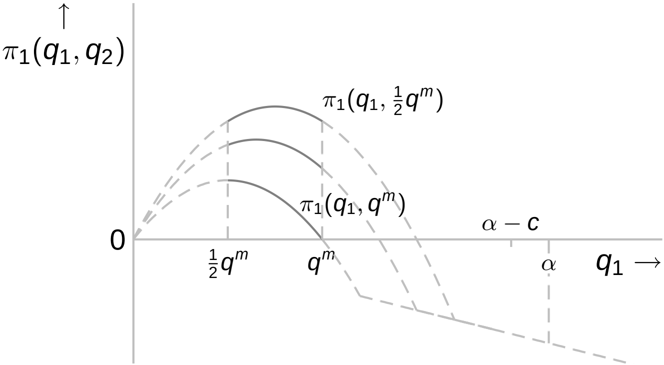

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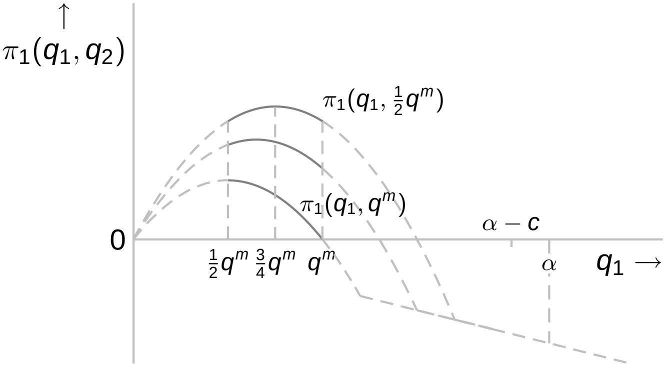

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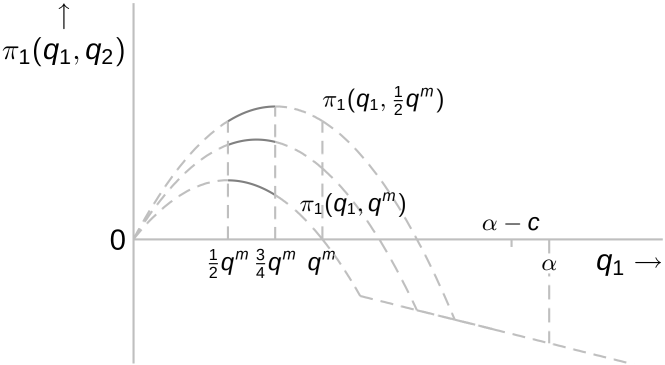

Cournot's duopoly game

Conclusion so far

- Every output $> q^m$ is strictly dominated (by q^m)

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  $\Rightarrow$ no rational firm chooses output $> q^m$

Cournot's duopoly game

Conclusion so far

- Every output $> q^m$ is strictly dominated (by q^m)
  $\Rightarrow$ no rational firm chooses output $> q^m$
- After outputs $> q^m$ are eliminated, every output $< \frac{1}{2}q^m$ is strictly dominated

Cournot's duopoly game

Conclusion so far

- Every output $> q^m$ is strictly dominated (by q^m)
  $\Rightarrow$ no rational firm chooses output $> q^m$
- After outputs $> q^m$ are eliminated, every output $< \frac{1}{2}q^m$ is strictly dominated
  $\Rightarrow$ a firm that believes the other firm is rational does not rationally choose an output $< \frac{1}{2}q^m$

Cournot's duopoly game

Conclusion so far

- Every output $> q^m$ is strictly dominated (by q^m)
  $\Rightarrow$ no rational firm chooses output $> q^m$
- After outputs $> q^m$ are eliminated, every output $< \frac{1}{2}q^m$ is strictly dominated
  $\Rightarrow$ a firm that believes the other firm is rational does not rationally choose an output $< \frac{1}{2}q^m$
- After outputs $< \frac{1}{2}q^m$ are eliminated, every output $> \frac{3}{4}q^m$ is strictly dominated
  $\Rightarrow$ a firm that believes the other firm is rational does not rationally choose an output $< \frac{1}{2}q^m$ or $> \frac{3}{4}q^m$

