Economics 316

Fall 2017 Martin J. Osborne

Solutions to problems for Tutorial 5

1. The payoffs are given in Figure 1. (The actions are the same as those in the game in which every expert is fully competent.)

	A	R
H	$\pi, -rE - (1-r)[sI + (1-s)E]$	$(1-r)s\pi, -rE' - (1-r)[sI + (1-s)I']$
D	$r\pi + (1-r)[s\pi' + (1-s)\pi], -E$	$0, -rE' - (1-r)I'$

Figure 1. A game between a consumer with a problem and a not-fully-competent expert.

Denote the probability that the consumer chooses A by q. Then the expert is indifferent between H and D if

$$q\pi + (1-q)(1-r)s\pi = q[r\pi + (1-r)(s\pi' + (1-s)\pi)]$$

or

$$q[(1-r-(1-r)s-(1-r)(1-s))\pi - (1-r)s\pi'] = -(1-r)s\pi$$

or

$$q(1-r)s\pi' = (1-r)s\pi$$

or

$$q = \frac{\pi}{\pi'}.$$

This number is always between 0 and 1.

Denote the probability that the expert chooses H by p. Then the consumer is indifferent between A and R if

$$\begin{aligned} &p[-rE - (1-r)(sI + (1-s)E)] + (1-p)[-E] \\ &\quad = p[-rE' - (1-r)(sI + (1-s)I')] + (1-p)[-rE' - (1-r)I'] \end{aligned}$$

or

$$p[-r(E-E') - (1-r)(1-s)(E-I')] = (1-p)(E - rE' - (1-r)I')$$

or

$$p[-(1-r)(1-s)(E-I') + (1-r)E - (1-r)I'] = E - [rE' + (1-r)I']$$

or

$$p = \frac{E - [rE' + (1-r)I']}{(1-r)s(E-I')}.$$

This number may be less than 1, equal to 1, or greater than 1. Consequently, three configurations are possible for the players' best response functions, as shown in Figure 2. The top panel of the figure shows a case in which s is large and the bottom panels show two cases in which s is small.

We see that when s is large the game has a unique mixed strategy Nash equilibrium, in which the probability the expert's strategy assigns to H is

$$p^* = \frac{E - [rE' + (1-r)I']}{(1-r)s(E-I')}$$

and the probability the consumer's strategy assigns to A is

$$q^* = \frac{\pi}{\pi'}.$$

The value of q^* is independent of s. That is, the degree of competence has no effect on consumer behavior: consumers do not become more, or less, wary. The fraction of experts who are honest is a decreasing function of s, so that greater incompetence (smaller s) leads to a *higher* fraction of honest experts: incompetence breeds honesty! The intuition is that when experts become less competent, the potential gain from ignoring their advice increases (since $I' < E$), so that they need to be more honest to attract business.

When s is small the game has a unique Nash equilibrium, which is pure. All experts are honest and all consumers are wary—they reject all advice to get a major repair. In this case, the experts are so incompetent the consumers fix all problems diagnosed as major themselves. The exact value of s in this range does not affect the nature of the equilibrium.

If s takes on the value given in the left panel of Figure 2, there is a continuum of equilibria. In all equilibria the expert is always honest; the probability that the consumer accepts her advice ranges from 0 to π/π'.

