Economics 316

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Solutions to Problem Set 5

1. If both E and E' decrease by the same amount, $E' - E$ remains constant, so the equilibrium value of p decreases: experts become less honest. The equilibrium value of q remains the same.

   If both E' and I' decrease, then the equilibrium value of p increases: experts become more honest. The equilibrium value of q remains the same.

2. The best response function of the unprincipled experts is exactly the same as it is in the original model. Now consider the best response of a consumer as a function of the fraction of unprincipled experts who choose *Honest*. If this fraction is p, then given that the fraction of experts who are principled and thus always choose *Honest* is α, the probability that the consumer faces an expert who chooses *Honest* is $\alpha + (1-\alpha)p$. Thus the consumer's best response to p is her best response to $\alpha + (1-\alpha)p$ in the original model: *Reject* if $\alpha + (1-\alpha)p < p^*$, *Accept* if $\alpha + (1-\alpha)p > p^*$, and any mixture of *Reject* and *Accept* if $\alpha + (1-\alpha)p = p^*$, where p^* is the probability the expert chooses *Honest* that makes the consumer indifferent between *Accept* and *Reject* in the original model. There are three cases.

   - If $\alpha < p^*$ then there exists p with $0 < p < 1$ such that $\alpha + (1-\alpha)p = p^*$, so that the consumer's best response function has the same shape as in the original model, with the value of p at which the consumer is indifferent being $(p^* - \alpha)/(1-\alpha)$. In this case the game has a mixed strategy Nash equilibrium in which the consumer chooses *Accept* with probability q^* (as in the original model) and the unprincipled expert chooses *Honest* with probability $(p^* - \alpha)/(1-\alpha)$.
   - If $\alpha = p^*$ then the consumer is indifferent between *Accept* and *Reject* if $p = 0$, so the consumer's best response function is vertical at 0. Thus the mixed strategy Nash equilibria are the strategy pairs $((p, 1-p), (q, 1-q))$ with $p = 0$ and $\pi/\pi' \le q \le 1$.

- If $\alpha > p^*$ then for all values of p (even $p = 0$) the consumer strictly prefers *Accept* to *Reject*, so that her best response to any p is $q = 1$. Thus in this case the game has a unique mixed strategy Nash equilibrium, in which $p = 0$ and $q = 1$. That is, all unprincipled experts choose *Dishonest* and every consumer chooses *Accept*.

3. (a) The game is given in Figure 1.

		Buyer 2: Seller 1	Buyer 2: Seller 2
Buyer 1	Seller 1	$\frac{1}{2}(1-p_1), \frac{1}{2}(1-p_1)$	$1-p_1, 1-p_2$
	Seller 2	$1-p_2, 1-p_1$	$\frac{1}{2}(1-p_2), \frac{1}{2}(1-p_2)$

Figure 1. The game in Problem 3.

(b) First consider the best response function of buyer 1. Denote by q the probability that buyer 2 chooses seller 1. Then buyer 1's expected payoff if she chooses seller 1 is

$$q \cdot \tfrac{1}{2}(1-p_1) + (1-q)(1-p_1)$$

and her expected payoff if she chooses seller 2 is

$$q(1-p_2) + (1-q) \cdot \tfrac{1}{2}(1-p_2).$$

Thus she prefers seller 1 if

$$\tfrac{1}{2}q(1-p_1) + (1-q)(1-p_1) > q(1-p_2) + \tfrac{1}{2}(1-q)(1-p_2)$$

or if

$$q(\tfrac{1}{2}p_1 + \tfrac{1}{2}p_2 - 1) > p_1 - \tfrac{1}{2}p_2 - \tfrac{1}{2}.$$

Given that $p_1 < 1$ and $p_2 < 1$ we have $\frac{1}{2}(p_1 + p_2) < 1$, so the inequality is equivalent to

$$q < \frac{1 - 2p_1 + p_2}{2 - p_1 - p_2}.$$

(Remember that if you divide both sides of an inequality by a negative number you need to reverse the inequality.) That is, buyer 1 prefers seller 1 if q is less than this cutoff. Hence if q is less than the cutoff, buyer 1's best response is $r = 1$, where r is the probability that buyer 1 chooses seller 1.

Similarly, if

$$q > \frac{1 - 2p_1 + p_2}{2 - p_1 - p_2}$$

then buyer 1 prefers seller 2 ($r = 0$) and if

$$q = \frac{1 - 2p_1 + p_2}{2 - p_1 - p_2}$$

then she is indifferent between the sellers.

Under the assumption that $2p_1 - 1 < p_2 < \frac{1}{2}(1 + p_1)$, the number on the right-hand sides of these conditions is between 0 and 1, so buyer 1's best response function is the blue function in Figure 2.

An identical calculation for buyer 2 (the game is symmetric) shows that her best response function is the red function in the figure.

We see that the game has three mixed strategy equilibria: two pure equilibria in which the buyers approach different sellers, and one mixed strategy equilibrium in which each buyer approaches seller 1 with probability $(1 - 2p_1 + p_2)/(2 - p_1 - p_2)$.

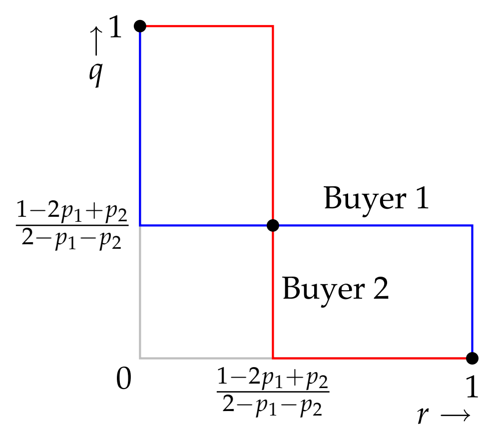

Figure 2. The players' best response functions in the game in Problem 3. The probability with which buyer 1 approaches seller 1 is r and the probability with which buyer 2 approaches seller 1 is q.

4. (a) The game has three Nash equilibria: $(Stop, Continue)$, $(Continue, Stop)$, and a mixed strategy equilibrium in which each player chooses *Stop* with probability

$$\frac{1-\varepsilon}{2-\varepsilon}.$$

Only the mixed strategy equilibrium is symmetric; the expected payoff of each player in this equilibrium is $2(1-\varepsilon)/(2-\varepsilon)$.

(b) The modified game also has a unique symmetric equilibrium. In this equilibrium each player chooses *Stop* with probability

$$\frac{1-\varepsilon+\delta}{2-\varepsilon}$$

if $\delta \leq 1$ and chooses *Stop* with probability 1 if $\delta \geq 1$. The expected payoff of each player in this equilibrium is $(2(1-\varepsilon)+\varepsilon\delta)/(2-\varepsilon)$ if $\delta \leq 1$ and 1 if $\delta \geq 1$, both of which are larger than her payoff in the original game (given $\delta > 0$).

After reeducation, each driver's payoffs to stopping stay the same, while those to continuing fall. Thus if the behavioral norm (the probability of stopping) were to remain the same, every driver would find it beneficial to stop. Equilibrium is restored only if enough drivers switch to *Stop*, raising everyone's expected payoff. (Each player's expected payoff in a mixed strategy Nash equilibrium is her expected payoff to choosing *Stop*, which is $p + (1-\varepsilon)(1-p)$, where p is the probability of a player's choosing *Stop*.)