Cournot’s duopoly game

Continuing the process

- The process continues … until only the Nash equilibrium $(\frac{1}{3}(\alpha - c), \frac{1}{3}(\alpha - c))$ remains

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Eventual conclusion

- The only action pair that survives iterated elimination of strictly dominated actions in the example of Cournot's duopoly game is the Nash equilibrium

Weakly dominated actions

	Q	F
Q	$2, 2$	$0, 3$
F	$3, 0$	$1, 1$

Prisoner's Dilemma

- In *Prisoner's Dilemma*, F strictly dominates Q

Weakly dominated actions

	Q	*F*
Q	2, 2	0, 3
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Prisoner's Dilemma

- In *Prisoner's Dilemma*, *F* strictly dominates *Q*

	Split	*Steal*
Split	£50,000, £50,000	0, £100,000
Steal	£100,000, 0	0, 0

Split or steal?

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Split or steal?

- In *Split or steal*, *Steal* does *not* strictly dominate *Split*
  - but *Steal* is never worse than *Split*
  - and is better if the other player chooses *Split*
- We say *Steal* weakly dominates *Split*

Weak domination

Definition

Player i's action a_i'' weakly dominates her action a_i' if

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a_i'' is at least as good for i as a_i' for *all* actions of the other players

and

a_i'' is better for i than a_i' for *some* actions of the other players.

Precisely,

$u_i(a_i'', a_{-i}) \geq u_i(a_i', a_{-i})$ for *every* list a_{-i} of other players' actions
$u_i(a_i'', a_{-i}) > u_i(a_i', a_{-i})$ for *some* list a_{-i} of other players' actions

Weak domination

Example

Lists of other players’ actions

		a'_{-i}	a''_{-i}	a'''_{-i}
Player i	T	1	6	0
	B	2	6	3

Player i’s payoffs

Weak domination

Example

Lists of other players' actions

		a'_{-i}	a''_{-i}	a'''_{-i}
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The action B weakly dominates T:

Weak domination

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The action B weakly dominates T:

- for every list of actions of the other players, B is at least as good as T for player i

Weak domination

Example

Lists of other players' actions

		a'_{-i}	a''_{-i}	a'''_{-i}
Player i	T	1	6	0
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Player i's payoffs

The action B weakly dominates T:

- for every list of actions of the other players, B is at least as good as T for player i
- for the list of actions a'_{-i} (and also for a'''_{-i}), B is better than T for player i

Weak domination and Nash equilibrium

- Can a weakly dominated action be used in a Nash equilibrium?

Weak domination and Nash equilibrium

- Can a weakly dominated action be used in a Nash equilibrium?

		Player 2	
		L	*R*
Player 1	*T*	$1, 1$	$0, 0$
	B	$0, 0$	$0, 0$

Weak domination and Nash equilibrium

- Can a weakly dominated action be used in a Nash equilibrium?

		Player 2	
		L	R
Player 1	T	$1, 1$	$0, 0$
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Weak domination and Nash equilibrium

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- (B, R) is a Nash equilibrium
- B is weakly dominated by T (and R is weakly dominated by L)

Weak domination and Nash equilibrium

- Can a weakly dominated action be used in a Nash equilibrium?

		Player 2	
		L	R
Player 1	T	$1, 1$	$0, 0$
	B	$0, 0$	$0, 0$

- (B, R) is a Nash equilibrium
- B is weakly dominated by T (and R is weakly dominated by L)
- So a weakly dominated action *can* be used in a Nash equilibrium

Weak domination

- Case against choosing *weakly* dominated action isn’t as strong as case against choosing *strictly* dominated action

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- In *Split or steal?*, for example, *Split* is optimal if player is *certain* that other player will choose *Steal* (and *Steal* is also optimal in this case)
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- In *Split or steal?*, for example, *Split* is optimal if player is *certain* that other player will choose *Steal* (and *Steal* is also optimal in this case)
- But any doubt, however small, makes *Split* suboptimal
- There is no *positive* incentive to choose a weakly dominated action

	Split	*Steal*
Split	£50,000, £50,000	0, £100,000
Steal	£100,000, 0	0, 0

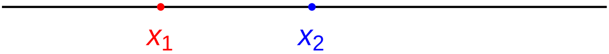
Split or steal?