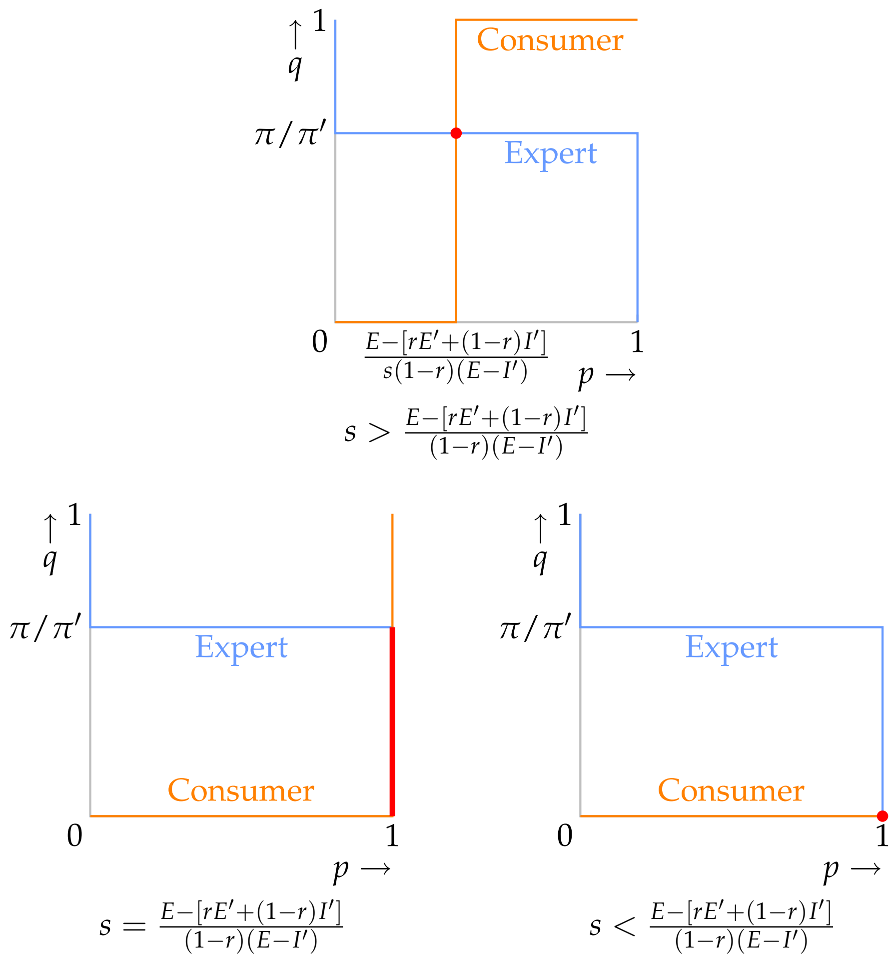

Figure 2. The players' best response functions in the game in Problem 1. The probability assigned by the expert to H is p and the probability assigned by the consumer to A is q.

2. Denote by p_i the probability with which each witness with cost c_i reports the crime, for $i = 1, 2$. For each witness with cost c_1 to report with positive probability less than one, we need

$$\begin{aligned} v - c_1 &= v \cdot \Pr\{\text{at least one other person calls}\} \\ &= v\left(1 - (1-p_1)^{n_1-1}(1-p_2)^{n_2}\right), \end{aligned}$$

or

$$c_1 = v(1-p_1)^{n_1-1}(1-p_2)^{n_2}. \tag{1}$$

Similarly, for each witness with cost c_2 to report with positive probability less than one, we need

$$\begin{aligned} v - c_2 &= v \cdot \Pr\{\text{at least one other person calls}\} \\ &= v\left(1 - (1-p_1)^{n_1}(1-p_2)^{n_2-1}\right), \end{aligned}$$

or

$$c_2 = v(1-p_1)^{n_1}(1-p_2)^{n_2-1}. \tag{2}$$

Dividing (1) by (2) we obtain

$$1 - p_2 = c_1(1-p_1)/c_2.$$

Substituting this expression for $1 - p_2$ into (1) we get

$$p_1 = 1 - \left(\frac{c_1}{v} \cdot \left(\frac{c_2}{c_1}\right)^{n_2}\right)^{1/(n-1)}.$$

Similarly,

$$p_2 = 1 - \left(\frac{c_2}{v} \cdot \left(\frac{c_1}{c_2}\right)^{n_1}\right)^{1/(n-1)}.$$

For these two numbers to be probabilities, we need each of them to be nonnegative and at most one, which requires

$$\left(\frac{c_2^{n_2}}{v}\right)^{1/(n_2-1)} < c_1 < \left(v c_2^{n_1-1}\right)^{1/n_1}.$$