Hotelling's model of electoral competition

- Two parties

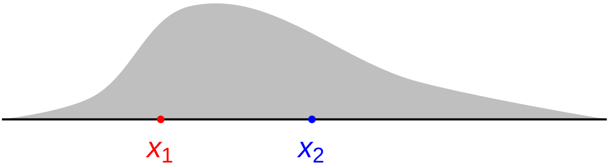
Hotelling's model of electoral competition

- Two parties
- Each party chooses a position on the line



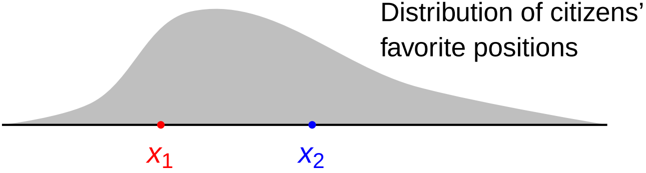
Hotelling's model of electoral competition

- Two parties
- Each party chooses a position on the line
- Large number of citizens



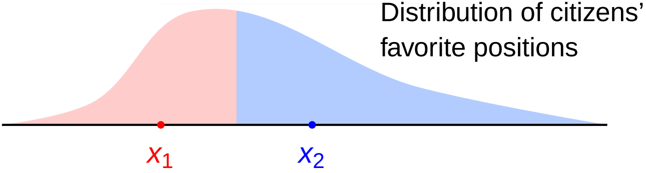
Hotelling’s model of electoral competition

- Two parties
- Each party chooses a position on the line
- Large number of citizens
- Each citizen has favorite position



Hotelling’s model of electoral competition

- Two parties
- Each party chooses a position on the line
- Large number of citizens
- Each citizen has favorite position
- Each citizen votes for party whose position is closest to her favorite position



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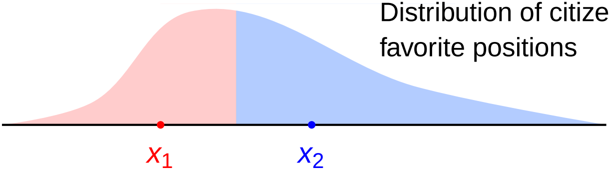
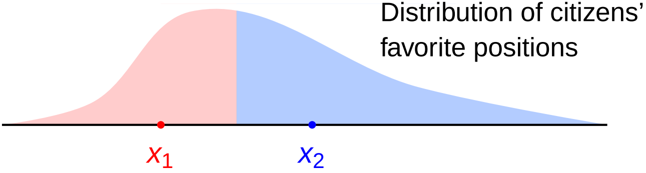
- Each citizen has favorite position
- Each citizen votes for party whose position is closest to her favorite position
- Party who gets most votes wins



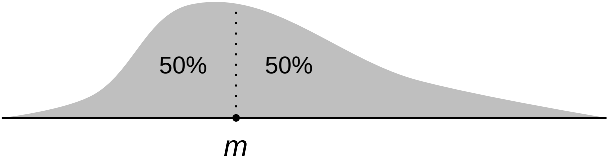
Distribution of citizens’ favorite positions

Hotelling's model of electoral competition

- Two parties
- Each party chooses a position on the line
- Large number of citizens
- Each citizen has favorite position
- Each citizen votes for party whose position is closest to her favorite position
- Party who gets most votes wins
- Each party prefers to win than to tie than to lose

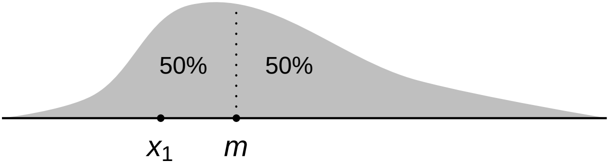


Hotelling's model of electoral competition



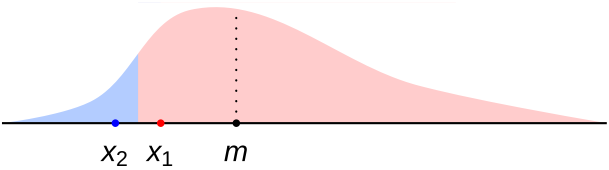
- Is any position weakly dominated?

Hotelling's model of electoral competition



- Is any position weakly dominated?
- Let $x_1 < m$ (median favorite position)

Hotelling's model of electoral competition



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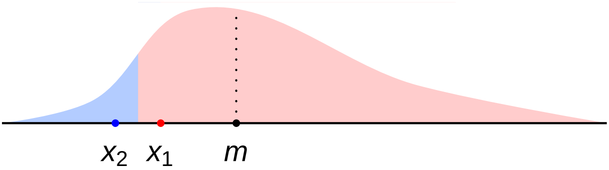
		Party 2				
		$x_2 < x_1$	$x_2 = x_1$	$x_1 < x_2 < m$	$x_2 = m$	$x_2 > m$
Party 1	x_1					
	m					

Outcomes for party 1

Hotelling's model of electoral competition



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Party 2

		$x_2 < x_1$	$x_2 = x_1$	$x_1 < x_2 < m$	$x_2 = m$	$x_2 > m$
Party 1	x_1	win				
	m					

Outcomes for party 1

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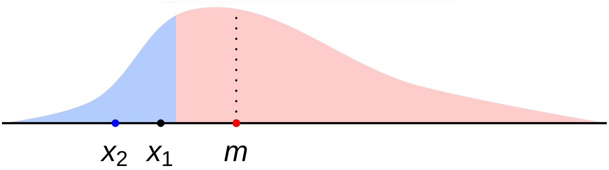

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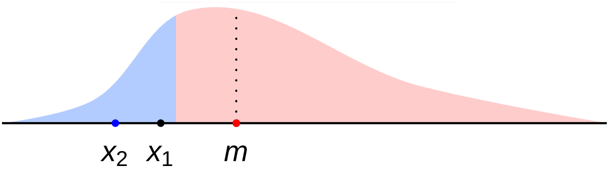

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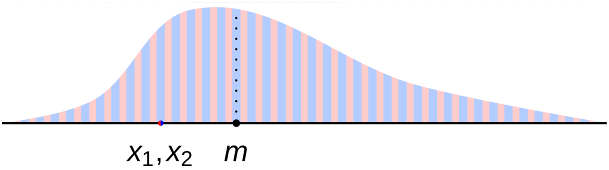
	m	win				

Outcomes for party 1

Hotelling’s model of electoral competition



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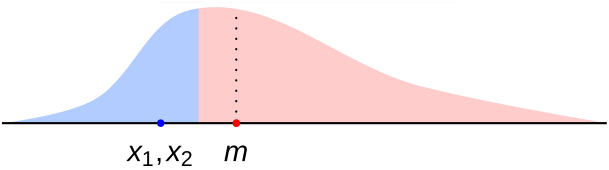
Party 2

Party 1		$x_2 < x_1$	$x_2 = x_1$	$x_1 < x_2 < m$	$x_2 = m$	$x_2 > m$
	x_1	win	tie			
	m	win				

Outcomes for party 1

Hotelling's model of electoral competition



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Outcomes for party 1

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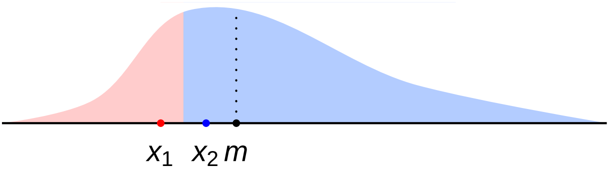

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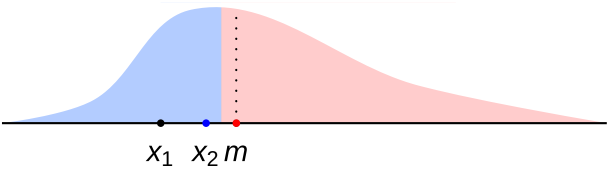
Party 1	x_1	win	tie	lose		
	m	win	win			

Outcomes for party 1

Hotelling's model of electoral competition



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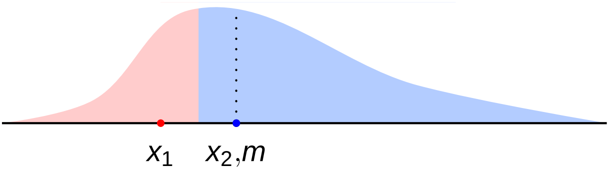
Outcomes for party 1

Hotelling’s model of electoral competition



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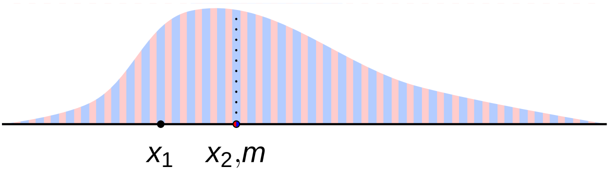
Outcomes for party 1

Hotelling's model of electoral competition



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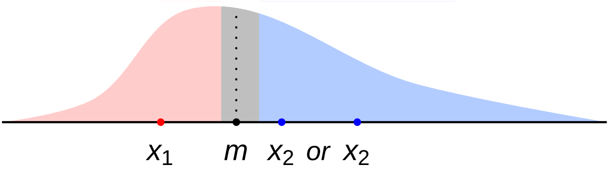
x_1	win	tie	lose	lose	
m	win	win	win	tie	

Outcomes for party 1

Hotelling's model of electoral competition



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Party 2

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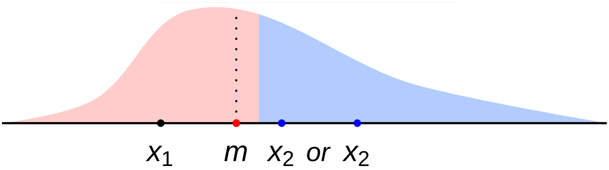
Party 1	x_1	win	tie	lose	lose	l, t, or w
	m	win	win	win	tie	

Outcomes for party 1

Hotelling's model of electoral competition



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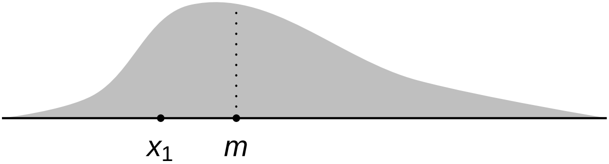
m	win	win	win	tie	win

Outcomes for party 1

Hotelling's model of electoral competition



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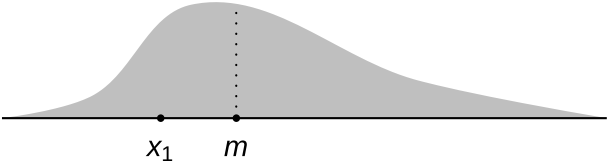
	m	win	win	win	tie	win

Outcomes for party 1

- So m weakly dominates *every* $x_1 < m$

Hotelling’s model of electoral competition



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		Party 2				
		$x_2 < x_1$	$x_2 = x_1$	$x_1 < x_2 < m$	$x_2 = m$	$x_2 > m$
Party 1	x_1	win	tie	lose	lose	l, t, or w
	m	win	win	win	tie	win

Outcomes for party 1

- So m weakly dominates *every* $x_1 < m$
- Similarly m weakly dominates every $x_1 > m$

Hotelling’s model of electoral competition

Conclusion

- For each player, m weakly dominates every $x < m$ and every $x > m$

Hotelling's model of electoral competition

Conclusion

- For each player, m weakly dominates every $x < m$ and every $x > m$
- . . . so m weakly dominates *every* other action

Hotelling's model of electoral competition

Conclusion

- For each player, *m* weakly dominates every $x < m$ and every $x > m$
- … so *m* weakly dominates *every* other action
- *m* is not weakly dominated

Hotelling's model of electoral competition

Conclusion

- For each player, m weakly dominates every $x < m$ and every $x > m$
- $\ldots$ so m weakly dominates *every* other action
- m is not weakly dominated
- Thus m is the only action that is not weakly dominated

Bertrand’s model of duopoly

- Two firms

Bertrand's model of duopoly

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- Each firm chooses a price

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Bertrand's model of duopoly

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- If $p_i < p_j$ then firm i gets all the demand and firm j gets no demand
- If $p_i = p_j = p$ then each firm gets the demand $\frac{1}{2}D(p)$
- Payoff of firm i

$$\begin{cases} (p_i - c)D(p_i) & \text{if } p_i < p_j \\ \frac{1}{2}(p_i - c)D(p_i) & \text{if } p_i = p_j \\ 0 & \text{if } p_i > p_j \end{cases}$$

Bertrand’s model of duopoly

- Is any price weakly dominated?

Bertrand’s model of duopoly

- Is any price weakly dominated?
- Let $p_1 < c$

		Firm 2	
		$p_2 < p_1$	$p_2 \geq p_1$
Firm 1	$p_1 < c$		
	c		

Payoffs for firm 1

Bertrand’s model of duopoly

- Is any price weakly dominated?
- Let $p_1 < c$

Firm 2

		$p_2 < p_1$	$p_2 \geq p_1$
Firm 1	$p_1 < c$	0	
	c	0	

Payoffs for firm 1

Bertrand’s model of duopoly

- Is any price weakly dominated?
- Let $p_1 < c$

		Firm 2	
		$p_2 < p_1$	$p_2 \geq p_1$
Firm 1	$p_1 < c$	0	< 0
	c	0	0

Payoffs for firm 1

Bertrand’s model of duopoly

- Is any price weakly dominated?
- Let $p_1 < c$

Firm 2

		$p_2 < p_1$	$p_2 \geq p_1$
Firm 1	$p_1 < c$	0	< 0
	c	0	0

Payoffs for firm 1

- So c weakly dominates any $p_1 < c$

Bertrand's model of duopoly

- Let $p_1 > c$ with $D(p_1) > 0$

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Payoffs for firm 1

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$\Rightarrow$ in Nash equilibrium, both firms use weakly dominated action!

Bertrand's model of duopoly

Summary

- Every $p < c$ is weakly dominated by c
- The price c is weakly dominated by every $p > c$ for which $D(p) > 0$
- Is any price $> c$ weakly dominated? [Problem Set 6]

Bertrand's model of duopoly

Version with discrete prices

- Suppose that a price has to be an integral number of cents
- Then two Nash equilibria, (c, c) and $(c+1, c+1)$ [Problem Set 2]
- Is $p = c + 1$ weakly dominated? [Problem Set 6]

Strict domination and rationality: summary

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- In many games, a large number of action profiles survive iterated elimination of strictly dominated actions
- But in some key economic models, the Nash equilibrium is the only action profile that survives

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- Using a weakly dominated action is not ruled out by rationality
- But is optimal only for beliefs that assign probability one to specific actions
- A weakly dominated action is not optimal for beliefs that assign positive probability to every list of actions of the other players
- In many economic models, restricting players to actions that are not weakly dominated makes sense

Domination and Nash equilibrium: summary

- A strictly dominated action is not used with positive probability in any mixed strategy Nash equilibrium

Domination and Nash equilibrium: summary

- A strictly dominated action is not used with positive probability in any mixed strategy Nash equilibrium
- A weakly dominated action may be used in a Nash equilibrium